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Bengal Engineers

Nov/29/61

A
TREATISE
ON
HYDRAULICS,
FOR THE USE OF
ENGINEERS,

BY

J. F. D'AUBUISSON DE VOISINS.

TRANSLATED FROM THE FRENCH,
AND ADAPTED TO THE ENGLISH UNITS OF MEASURE,
BY JOSEPH BENNETT,
CIVIL ENGINEER.

NEW EDITION.

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TRANSLATOR'S

DEDICATORY PREFACE.

To the Boston Society of Civil Engineers:

IN dedicating this translation to your Society, I take this occasion to express my thanks for the interest you have manifested in it, and for the friendly aid you have given me. I wish you to regard it as my quota towards the contributions of scientific or other matter, which it was our main object to elicit from each member. I trust it may be found worthy of your attention, and if I should succeed in imparting to others a portion of the delight and profit derived by me from the study of the original, I shall think I have labored to some purpose.

When I had made some progress in this translation, I received an interesting letter from FRANKLIN FORBES, Esq., accompanying a translation of the first half of the book. It was his intention to have finished the same for publication, but his professional engagements prevented him. For so free and generous a gift from an entire stranger, I would offer this expression of my sincere thanks.

Considerations connected with the cost of the work have changed one proposed feature of the translation; which was, to present before the reader, side by side, the metrical formulæ, with their reductions to the English units of measure, to enable him to judge for himself of the accuracy of these reductions, which are often of a complicated character. In case of any error, the means of its correction, on this plan, might be close at hand. My friends overruled this, and insisted upon the necessity of presenting the English measures solely.

Had I adopted this course from the first, I should have selected round numbers, and not the fractional numbers, as they are now necessarily given, as the equivalents of the original metrical quantities. Aware of the responsibility of presenting these English reductions, and of the absolute necessity that they should be *correctly* given, I have revised most carefully all the formulæ, as far as the subject matter of motors and their effects. In the Appendix will be found the method of their reduction, not as any thing new to mathematicians, but to save the unskilful the unnecessary labor, which I myself should have avoided, had I been as familiar with the process as at present.

A direct reduction of the formulæ, as given by D'Aubuisson, has not been made in all cases. When, for instance, in following implicitly his steps, with English measures, I have arrived at results not exactly equivalent to his, I have adhered to the results thus found, as a true fulfilment of the purposes of the author. In case, therefore, of an apparent discrepancy, the reader will do well to retrace the steps I have taken, before pronouncing them erroneous.

No particular pains were taken as to the construction of sentences, nor as to elegance of diction. The reductions being four fifths of the labor of the work, received my chief attention. For many improvements and corrections of the original manuscript, I am much indebted to my friends JAMES B. FRANCIS and E. S. CHESBROUGH; and to the interest which they, with some other members of your Society, have taken in this matter, I am indebted for its publication.

JOSEPH BENNETT.

AUTHOR'S PREFACE.

My purpose in composing this work has been to present to engineers, (that is, to all who have to propose or to execute great constructions,) the rules which should guide them in the plans they project for the conveyance of water, and for hydraulic works and machines. I wished, at the same time, to impart a full understanding of these rules, to fix the degree of confidence which they ought to inspire, and to show their application.

Hydraulics, as I proposed to treat it, being a science of facts, it was my duty to explain the facts and the circumstances proper to make them well understood. Guided, then, by simple reasonings, or by the principles of physics and elementary mechanics, I sought to deduce from them the rules which I have given. Many of them could be expressed by algebraic formulæ, and I have not failed to make use of that most exact and concise of languages. A single glance thrown on an algebraic result shows at once all the quantities relative to the question in hand, as well as the operations to be performed on them in order to arrive at the solution. Whenever the formulæ did not flow immediately from the facts observed, I have always been careful to compare their results with those of experiment. Both classes of results have been, as much as possible, placed in the form of tables before the eye of the reader, so that he might judge for himself as to the modifications which that comparison required in the formulæ, as well as of the degree of exactness which they promised in applying them to practice.

Examples, showing the manner of effecting such applications, serve as a commentary on the rules, and have, moreover, enabled me to make mention of the cases which most frequently occur in practice.

Engineers occupied exclusively in their department for a long series of years, may have lost the ready use of formulæ, and may find themselves under temporary embarrassment as to the acceptation to be given to some of the characters employed in them; one example, on a problem analogous to that which they desire to solve, will free them from the embarrassment.

It may further be said, on the use which I have often made of algebraic expressions, what has already been said on the occasion of another of my works, nearly of the nature of the present, that in using a language unknown to many persons employed in constructions, I make my work less generally useful. I have noticed some of the advantages of this language, and I do not believe that in sacrificing them I should gain rather than lose in respect to utility. I will also remark, that if any one would confine himself to what is strictly necessary, the use of this treatise only demands an ability to read a most elementary algebraic formula, and to perform by logarithms the operations which it indicates. But this knowledge is indispensable to the solution of questions in hydraulics: let it be required, for example, to fix the diameter of a series of pipes designed to convey a given volume of water; it would be necessary, among other operations, to extract the fifth root of the square of that volume; and such an extraction can scarcely be effected otherwise than by logarithms.

On the other hand, some will certainly reproach me for having too much neglected the use of analysis, and especially the infinitesimal analysis. But this book, a kind of manual of *hydraulics for the use of engineers*, is not a mathematical work, nor an application of mathematics, like the *Idraulica* of Venturoli. A very great number of the rules and precepts which it contains, for example, on the good arrangement to be given to a system of pipes, on the subject of sluice-gates, on bucket-wheels, &c., are foreign to that science; in this treatise, mathematics are only accessory; whenever they have offered me a means of arriving at my object without being confined to geometrical rigor, as I might otherwise have been, I have taken the most direct, most easy, and most beaten path. Hence it follows, that I have not used the *principle of the vis viva*—a principle so fruitful, and now become almost the only instrument which geometricians use in questions relating to hydraulics and to machines; I feared that it would not be sufficiently familiar to most of those for whom this book was designed; besides, the method which I have followed, preserving in the problems to be solved the immediate data of the observation, the height of the head in the flow of fluids, the amount of fall in machines, &c., appeared to lead me more directly to practical applications. Thus my work, by its nature, falls more properly in the province of the sciences of observation, of the physical sciences, than in that of the mathematical sciences; it is a treatise on experimental and applied hydraulics, and not on theoretical hydraulics.

I have no more to say respecting the plan which I have followed; the table of contents at the beginning of the volume sufficiently indicates it; and the short headings of the sections and chapters explain their character. I have distinguished, by means of a smaller type, the examples, the details of experiments, when it has been convenient to give them, some particular remarks, and some developments not found elsewhere; for example, on conduits and distributions of water.

Our metrical system of weights and measures offers too many advantages in calculations, and especially in the calculations of hydraulics, by the extreme facility with which the weights of water can be converted to volumes and reciprocally, to be neglected; thus I have adopted that system, with its decimal division, exclusively and in all its purity. Consequently, I have taken but one unit for measures, the metre; and one for weights, the kilogramme. In measures of length, I indicate the place of the unit figure by an ^{m}, placed as an exponent; the comma then being useless, I neglect it; thus, I write $17^{m}38$ and $0^{m}037$; I put two (^{mm}) in the measures of surface, and three in those of capacity: I write, for example, $8^{mm}42$ for 8,42 square metres, and $0^{mmm}0594$ for 0,0594 of a cubic metre. In being thus confined to a single unit, we must often employ a great number of zeros, it is true; but this method is infinitely the most suitable for comparisons; it spares the reader that confusion in which the mind is continually held, when sometimes the metre, sometimes the centimetre, and sometimes the millimetre is taken for unity.

The second will always be our unit of time.

Finally, I have preserved the division of the circle into 360°; a division which goes back to the remotest antiquity, and which is exclusively adopted by all nations. I feared to disturb this happy uniformity in the language of all times and of all countries.

Permit me here to claim indulgence for this work, most probably the last which I shall be able to write. The single desire of propagating scientific knowledge and its applications in France, so that our hydraulic works and machines might for the future be more fitly arranged, induced me to undertake it; some of those which I have already published have, perhaps, not been without some utility, and, addressing myself to the genius which inspired them, I said:

"*Extremum hunc, Arethusa, mihi concede laborem.*"

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ERRATA.

Page 4, line 33, *for* .00010 and .00013, *read* 0.0010 and 0.0013.

Page 19, " 13, the expression $380d\sqrt{h}$, in the original, is manifestly erroneous; and, consequently, its equivalent in feet, $209d\sqrt{h}$. Possibly, the coefficients should be 3.80 and 2.09.

Page 37, line 17, *for* 4.43, *read* .443.

" 38, " 23, *for* $216d^2\sqrt{H}$, *read* $2.16d^2\sqrt{H}$.

" 40, " 21, *for* 0.889, *read* .0889.

" 47, " 14, *for* quality, *read* quantity.

" 47, last line, *for* $.0885S\sqrt{2gH}$, *read* $0.885S\sqrt{2gH}$.

" 51, line 11, *for* one example, *read* our example.

" 55, table, 4th column, 5th line, *for* 6.5782, *read* 6.5882.

" 58, table, 1st column, 7th line, *for* 100°, *read* 180°.

" 75, line 14, *for* sensible (*so in original*), *read* insensible.

" 83, " 5, *for* above .8202 ft., or a quarter of, &c., *read* above 0.25 (or a quarter) of, &c.

" 87, line 3, *for* .664, *read* .663.

" 93, " 22, *for* $sV\tau$, *read* $mSV\tau$.

" 99, " 12, *for* in one second x, and the descent, &c., *read* in one second, and x the descent, &c.

" 114, line 3, after *ac*, *insert*—calling *ab* the entire force of gravity, or *g*.

" 119, line 25, *for* after, *read* according to.

" 121, " 5, *for* .022332, *read* .022449.

" 125, " 11, *for* berms, *read* banks.

" 131, " 12, *for* $a = 0.000024265$, *read* $a = 0.000111415$.

" " " 13, *for* $b = 0.000111415$, *read* $b = 0.217785$.

" 174, " 11, *for* Registrar of the States, *read* registers of the stages.

" 176, line 9, *for* Q =, *read* H =.

" 189, " 6, *for* *m* 0.85, *read* *m*, 0.85.

" 206, " 6, *for* supplied, *read* replaced.

" " " 25, *for* $.0001333\,\frac{Lv^2}{D}$, *read* $.00043738\,\frac{Lv^2}{D}$.

" 247, " 12, *for* $D = .2349\sqrt{\&c.}$, *read* $D = .2349\sqrt[5]{\&c.}$

" 252, " 15, *for* dimensions, *read* diminutions.

" 286, " 25, and in marginal note, line 3, *for* immersion, *read* emersion.

" 346, line 29, *for* 0.049 ft., *read* 0.490 ft.

" 350, " 17, *for* dynamic, *read* dynamometric.

" 417, " 29, *for* expense, *read* expenditure.

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TREATISE

ON

HYDRAULICS.

1. HYDRAULICS has for its object the knowledge of the phenomena presented by fluids in motion, and of the laws which nature follows in the production of those phenomena. It has principally in view the application of this knowledge to the means of directing, conveying and raising fluids, in the manner best suited to the end proposed.

2. Fluids are bodies whose particles, in consequence of an extreme mobility, yield to the slightest impression which they experience. Their independence, however, is not perfect; an adhesion binds them, to a certain extent, to each other.

3. These bodies are divided into two classes:—*incompressible fluids*, or fluids properly so called, to which philosophers sometimes give the name of *liquids;* and *compressible* or *elastic* fluids. Water is the type of the former, and atmospheric air of the latter.

4. Although all fluids, as well as all bodies in nature, are strictly compressible and elastic, yet some are so slightly so, in comparison with others, and the difference in this respect is so essential in the expression of the laws of their motions, that we have preserved this distinction.

PART FIRST.

HYDRAULICS, PROPERLY SO CALLED.

5. Water in motion presents itself in four different ways: as passing out of a reservoir; or flowing in a bed; acting as a motor; or in a passive state, raised by machines. Hence our four sections of hydraulics.

Before commencing them, let us fix the true value of two quantities, which are found in all calculations relating to this science—the weight of water and the intensity of gravity. These quantities are variable, but almost always supposed constant. What follows will enable us to judge of the error which may result from this supposition, in the different cases which will be treated of.

Weight of Water.

6. When water is entirely pure, and is taken at its *maximum* density, it weighs 62.4491 lbs. per cubic foot: such is its *specific weight*.

It may vary from three causes.

The most powerful is the temperature. We know that heat expands all bodies, and this diminishes their density or specific weight. From the most accurate experiments, the density of pure water, at different degrees of the Centigrade and Fahrenheit thermometers, would be as indicated in the following table:—

TEMPERATURE.		Weight of a Cubic Metre.	Weight of a Cubic foot in lbs.
Centigrade.	Fahrenheit.	kil.	
4	$39\frac{1}{5}$	1000.	62.449
6	$42\frac{4}{5}$	999.95	62.446
8	$46\frac{2}{5}$	999.87	62.441
10	50	999.72	61.432
12	$53\frac{3}{5}$	999.54	62.420
15	59	999.14	62.396
20	68	998.24	62.339
25	77	997.99	62.268
30	86	995.73	62.182
50	122	987.58	61.673
100	212	956.70	59.745

Below 4° Centigrade or 39° Fahrenheit, the density, instead of continuing to increase, diminishes; this diminution, at first very slow, rapidly progresses towards the limit of congelation, and the weight of a cubic foot of ice is only 58.078 lbs.

The effects of pressure are much less sensible. Water was, for a long time, considered wholly incompressible; but experiments, lately made, have shown that, under very heavy loads, it is really compressed, although but a very small quantity; about 0.000046 of its volume under the weight of one atmosphere; that is, under a pressure represented by the height of a column of mercury in a barometer, a height estimated at 29.922 inches, and which is equivalent to the height of a column of water about 33.793 feet; so that the specific weight of the lower part of a lake 328 feet deep would be 2206½ lbs., that of the upper part being 2205½ lbs.

But as, in common practice, we shall not have to calculate upon such depths or heights of water, we may, without sensible error, entirely neglect the effects of pressure.

What proceeds from saline or earthy substances contained in the waters which run on the surface of the globe, may also, in most cases, be omitted, the specific weight of the water of rivers being only one or two ten-thousandths greater than that of distilled water, which is taken as the standard of perfectly pure water.

Professor Boisgaraud found, by many trials, made with great care, 1000^{k}.149 for the specific gravity of the water of the Garonne, that of distilled water being 1000 kilogrammes to the metre, or 62.449 pounds to the cubic foot. Brisson has nearly an equal result for the Seine.

Moreover, a mass of water, when surrounded by air, loses, like all other bodies, a part of its weight equal to the weight of air whose place it occupies; and this loss, which is seldom below $\frac{10}{10000}=.00010$, may be even $\frac{18}{10000}=.00018$.

Finally, in our mean temperatures, and according to different circumstances, the weight of a cubic foot of water will be only from 62.35 lbs. to 62.39, or the cubic metre from 998^{k}4 to 999^{k}. We shall, however, in this treatise, constantly admit 1000^{k}, this value rendering the conversion of cubic metres of water into kilogrammes, and *vice versa*, extremely easy.

Numeric Expression of Gravity.

7. Experiments made with extreme care at the observatory of Paris, gave 0^{m}9934=39.128 inches, or 3.2606 feet, for the length of a pendulum vibrating seconds, this length being reduced to

the level of the sea. Whence we conclude, that in that place, a heavy body descends $4^{m}9044$ $(=\frac{1}{2}\times 0.99384\pi^{2})=16.091$ feet, during the first second of its fall. If, at the end of that time, gravity ceased to act upon it, it would continue to descend, but with a uniform motion, running through double the space, or 32.182 feet per second; this number, which expresses the velocity impressed by gravity in the unit of time, represents, for Paris, the intensity of that accelerating force; we generally designate that intensity or velocity by g, the initial letter of the word *gravity*. It augments, however, with the latitude, and diminishes with the elevation above the level of the sea, and generally we have

$$\begin{array}{r} \text{In feet} \\ \text{In metres} \end{array} \left\{ \begin{array}{l} g=32^{f}16954\,(1-0.00284\cos 2l)\left(1-\dfrac{2e}{r}\right), \\ g=9^{m}8051\,(1-0.00284\cos 2l)\left(1-\dfrac{2e}{r}\right), \end{array} \right.$$

l being the latitude of the place, e its elevation above the level of the sea, r the radius of the terrestrial spheroid at the level of the sea in that place:

$\{r=6366407^{m}(1+0.00164\cos 2l)\}=20887510^{f}(1+0.00164\cos 2l)$

Thus, at Toulouse, where $l=43^\circ\ 36'$ and $e=146^{m}=479^{f}$ we have $g=9^{m}8032=32.1633^{f}$; at Montlouis, where $l=42^\circ\ 30'$ and $e=1620^{m}=5315^{f}$ (the mean height of the barometer being $23^{p}\ 2\frac{1}{8}^{l}$ $=24.72$ inches) (*Journal des Mines*, tom. 23, p. 318), $g=9^{m}7977=32.1453^{f}$.

Notwithstanding these variations, I shall constantly take $g=9^{m}8808=32.1817$; but I shall remark, at the same time, and according to the examples we have just seen, that the results of calculations into which this quantity shall enter, may be in error, even for France, more than one-thousandth.

8. The value of g will very often appear under two forms, of which I will show the origin.

According to the first principle of the fall of heavy bodies, and of uniformly accelerated motion in general, the velocities acquired are as the times occupied in acquiring them; so that if v is the velocity acquired by a body at the end of the time t, g being, as we have just seen, the velocity acquired in 1″, we shall have

$$v : g :: t : 1,\ \text{or}\ v=gt.$$

According to the second principle, the spaces passed through, or the heights of the falls, are as the squares of the times occupied in passing through them; then if h is the height through which the same body has fallen in the time t, $\frac{1}{2}g$ being the fall corresponding to 1″, we shall have

$$h : \tfrac{1}{2}g :: t^2 : (1'')^2, \text{ or } h=\frac{gt^2}{2}.$$

Taking the value of t in this latter equation, and substituting it in the first, we have

$$v=\sqrt{2gh}, \text{ and consequently } h=\frac{v^2}{2g}.$$

Since $g=9^m8088=32.182^{ft}$

$$\sqrt{2g}=\sqrt{64.364}=8.0227$$

$$\text{and } \frac{1}{2g}=.015536.$$

Consequently, $v=8.0227\sqrt{h}$; and $h=.015536\,v^2$.

We call v *the velocity due to the height* h, *and* h *the height due to the velocity* v.

The Greek letter π, which we have taken above, as it expresses the ratio of the circumferance to the diameter (3.1416), it will have no other acceptation in this work. The fourth of that quantity, (.7854,) which is the ratio of the circle to the circumscribed square, presenting itself very frequently in our calculations, we shall designate by π'.

SECTION FIRST.

ON THE FLOWING OF WATER CONTAINED IN A RESERVOIR.

9. The reservoir from which water flows may be kept constantly full; or it may receive no additional water, and then empty itself; the opening through which it flows, instead of emitting the fluid into the atmosphere, may pour it into a second reservoir, more or less filled. These three cases give place to the division of this section into three chapters.

CHAPTER FIRST.

ON THE FLOWING, WHEN THE RESERVOIR IS CONSTANTLY FULL.

10. The opening through which the water flows is made in the bottom, or one of the sides of the reservoir. In the latter position, (and this is of most frequent occurrence,) the surface of the fluid in the basin may be kept above the upper edge of the opening, which is then surmounted, and, as it were, bounded by the fluid throughout its perimeter; in this case, it takes more particularly the name of *orifice*. This orifice is Definitions.

either simply made in a thin side, that is to say, in a side whose thickness is not half of the smallest dimension of the opening; or it is supplied with an *ajutage*, or short tube, sometimes *cylindrical*, more often *conical*, *converging* towards the exterior of the basin, and rarely *diverging;* an opening made in a very thick side would evidently be equivalent to an orifice in a thin side, with an ajutage. The surface of the fluid may also be below the upper edge of the opening; this edge is then as if it were not, and generally it does not exist; the opening is no longer limited on the upper part, and it takes the name of *weir*. The laws of flowing, in this second case, as they present peculiar circumstances, will be the object of a special article. The third case is intermediate between the two preceding, as when the fluid surface is kept at a very small elevation above the orifice. We shall precede the three articles, whose object we have just indicated, by an article, in which we shall expose the general principles of flowing, and the modifications which affect it from the *contraction* which the fluid vein experiences in passing through the different openings just mentioned.

The vertical distance or height of the fluid surface in the reservoir above the centre of gravity of the orifice, a distance sometimes elliptically designated by the simple phrase *height of the reservoir*, is the *head* of water on the orifice, or the head under which the flowing takes place.

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ARTICLE FIRST.

General principles of flowing and modifications due to contraction.

1. PRINCIPLES.

Theorem of Torricelli. Fig. 1.

11. Let X be a vessel kept constantly full of water up to AB. If on the horizontal faces CD and EF are made the orifices M and N, the fluid will pass out in the form of vertical jets, which will rise nearly to the level AK of the water in the reservoir; they would quite attain that level, if certain causes, to be investigated in the sequel, opposed no obstacle.

Poisson Mech. § 130.

Now, from the first principles of dynamics, in order that a body thrown vertically may attain a certain height, it is necessary that at its point of departure, it receive a velocity equal to that which it would have acquired by falling freely from the same height. Consequently, since the fluid particles which pass from the orifices M and N are raised to the respective heights M G and N H on passing out, they must have been impelled with velocities due to those heights, which are the heights of the surface of the reservoir above the orifices. In like manner, if on a vertical face FR an opening O be made, we shall hereafter see (36) that, according to the respective values of the lines OP and PQ, the fluid passes out at O with a velocity due to the height OK. It would pass out with a velocity due to KR, if the orifice were opened on the bottom RT of the vessel.

It will always be thus with these different orifices, whatever be their magnitude compared to the transverse section of the vessel, provided, however, that the fluid surface, preserving a constant level, remain even and *tranquil;* a condition which could not be fulfilled, if the size were very large, the water flowing out producing violent commotion in the vessel.

Generally, and making abstraction of every obstacle or all cause of perturbation, *the velocity of a fluid, at its passage through an orifice made in the side of a reservoir, is the same as a heavy body would acquire in falling freely from the height comprised between the level of the fluid surface in the reservoir and the centre of that orifice.*

This theorem, known under the name of Toricelli's theorem, was established and published by that celebrated philosopher in 1643, as a consequence of the laws of the fall of heavy bodies; laws which had just been discovered by his master, the illustrious Galileo.

If we designate by v the velocity of issue, and by H the height or head of water in the reservoir, it will give (8)

$$v=\sqrt{2gH}.$$

General Principles.

12. We have just seen that water passing from the openings M and N did not quite attain the level of the fluid in the reservoir. If to these openings we adapted two perfectly equal tubes, the water would rise still less high; but the diminution of height would follow exactly the same ratio. For example: if the jet which issues from the tube at M were only two thirds of MG, that which would pass from the tube at N would be only two thirds of NH. In general, let n be the ratio between the height of the jet and that of the reservoir for a tube of a certain form, H and H′ two heights of the reservoir, and v and v' the corresponding velocities, we shall have

$v=\sqrt{2gnH}$ and $v'=\sqrt{2gnH'}$; whence

$$v : v' :: \sqrt{H} : \sqrt{H'};$$

that is to say, the openings being of the same form, *the velocities are always as the square roots of the heads.*

Experiments made by Mariotte, 150 years ago, and repeated a hundred times since, leave no doubt as to this principle. I will here give the results of some of them; this will fix the degree of confidence with which the principle may be received; other details from the series of experiments which furnished these will be given at No. 25. The first series was made by M. Castel and myself; the second, by Bossut; the third and fourth, by Michelotti, and the last, by MM. Poncelet and Lesbros.

It will be remarked, that the heads were varied in the ratio of 1 to 200 and more, and the sections of the orifices from 1 to 500; and yet, in all, the velocities followed the ratio of the square roots of the heads; the small differences which are seen, sometimes in excess, sometimes deficient, may be neglected;—small errors are inevitable in such experiments. Their direct object was the determination of the discharges; but it is evident that when the orifice is the same, the discharge varies only with the velocity, that it is exactly proportional to it, and that the series of ratios of one is also the series of ratios of the other.

Diameter of Orifice.	Head of Orifice.	SERIES OF	
		Sq. roots of Heads.	Discharges or velocities.
Inches.	Inches.		
0.3937	1.024	1.000	1.000
	1.181	1.074	1.064
	1.575	1.241	1.244
	1.969	1.386	1.393
	2.362	1.519	1.524
	ft.		
1.063	4.265	1.000	1.000
	9.580	1.500	1.497
	12.500	1.713	1.707
3.189	7.677	1.000	1.000
	12.500	1.305	1.301
	22.179	1.738	1.692
6.378	6.923	1.000	1.000
	12.008	1.316	1.315
squares.	1.312	1.000	1.000
—	2.297	1.323	1.330
7⅞ in.	2.281	1.581	1.590
by	4.265	1.803	1.806
7.⅞ in.	5.249	2.000	2.000

Answers for fluids of all kinds.

13. The general principle that the velocities are as the square roots of the heads, as well as the theorem of Toricelli for cases where it is applicable, extends to fluids of all kinds; to mercury, oil, and even aeriform fluids. So that the velocity with which each of them passes an orifice, is independent of its nature and of its density; it depends only on the head; experience proves it.

Simple reasoning, also, can show that it must be so. Take mercury, for example; the particles placed before the orifice, and on which it is necessary to impress a certain velocity, are, it is true, fourteen times more dense than those of water, and therefore they oppose fourteen times as much resistance to motion; but as the mass which presses and which produces the velocity of passing out, (being fourteen times greater,) exerts a motive effort fourteen times greater, there is a compensation, and the impressed velocity remains the same.

Case of foreign pressure.

14. To the pressure which a fluid contained in a vessel exerts by its weight on the orifice of exit, may be added a foreign pressure, and the velocity of flowing is augmented. What will be its increase and its definite value?

Let P be the weight of body which produces the pressure, and s the fluid surface or portion of the fluid surface on which it immediately acts, namely, that which is in contact with it; h the elevation of that surface above the orifice, and p the weight of a cubic foot of the fluid contained in the vessel. For the given body substitute, in imagination, a column of that fluid, which would have s for its base, and whose height h' would be such that the weight of the column would be equal to that of the body; we should thus have $P = psh'$ from which to deduce h'; substituting thus one body for another of equal weight, we should not change the pressure experienced by the particles contained in the vessel. Suppose, further, that after having withdrawn the body, we add in the vessel (whose sides we may suppose to be prolonged to an indefinite height) a quantity of the same fluid as that already contained, until its level has attained the summit of the column;

according to the laws of hydrostatics, all the mass of the fluid added would only produce a pressure equivalent to that of a single column; so that the particles situated before the orifice would experience a pressure exactly equal to what they first experienced, and will always tend to pass out with the same velocity. Now, in the new state of things, the height of the reservoir above the orifice, the height generating the velocity of exit, is evidently $h'+h$, and consequently this velocity will be

$$\sqrt{2g\,(h+h')}$$

Take, for example, a vessel closed on all sides and filled with alcohol, whose specific gravity is 0.837; on the cover is a circular opening of 1¼ inch diameter, in which is a piston loaded with $18^{\text{ozs.}}$; the orifice of exit is 10 inches beneath that opening. To determine the velocity with which the alcohol will run out. We admit that the friction of the piston on the edges of the opening is balanced by the weight of the piston itself.

We then have $P=18^{\text{ozs.}}=1.125^{\text{lbs.}}$; $s=.7854\times(1.25)^2=1.227^{\text{sq.in.}}=.0085^{\text{sq.ft.}}$; $p=.837\times62.429=52.271^{\text{lbs.}}$ and $h=10^{\text{in.}}=.833^{\text{ft.}}$: for h', the equation $P=psh'$ or $1.125=52.271\times.0085\,h'$, gives $2.5329^{\text{ft.}}$. Thus the alcohol will issue with a velocity of $\sqrt{2g(2.5329+.833)}=\sqrt{64.364\times3.3659}=14.718^{\text{ft.}}$.

If the vessel were not kept constantly full, this velocity would gradually diminish, and in such a manner as we shall see in the following chapter.

Theoretic Discharge.

15. After having given the expression of the velocity with which any fluid issues from an orifice, we pass to the use made of it in determining the discharge.

We call the *discharge* of an orifice, the volume of fluid which runs out of it in the unit of time, the second.

If the mean velocity of all the fluid particles were that due to the whole head, H, this velocity, which is then called *theoretic velocity*, would be $\sqrt{2gH}$; if, at the same time, the particles passed out from all points of the orifice, and in parallel lines, it is evident that the volume of water running out in one second would be equal to the volume of a prism which had the orifice for a base, and that velocity for its height; it would be, calling S the area or section of the orifice,

$$S\sqrt{2gH}.$$

This is the *theoretic discharge.*

Real Discharge.

16. But the *actual discharge* is always less.

To give an accurate idea of the state of things, let us consider the fluid vein a little after its passage from the orifice, and let us cut it by a plane perpendicular to its direction. It is manifest that the discharge will be equivalent to the product of the section by the mean velocity of the lines, at the instant of their crossing the section: if this section were equal to that of the orifice, and if this velocity be equal to that due to the head, the actual discharge would be equal to the theoretic discharge. But it happens, either that the section of the vein is sensibly smaller than that of the orifice, as in flowing through orifices in a thin side; or that the velocity at the section is sensibly less than that due to the head, as in cylindrical tubes; or even that there is a diminution both in the section and in the velocity, as in certain conical tubes. So that the actual discharge will, in all these different cases, be less than the theoretic; and in order to reduce the theoretic to the actual, it must be multiplied by a fraction. If m represent that fraction, and Q the actual discharge, we shall have

$$Q = m\,S\sqrt{2gH}.$$

Designating by Q′ the volume of water flowing in any time T, we should also have

$$Q'=m\ ST\ \sqrt{2gH}.$$

Whether the diminution in the discharge proceed from a diminution in the section of the vein, or from a diminution in the velocity, it is always a consequence of the contraction which the vein experiences on passing through the orifice; thus the multiplier m, *or coefficient of reduction of the theoretic discharge to the actual discharge*, is commonly called *the coefficient of the contraction of the fluid vein*, or simply, *coefficient of contraction.* Its determination is one of very great importance: on its accuracy depends that of the results obtained when the formula for the flow of fluids is applied to practice; it has also been the great object of the experimental researches of hydraulicians. We will make known the results to which they have arrived, after making some preliminary observations.

2. ON CONTRACTION AND ITS EFFECTS.

Cause of Contraction.

17. Take a transparent vessel, let water flow through an orifice in its side, and make the motion of the particles of the fluid visible by mixing with them small substances of a specific gravity about equal to that of the water, such as saw-dust of certain kinds of wood; or, better still, by introducing light chemical precipitates, such, for example, as take place when drops of the solution of nitrate of silver are poured into water slightly salted; at a small distance from the orifice, say from 1 inch to 1¼ inch for an orifice of ½ inch diameter, the fluid particles directed from all parts towards the orifice are seen to describe curved lines, and to termi-

Figs. 2 and 3.

nate by passing towards the orifice with a very accelerated motion, as towards a centre of attraction.

The convergence of the directions which they take in the interior of the vessel, on the instant of their arrival at the orifice, still continues for a little distance after they have passed through it; so that the fluid vein, at its passage from the orifice, is gradually contracted up to a point where its particles, by the effect of their reciprocal action, and of the motions impressed upon them, take a parallel direction, or other directions. The vein thus forms a kind of truncated pyramid or cone, whose greater base is the orifice, and whose smaller is the fluid section at the point of greatest contraction—a section which is often called *the section of the contracted vein.* This figure, and all the phenomena of contraction, are thus a consequence of the convergence of the lines, when they arrive at the orifice, or of the obliquity of the direction of some in respect to others.

Nature of its effects.

18. When the orifice is in a thin side, the contraction takes place below the plane of that orifice; it is *exterior;* it is seen; its dimensions can be measured, and they have actually been measured. We shall soon tell what has been done in this respect; we shall here simply remark, that in circular orifices, beyond the section of the greatest contraction and up to a certain distance, the vein continues in the form of a cylinder, of which that section would be the base, and with a velocity nearly that due to the height of the reservoir. The discharge, then, will be the product of that section by that velocity; so that the contraction will be limited to reducing the section which is to enter into the expression of the discharge. The flowing takes place as if, for the real orifice, another had been substituted,

of a diameter equal to that of the contracted section, and as if there had been no contraction.

19. If to the orifice AB, a cylindrical tube ABCD be fitted, the fluid lines will arrive at AB converging, and consequently the fluid will be contracted at the entrance of the tube. Experiments, to be given hereafter (44), will indicate that the contraction there is equal to that which takes place in orifices with thin sides; it would be only *interior* in relation to the mouth of the outlet. Moreover, beyond the contracted section, the attraction of the sides of the tube occasions a dilation of the vein; the threads are carried against the sides, they follow the sides, and pass out parallel to each other and to the axis of the tube; so that the section of the vein at its exit is quite equal to that of the orifice, but the velocity is not that due to the head of the reservoir. If the flowing were produced only by the simple pressure of the fluid contained in the reservoir, probably the velocity, at the section of greatest contraction, would be that due to the head; then it would diminish in proportion as the vein dilates, in virtue of the law or axiom of hydraulics, *when an incompressible fluid in motion forms a continuous mass, the velocity, at its different sections, is in the inverse ratio of the area of the section;* the diminution would cease when, the vein having attained the sides, its section would become equal to that of the orifice. Since m is the ratio of the section of greatest contraction to that of the orifice, the velocity along the sides, and consequently at the exit, would be $m\sqrt{2gH}$; and for the discharge, we should have $S \times m\sqrt{2gH}$.

In orifices in a thin side, it was $mS \times \sqrt{2gH}$; thus the discharge would be the same in both cases; the only difference is, that in the latter, the diminution would have affected the factor S, and in the tubes, it would have fallen on the factor $\sqrt{2gH}$; that is to say, on the velocity. But the attractive action of the sides changes this state of things; not only does it cause the lines to deviate from their direction, but it also increases their velocity; so that the velocity of exit is greater than $m\sqrt{2gH}$; it will be $m'\sqrt{2gH}$, m' being a fraction greater than m; and the discharge will become $S \times m' \sqrt{2gH}$.

We see by this, that in cylindrical tubes and in ajutages generally, the effect of contraction is involved in that of the attraction of the sides. Without being able to assign what belongs to the first alone, we will remark, that for every interior contraction, there is a corresponding diminution of velocity, and every exterior contraction produces a diminution of section.

Form of the vein, the orifice being circular.

20. Let us examine the form which contraction gives to the fluid vein passing from an orifice. Take first the most simple case, that of a circular orifice in a thin and plane side.

The direction as well as the velocity of the particles at the different points of the orifice being symmetrical, the contracted vein must also have a symmetrical form, and consequently be a solid of revolution, a conoid. It is so in fact, and observations about to be reported, give it the form represented by A B b a (Fig. 4). Beyond a b, the contraction ceases, and the vein continues under a form sensibly cylindrical for a certain length, and until it becomes entirely deformed, from the resistance of the air and other causes.

In the first part of that length, it is full, clear, sometimes like a bar of the most beautiful crystal; then it becomes disturbed, and, examined in a strong light, it presents a series of swellings and contractions. From the very ingenious experiments of M. Savart, the appearance of continuity of the disturbed part is only an optical illusion, arising from the rapidity of the motions; this part consists of a series of distinct drops, alternately large and small, leaving between each other a space eight or ten times greater than their mean diameter, the form of which, oscillating round that of a sphere, is alternately an elongated and an oblate spheroid.

The same philosopher observed, that the length of the clear part, as well as that of the swellings in the disturbed part, increased proportionally to the diameter of the orifice and the head; for the clear part, it was nearly 380 $d\sqrt{h}$ in metres, or 209 $d\sqrt{h}$ in feet. The formation of drops, that is to say, their detachment from the clear part, is not, even in descending jets, an effect of the acceleration of velocity due to gravity; for it takes place equally in jets thrown upwards. It appeared to Savart to be an immediate effect of the oscillation, which occurred in the fluid of the reservoir, in consequence of which, the particles of the jet, being sometimes more and sometimes less pressed at their exit from the orifice, moved with a velocity alternately greater and less. I have discovered such alternations in most of the motions of fluids which I have been enabled to observe; I have seen them also, in a very marked manner, during my experiments upon the resistance which the air experiences in conduit pipes; I have seen the air advance irregularly and as by undulations; the waves, as they spread, would accelerate and retard the velocity periodically.*

M. Savart also showed the very singular influence of the waves of sound on the liquid veins; for example, if the disturbed part be received on the bottom of a vessel, there is heard a sound due to the impulse of successive drops; if then a note be produced on a violin in unison with this sound, the clear part of the jet is immediately seen to become shortened, and sometimes even to disappear entirely; the swellings of the troubled part become bigger and shorter, and the space which separates them is greater.

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* Annals des Mines, tom. III., p. 401. 1828.

I refer to the paper of the author (*) for other effects of sonorous undulations on fluid veins; I confine myself here to remarking, that these undulations have no influence on the discharge.

Dimensions of the contracted vein.

21. To return to the commencement of the jet, to the contracted vein properly so called, the conoid AB b a. Attempts have been made to determine its respective dimensions, and particularly the ratio between the diameters of the two bases, by direct measurements. Newton, who discovered the phenomenon of contraction and its effects on the discharge, and first attempted such an admeasurement; he concluded that the ratio of the section of the orifice to the contracted section was that of $\sqrt{2}$ to 1; and consequently, that of the diameter was as 1 to 0.841; but we believe that theoretical considerations, rather than a physical measurement, led him to adopt that result. Since then, several philosophers have made like measurements; thus AB being 1, Poleny found for a b 0.79; Borda, 0.804; Michelotti, 0.792; Bossut, from .812 to .817; Eytelwein, .80; Venturi, .798; finally, Brunaci, .78. Nearly all these numbers, whose mean term is .80, are very probably a little too large; they were found by measurements taken with callipers; if closed too much, the points were thrust into the body of the stream and the disturbance indicated it; but if too much open, the eye could not exactly appreciate how much it was so; hence an error in excess might be made, but not one in deficiency.

Michelotti the younger, took up this question, which had already been treated by his father. Large jets obtained under great heads, gave him the following results:—

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* De la constitution des veines liquides lancés par des orifices circulaires en mince parol, par M. Felix Savart. 1833.

Head above the orifice, in feet.	Diameter in Inches.		Ratio between Diameters.	Distance from orifice to contraction, in incs.	Ratio of the distance to the contracted diameter.
	At the orifice.	At the contraction.			
6.890	6.394	5.047	0.790	2.520	0.501
12.008	6.394	5.039	0,788	2.520	0.500
7.349	3.197	2.511	0.786	1.260	0.500
12.502	3.197	2.504	0.783	1.210	0.492
22.179	3.197	2.413	0.755	1.181	0.497

Abstracting the last number 0.755, which is entirely anomalous, the mean ratio between the two diameters is 0.787. From what has been said, I think it may be adopted, but only as a mean term; for, as we shall soon see, (26,) this ratio experiences variations, slight, to be sure, which depend upon the heads and the diameters of the orifices. The length of the contracted vein should be about half the diameter of the smallest section, or 0.39 of the diameter of the orifice. According to these experiments, the three principal dimensions, AB, a b and CD, of the contracted vein, would be respectively as the numbers 100, 79 and 39.

Eytelwein, chiefly increasing the last dimension, one very difficult to determine with accuracy, takes the numbers 10, 8 and 5; this ratio is quite generally admitted. As to the curves Aa and Bb, Michelotti refers them to a cycloid. In conclusion, the form of the fluid vein, at its passage from a circular orifice, has some resemblance to the bell-shaped end of a hunting horn.

Effect of the form upon the Discharge.

22. The ratio between the diameters being 0.787, that between the sections will be the square of 0.787, or 0.619; thus, if s is the section of the contracted vein and S that of the orifice, we shall have $s=0.619$ S. From the explanations made, (16 and 18,) the discharge will be $s\sqrt{2gH}$, or 0.619 S $\sqrt{2gH}$. So that m, or the coefficient of contraction given by physical

measurements of the vein, will be at a mean 0.619; and the measurements of the discharge indicate nearly the same (25).

If the velocity due to the head of the reservoir were really the *velocity at the passage* of the contracted section, and the flowing were produced through a tube which had exactly the form of the contracted vein, by introducing into the expression of the discharge, the exterior orifice of that tube or s, the calculated discharge would be equal to the real discharge, and the coefficient for reducing one to the other would be 1. Michelotti, in one of his experiments, by employing a cycloidal tube, found it 0.984; it is probable that it would have come up to 1, if the sides of the tube had been more exactly bent to the curvature of the fluid vein; and if the resistance of the sides, as well as that of the air, had not slightly retarded the motion.

Form, with Polygonal Orifices. 23. Orifices, whose perimeter is a polygon, or any figure other than a circle, do not present a form so simple, or leading to the same consequences.

The different parts of the orifices not being symmetrical, the fluid vein does not preserve the form which it had on coming out, and it changes from it continually as it removes from it. At its exit, the faces corresponding to the rectilinear sides of the orifice become more and more concave; the edges corresponding to the angles become truncated and terminate by disappearing. Fig. 5th. Thus Poncelet and Lesbros, having drawn, by aid of very exact means, the form of a vein which passed from a square orifice ACEG, whose sides were 7⅞ inches under a head of 5½ feet, had, at the distance of 5.9 inches from the orifice, the section $a\ c\ e\ g$; and at 11.81 inches, the section $b'\ d'\ f'\ h'$.*

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* Expériences hydrauliques sur les lois de l'écoulemont des eaux à traverse les orifices rectangulaires verticeaux et à grandes dimensions, par M. M. Poncelet et Lesbros, Capitaines du génie—1832. Pag. 120 et suivantes.

This last, one of the nine sections observed, was the smallest; its area was to that of the orifice in the ratio of 0.562 to 1, whilst that of the actual discharge to the theoretic discharge was found to be 0.605; they would have been equal, if the velocity of that smallest section had been due to the head of the reservoir.

Reversing of the vein.

24. Although the fluid particles at $b' \, c' \, d'$, &c., on this section, are those which came out at the points BCD, &c., of the orifice, and in removing from the reservoir have always remained on the line of intersection of the vein with the planes passing through its axis and those points respectively, it is nevertheless true, that the section $b' \, d' \, f' \, h'$ is a kind of square, the vertex of whose angles corresponds to the middle of the sides of the square of the orifice; and that the vein appears to have made an eighth of a revolution around its axis.

A phenomenon of this nature is produced on all the veins which come out of an orifice not circular; it is called the *reversing of the vein.* It is accompanied by very remarkable circumstances, which I will state in referring to the results of one of the numerous experiments of Bidone on this subject.*

The orifice was a regular pentagon A of 0.551 inches each side, made in a thin vertical plate of copper; (the figure representing it, with its accessories, is one quarter of the natural size); the flowing took place under a head of 6.463 feet. At the distance of 0.472 inches, the section perpendicular to the axis of the vein was a quite regular decagon. At 1.181 inches was the greatest contraction or *first knot.* Beyond, the vein entirely changed its form; it presented five fluid plates, disposed symmetrically around the axis, as is seen in the section B, made 3.74 inches from the orifice; the planes of the blades passed through the centres of the sides of the orifice. Their breadth continued to increase up

Fig. 6.

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* Expériences sur la forme et la direction des veines et courants d'eau lancés par diverses overtures, de George Bidone. Turin, 1829.

to the belly of the vein represented at C. Then it diminished, and the blades united anew in a *second knot*, at 2 feet 10 inches from the orifice. Beyond, the vein was twisted and irregular.

For the rectilinear pentagon of the orifice, were successively substituted pentagons with convex and concave sides, sides presenting salient and re-entering angles like the star D, and the vein always preserves the same form, the same five blades.

With orifices of 6 and 8 sides, we had 6 and 8 blades; and the reversing of the vein was a 12th and 16th of the circumference. When the opening was a rectangle, narrow and very long in the horizontal direction, at a certain distance, the vein consisted only of a broad vertical blade; the reversing seemed complete.

Often, beyond the second knot, the vein dilates again and divides a second time into the same number of blades; but their plane does not correspond to the middle of the sides of the orifice, but to the vertex of the angles; that is to say, the vein is again turned an equal quantity; or rather it returns to its place. The blades increase in breadth up to the second belly and diminish again to form a third knot, beyond which sometimes there is still a new dilation, a third belly and a fourth knot. Eytelwein produced similar series of knots and swells with orifices of different forms; he represented them in his German translation of Sperimenti idraulici of Michelotti, p. 19 et pl. iv.— 1808.

There are also hollow veins, &c.; but the examination of all these forms, as well as of the causes which may produce them, do not come in the province of this treatise; and I refer to the very interesting paper of Bidone for these particulars. I limit myself to the following observations. The first and principal cause of the forms and reversing of the veins is the oblique direction with which the different fluid lines arrive at the orifice of exit, a direction which has a tendency to continue beyond. The action of these lines on the form is stronger and more influential, the more acute the angles from which they issue; those from the acute angles compress the vein in some sort more strongly than the rest, and consequently, the blades are formed on the parts intermediate to those where they exert their action. Then the resistance of the air and the mutual attraction of the particles contribute to shrink up the blades and to the formation of the second knot.

The obliquity of the fluid lines, in respect to each other, on their arrival at and passage through the orifice, also produced an effect which I ought to mention. As long as the obliquity is equal on all parts, the axis of the vein, which is in the direction of the resultant of the reciprocal action of the filets, remains perpendicular to the plane of the orifice; but if the obliquity is destroyed on one of the sides, for example, by the aid of a board tangent to the side, and which passes into the interior of the reservoir, perpendicular to the plane, the oblique impulse of the lines which arrive on the other sides, not being counterbalanced on that side, will carry the vein over, and its axis will no longer be that of the orifice.

ARTICLE SECOND.

On flowing through Orifices.

We have distinguished four kinds of orifices; those in a thin side, cylindrical tubes, conical converging and conical diverging tubes. Let us examine the principal circumstances of the motion through each of them, particularly in what concerns their discharge.

1. ORIFICES IN A THIN PARTITION.

Determination of the coefficient of contraction.

25. We come to the direct determination of the coefficient of reduction, from the theoretic to the actual discharge.

We will measure with care the volume of water passing from a given orifice, under a constant head, and during a certain time; and we shall derive from it the product of the flow in one second or the actual discharge; we will divide it by the theoretic discharge corresponding to that orifice and to that head, and the quotient will be the coefficient sought.

Many hydraulicians have applied themselves to this investigation; I give, in the following table, the principal results obtained up to the present time; those which appear to have been made under the most favorable circumstances or which were generally admitted.

CIRCULAR ORIFICES.

Observers.	Diameter, in incs.	Head, in ft.	Coefficient.
Mariotti,	0.268	5.873	0.692
Do.	0.268	25.920	0.692
Castel,	0.394	2.133	0.673
Do.	0.394	1.017	0.654
Do.	0.590	0.453	0.632
Do.	0.590	0.984	0.617
Eytelwein,	1.027	2.372	0.618
Bossut,	1.067	4.265	0.619
Michelotti,	1.067	7.317	0.618
Castel,	1.181	0.223	0.629
Venturi,	1.614	2.887	0.622
Bossut,	2.126	12.500	0.618
Michelotti,	2.126	7.218	0.607
Do.	3.189	7.349	0.613
Do.	3.189	12.500	0.612
Do.	3.189	22.179	0.597 ?
Do.	6.378	6.923	0.619
Do.	6.378	12.008	0.619

SQUARE ORIFICES.

Observers.	Side of square in incs.	Head, in ft.	Coefficient.
Castel,	0.394	0.164	0.655
Bossut,	1.063	12.500	0.616
Michelotti,	1.063	12.500	0.607
Do.	1.063	22.409	0.606
Bossut,	2.126	12.500	0.618
Michelotti,	2.126	7.349	0.603
Do.	2.126	12.566	0.603
Do.	2.126	22.245	0.602
Do.	3.228	7.415	0.616
Do.	3.189	12.566	0.619
Do.	3.189	22.376	0.616

RECTANGULAR ORIFICES (Bidone).

Rectangle.		Head, in incs.	Coefficient.
Height in inches.	Base in incs.		
0.362	0.728	13	0.620
0.362	1.457	13	0.620
0.362	2.909	13	0.621
0.362	5.818	13	0.626

The most remarkable of all these experiments, as well for the great size of the jets as for the greatness of the head, are those which Michelotti executed in 1764, at the fine hydraulic establishment constructed for that purpose at about two miles from Turin; the reservoir consisted of a tower twenty-six feet three inches high, whose interior, which is a square of 3.182 feet per side, receives through a canal the waters of the Doire. On one of the faces were fitted, at the different heights, the orifices or tubes which were thought proper; arrangements were made to receive them, and on the ground, which is at the base, were several measuring basins.* These experiments were repeated in 1784 by Michelotti the younger, and they are the last introduced into the table. I shall remark, on this subject, that the coefficients obtained with the great orifices were larger than the rest, and that, contrary to the rule deduced from the observations collectively; some peculiar circumstances must have produced this anomaly. The results given by Bossut are generally greater than those of Michelotti, and seem to be erroneous by excess.

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* Sperimenti idraulici, etc., de F. D. Michelotti. Turino, 1767 et 1771.

As to the experiments which M. Castel and myself made at Toulouse, notwithstanding all our pains bestowed upon them, the smallness of the orifices does not permit us to vouch for the determined coefficients to within hundredths. We were principally engaged with the orifice of $0^m01 = 0.394$ inch, as being, in some respects, the point of departure in the distribution of water made according to the metrical system of weights and measures.

Experiments of MM. Poncelet and Lesbros.

26. The experiments just reported and those made by other authors, by M. Hachette in particular, have shown that the coefficient of contraction is generally greater for small orifices and small heads; but they furnished only vague and almost contradictory notions in this respect. It would have been impossible to deduce from them the series of coefficients from great orifices to the smallest and from great heads to the smallest; this deficiency has recently been supplied by MM. Poncelet and Lesbros. They made, in 1826 and 1827, at Metz, a series of experiments on a *very* great scale, and with care and means which had not before been employed.

They appear to me to have nearly solved the great and useful problem of the contraction of the vein in a thin partition, perhaps as nearly as the nature of the subject admits; and in a manner, if not entirely theoretical, at least, very suitable to applications.*

In these experiments, the orifices were rectangular, and all of $0^m2 = 7.874$ inches base; the heights were successively 7.874

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* Expériences hydrauliques, etc.

inches, 3.937 inches, 1.968 inches, 1.18 inches, 0.787 inch, 0.394 inch; the heads varied from 0.394 inch to 5.577 feet. For each of these orifices, the discharge was measured, with several repetitions, under seven or ten heads, of which the two extremes were taken, the one nearly as small and the other as large as the apparatus allowed; and the corresponding coefficients were calculated.

Taking, then, the heads for abscissas and their coefficients for ordinates, the curve relating to that orifice was traced; and by its aid, they determined the ordinates or coefficients intermediate to those directly given by experiment. In this manner, the authors were enabled to arrange a large table of coefficients for each orifice, from which I extract the following:

HEAD on centre of orifice.	HEIGHT OF ORIFICES (base of each 7.874 inches).					
	$7.874^{\text{in.}}$	$3.937^{\text{in.}}$	$1.968^{\text{in.}}$	$1.181^{\text{in.}}$	$.787^{\text{in.}}$	$.394^{\text{in.}}$
Inches.						
.394						0.709
.787					0.660	0.698
1.181				0.638	0.660	0.691
1.575			0.612	0.640	0.659	0.685
1.968			0.617	0.640	0.659	0.682
2.362		0.590	0.622	0.640	0.658	0.678
3.150		0.600	0.626	0.639	0.657	0.671
3.937		0.605	0.628	0.638	0.655	0.667
4.725	0.572	0.609	0.630	0.637	0.654	0.664
5.906	0.585	0.611	0.631	0.635	0.653	0.660
7.874	0.592	0.613	0.634	0.634	0.650	0.655
11.811	0.598	0.616	0.632	0.632	0.645	0.650
15.748	0.600	0.617	0.631	0.631	0.642	0.647
Feet.						
1.640	0.602	0.617	0.631	0.630	0.640	0.643
2.297	0.604	0.616	0.629	0.629	0.637	0.638
3.281	0.605	0.615	0.627	0.627	0.632	0.627
4.265	0.604	0.613	0.623	0.623	0.625	0.621
5.250	0.602	0.611	0.619	0.619	0.618	0.616
6.582	0.601	0.607	0.613	0.613	0.613	0.613
9.843	0.601	0.603	0.606	0.607	0.608	0.609

All the numbers in this table are the respective values of *m* in the formula $Q = mS\sqrt{2gH}$. But those which in each column are found above the transverse line, are not the true coefficients of reduction from the theoretic to the actual discharge, as we shall see in a following article. (64)

Glancing over the numbers of each column, we see that they increase as the head increases, but only up to a certain point, beyond which they diminish, although the head still augments. However, in small orifices, those below 1.181 inches, the increasing part of the series is very limited; and even in very small ones it is nothing. We see also that the terms of the decreasing part of all the series approach equality in proportion as the head increases in value.

The same coefficients answer for all forms of orifices.

27. Although the coefficients in the table above are deduced from experiments made on rectangular orifices, they may serve for all others, whatever be their form; the height of the rectangle noted in the table will express the smallest dimension of the orifice which should be used. For it is generally admitted, that the discharge is entirely independent of the figure of the orifice, and that it always remains the same, while the area of the opening is unchanged; always provided, in accordance with an observation made by M. Hachette, that this figure presents no reëntrant angles.

Experiments on Sluice Gates.

28. Although some of the orifices on which Poncelet and Lesbros made their experiments are very large, still there are those which discharge twenty or thirty times as much water; such are the openings of sluice gates in canals of navigation, and it was important to establish directly the coefficient of their discharge. In 1782, Lespinasse, a skilful engineer, made for this purpose several experiments on the canal of Languedoc, to which, ten years after, Pin, engineer of the same canal, added some others.* The principal results of these, like the former, are placed in the following table.

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* Anciens Mémoires de l'Académie des Sciences de Toulouse. Tom. II. 1784.—Historie du canal du Midi ou Languedoc, par le général Andréossy. Tom. I., pag. 251.

The breadth of the opening is nearly 4.265 feet; the form not being exactly a rectangle, the heights are to be regarded as only approximate.

OPENINGS.		Head on the centre.	Discharge in one second.	Coefficient.
Area.	Height.			
sq. feet.	feet.	feet.	cubic feet.	
7.745	1.805	14.554	145.292	.613
6.992	1.640	6.631	92.635	.641
6.992	1.640	6.247	88.221	.629
6.466	1.509	12.878	138.937	.641
6.723	1.575	13.586	128.764	.647
6.723	1.575	6.394	83.948	.616
6.723	1.575	6.217	79.857	.594
6.717	1.575	6.480	85.219	.621
		Mean term, . . .		.625

This mean coefficient, exactly equal to that obtained from an experiment made on a sluice of the basin of Havre* is a little greater than that indicated by the table of M. Poncelet (26); probably the cause of it is, that on all the perimeter of the opening, the flowing did not occur as in a thin side, and that on some point, the contraction was suppressed. It may be remarked on this subject, that the wood work which surrounded this orifice was 0.27^{m}=.886 ft. thick, and even 0.54^{m}=1.772 feet thick on the lower edge. Also, when the gate was raised only a small quantity, the contraction ceased on the four sides and the coefficient increased considerably. For example, Lespinasse having raised the gate only 0.12^{m}=.394 ft., had for a coefficient .803, while with 1.509 feet opening, he had a coefficient of only .641.

Effect of two orifices near each other.

29. The experiments of this engineer presented a very remarkable fact, of which no mention was made, and which reäppeared in those of Pin. A sluice gate had two parts, and each had an opening in it; if, while the water was flowing through one, the second was opened, the discharge of the first was diminished; if both

* Architecture hydraulique, par Bélidor et Navier. Tom. I., pag. 289.

were opened together, the discharge was not double of the two taken separately, although each had the same area and head. The difference is about one eighth, as may be seen by the following comparison of the coefficients of reduction, for the two cases.

The interval between the two openings is $2^{m}.92 = 9.58^{ft}$, and their plane forms an angle of 60° with the direction of the canal.

COEFFICIENT	
with one opening.	with two openings.
0.641	0.550
0.689	0.555
0.616	0.554
0.594	0.526
0.621	0.555
0.620	0.548

30. But it is very worthy of remark, that this fact, which appeared positive for the sluices of the canals, did not take place at all in a series of experiments which M. Castel and I made on a small scale, but with very great care, for the purpose of verifying it. We had, side by side, three rectangular orifices of $.328^{ft}$ base by .033 height, and separated by an interval of only $.033^{ft}$. We measured the water passing the middle orifice first, keeping the two side orifices closed, then opening one and finally opening both; the mean results are given in the following table:—

Head on the orifice.	Discharge from Middle Orifice.			Coefficient.
	Middle orifice alone open.	Middle orifice, with 1 lateral orifice, open.	Middle orifice, with the 2 lateral orifices, open.	
feet.	cubic feet.	cubic feet.	cubic feet.	
.0656	.01607	.01606	.01614	0.728
.0984	.01946	.01946	.01942	0.720
.1312	.02242	.02246	.02250	0.719
.1640	.02497	.02497		0.715
.1969	.02723	.02716		0.710

Supposing that these unexpected coefficients might have been influenced by the very small interval from one orifice to the other, we increased the interval five fold, that is, from .394 inch to 1.968 inches, and the coefficients remained the same.

31. Surprised at the difference between our results and those found on the canal of Languedoc, and fearing that it arose from the particular form of our orifices and apparatus, I requested M. Castel to make new experiments; and in 1836 he had the kindness to perform a series, by the aid of the great apparatus which he had just been using for his great work on wiers (No. 72 and seq.). He dammed up a canal $0^{m}.74=2.428$ feet broad, with a thin copper plate, in which he opened, on the same horizontal strip, three rectangular orifices, each 3.94 inches wide by 2.36 inches high, and separated from each other by an interval of 3.15 inches. The flowing took place under a constant head of 4.213 inches above their centre, and the coefficients of contraction were as follows:

One orifice open	for the middle	.6198
	" right	.6193
	" left	.6194
Two orifices open	the two outsides	.6205
	middle and right	.6205
	" " left	.6207
The three orifices all open		.6230

Here, in proportion as the orifices were open, instead of a diminution in the coefficients, there was an increase, very small, to be sure. As it depended on a particular cause, a greater velocity of water in the canal, in consequence of a greater discharge (See Nos. 38 to 79), we shall make deduction of that, and conclude that, when in the dam of a reservoir or course of water, new orifices are opened by the side of an orifice already existing, the discharge through that orifice is not diminished by it.*

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* Some persons thought that such a consequence would not extend to the case when two orifices were situated in planes making a certain angle, as in the openings of the sluice gates. M. Castel has just solved this question. He took two plates joined at an angle of 120° (that of sluice gates is generally from 10° to 20° more open) ; in each he made two rectangular orifices of 3.94 inches wide by 2.36 inches high ; one 4.72 inches and the other 11.02 inches distant from the angle that joined them ; he fitted this partition to the extremity of his canal, and let the water flow under a head of $0^{m}14$=5.51 inches. He first opened successively each of the four orifices ; then two at a time, differently combined ; then three differently combined, and finally four. The following table presents the mean results obtained.

That given in the second line was obtained by the two extreme orifices, which were disposed like those of the sluice of the canal of Languedoc.

No. orifices.	Coefficient.
1	.618
2	.619
3	.620
4	.622

As a last objection, it was said that the heads at the sluice of the canal of Languedoc were from 2^{m}=6½ feet to 4^{m}=13 feet. To obtain an analogous case, M. Castel adapted to the experimental apparatus cited in article 49, two orifices of 1.97 inches wide by 1.18 inches high, and had the following results.

It is always the same coefficient, with the insignificant increase due to the number of orifices open.

Head.	No. orifice.	Coefficient.
$3.379^{ft.}$	1	.621
	2	.622
$6.693^{ft.}$	1	.619
	2	.621

These experiments, often repeated, with apparatus free from every exceptionable circumstance, and where any sensible error was impossible, by the most accurate and conscientious observer, induce me, if not to call in doubt the facts announced in No. 29, at least to regard them as anomalous, and to reject the general consequence which I had drawn from them. [15th November, 1838.]

Case of the contraction being destroyed on any part of the orifice.

32. In the different cases hitherto investigated, it is admitted that the fluid of the reservoir arrives equally at all parts of the orifice, but often it is not so; for example, when the orifice is at the bottom of a vertical side, and its lower edge is in the plane of the bottom of the reservoir, the contraction is then destroyed on that side, and consequently, the discharge is greater. What will be the increase in discharge for a certain length of suppression in the contraction? This question has recently been nearly solved by M. Bidone, by the aid of numerous experiments made for that purpose at the water-works of Turin.*

The orifices were made in thin vertical copper-plates; on their interior surface were fixed, perpendicular to their plane, small plates, on a level with certain sides of the orifice; as it were, the prolonging of these sides into the interior of the reservoir. During the flowing, the water running along the plates passed through the adjacent sides without any contraction, while a contraction occurred on the other sides. The form and size of these orifices were various. I shall limit myself to giving the results of experiments with a rectangular orifice of $0^{m}054 = 2\frac{1}{10}$ inches base and 1.06 inches in height; the plates adapted to them, sometimes on one side and sometimes on two or three, were 2.638 inches long; they thus extended that length into the reservoir. The flowing having been produced under heads varying from 6.562 feet to 22.573 feet, we have the following coefficients:

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* Recherches expérimentales et théoriques sur les contractions partielles des veines d'eau, etc., par George Bidone. Turin, 1836.

The contraction being suppressed on	Part of orifice without contraction.	Coefficient.	Ratio.
Neither side	0	.608	1.000
a small "	$\frac{1}{6}$	.620	1.020
a great "	$\frac{2}{6}$	.637	1.049
a great and a small	$\frac{3}{6}$	.659	1.085
two small and one great	$\frac{4}{6}$	.680	1.119
two great and one small	$\frac{5}{6}$	.692	1.139

M. Bidone, taking the mean result of all the experiments made on rectangular orifices, admits for the numbers of the last column, which indicates the increase of the coefficient and consequently of the discharge, that for the orifice entirely free being taken for unity, the general expression $1+0.152\frac{n}{p}$, in which n represents the length of the part of the perimeter when the contraction is suppressed, and p the length of the whole perimeter. The greatest error which this formula gave M. Bidone being only $\frac{1}{35}$, we may adopt for the value of the discharge in rectangular orifices when there is no contraction on a part of the perimeter,

$$mS\sqrt{2gH}\,(1+0.152\tfrac{n}{p}).$$

The same author also made experiments on circular orifices. He took one of 1.575 inches diameter, and by the aid of curved cylindrical plates, he destroyed the contraction, first, on an eighth of the circumference; then successively on 2, 3, 4, 5, 6 and 7 eighths. I indicate the results obtained in the following table.

$\frac{n}{p}$	Coefficient.	Ratio.
0	0.597	1.000
$\frac{1}{8}$	0.603	1.011
$\frac{2}{8}$	0.615	1.032
$\frac{3}{8}$	0.625	1.048
$\frac{4}{8}$	0.639	1.072
$\frac{5}{8}$	0.649	1.087
$\frac{6}{8}$	0.664	1.112
$\frac{7}{8}$	0.670	1.123

We see here that the numbers of the last column increase a little less rapidly than in the case of the rectangular orifices, so that the general expression from these numbers would be only $1+0.128\,\frac{n}{p}$.

M. Bidone, after having circumscribed seven-eighths of his circular orifice, wished to circumscribe it entirely; and for this purpose, he fitted to the orifice a cylindrical tube of 0^m04=1.575 inches diameter, which ran 0^m067=2.638 inches into the interior of the reservoir; he had 0.767 for the coefficient, and consequently, 1.285 for the number of the last column. The expression above would have given 1.128—a number in which the increase is not even half of that really obtained. Whence we conclude, that the phenomena of flowing through interior tubes, the case where the contraction is entirely suppressed at the edges of the exterior orifice, is no longer of the same kind as that where it is destroyed only in part, however great that part may be; there is no passing from one case to the other.

Orifices in sides not plane.

33. We have always supposed the sides in which the orifices were, to be plane, but they may be of another form. To give an idea of the effect which may result upon the product of the flowing, it is necessary to remember, that if the fluid lines arrive at the orifice parallel to each other, the actual discharge would be equal to the theoretic discharge, and that it is less only in consequence of the obliquity with which they unite, from which obliquity necessarily results, at the point of contact, the destruction of a part of the motion acquired. This being established, if around the orifice we imagine a spherical surface or cap, of a radius equal to that of the sphere of activity of the orifice, and limited by the sides of the vessel, it would be traversed at each of its points, and in a direction nearly perpendicular, by the arriving lines; the more extended the spherical cap, the more oblique will be their directions, and the more opposed to each other; and consequently, the more will their motion be destroyed at the orifice, and the less

Fig. 7.

Fig. 3.

considerable the discharge. When the side is plane, the cap is the surface of a hemisphere (Fig. 3), and is found in the case to which belong the coefficients of discharge given above (26). But if it is disposed in the form of a funnel, or if it is simply concave towards the interior of the vessel, then the cap is smaller and the discharge greater, without, however, exactly following the ratio of the spherical surface. If, on the contrary, the side is convex, the product is less; it will be smaller still in the case represented at Fig. 7. Finally, it would be a *minimum*, if the cap became an entire sphere; and this would happen, if it were possible to transport an orifice to the middle of the fluid mass inclosed in the vessel.

Interior Tubes.

Fig. 8.

34. Borda succeeded in almost entirely realizing this case. He introduced into a vessel a tin tube $0^{m}135$ $=4.43$ feet long and $0^{m}032=.105$ feet diameter; and under a head of 0.820 feet, he caused the flowing to take place in such a manner that the effluent water in no way touched the sides of the tubes; the actual discharge was only 0.515 of the theoretical discharge, and several considerations led Borda to admit that it might have been reduced to .50.*

Having afterwards surrounded the orifice of entrance of the tube with a large border, thus putting it, although in the middle of the fluid, into the same circumstances as when it is perforated through a thin side of a vessel, the coefficient was raised to 0.625. He might have obtained the same result by employing simply a tube with very thick sides.

If the sides of the tube had a sensible thickness, without being too considerable, 0.394 inch or even 0.788

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* Mémoires de l'Académie des Sciences de Paris. Année 1766.

inch, for example, and were also cut quite square off at the extremity, so that the zone formed by the thickness should be plane, with sharp edges, the fluid winding round the exterior edge would enter the tube without touching the rest of the zone (Fig. 8 *a*); so that every part of the side inside of the exterior surface would be without effect, and the flowing would take place as if that surface alone existed. This, therefore, will be its diameter; that is to say, the exterior diameter of the tube, which must be introduced into calculations relating to interior tubes. By taking this, Bidone found, by two experiments, that the action of the vein running in the tubes without touching the interior, was very nearly half the section of the tube, and that the coefficient of contraction was nearly 0.50.

Limit of Coefficients.

35. Thus 0.50 and 1 (22) will be the limits of the coefficients of contraction; limits which may be approached very nearly, but never quite attained. For orifices in a plane side, they seldom descend below .60 or rise above .70; and even in ordinary practice, they are confined between .60 and .64; as a mean approximate term, .62 is usually taken, and we have—

Ordinary Formula.

In Metres, $Q=0.62\,S\sqrt{2gH}=2.75\,S\sqrt{H}=216d^2\sqrt{H}$; or,

In Feet, $Q=0.62\,S\sqrt{2gH}=4.974\,S\sqrt{H}=3.9066d^2\sqrt{H}$,

d being the diameter of a circular orifice. But, whenever accuracy is required, we should have recourse to the coefficient of No. 26.

Real velocity of issue.

36. In the velocity with which water flows from orifices in a thin side, as we have admitted exactly that due to the head of the reservoir, is it $\sqrt{2gH}$? We will examine it.

We may ascertain the velocity with which water runs from an orifice, by the height to which a vertical jet,

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starting from that orifice, is thrown; it is at least $\sqrt{2gh}$, h being that height. Now, from what will be seen in the chapter on *spouting fluids*, h differs from H only 1, 2, 3, &c. hundredths of the square of its value, according as H is 1^{m}, 2^{m}, 3^{m}, &c.; and the velocities being as the square roots of the heights, the *actual* velocities will differ in the same cases only 1, 2, 3, &c. half-hundredths of the *theoretic velocity*. Another mode of determining the actual velocity indicates still less difference. I will present it, before making an application of it.

37. When a body is thrown in any direction AY, with a certain velocity, by the combined influence of that velocity and of gravity, it describes a curve AMB; if the velocity, and consequently the resistance of the air, is not very great, that curve is a *parabola*. Fig. 9.

The demonstration of this fact being found in all treatises of mechanics and physics, I shall not dwell upon it, but confine myself to what concerns the fundamental principle which we are to employ. Let v be the velocity with which a body is impelled along AY, and t the time spent in arriving at N, in this direction, if the force of projection acted alone upon it; the motion would then have been uniform, and we should have had $AN=vt$; on the other hand, had the body been subjected to the action of gravity alone, it would have descended from A to P during the same time, so that we should have had $AP=\frac{gt^2}{2}$ (8). Draw the parallelogram APMN; at the end of the same time, it really will arrive at M, and will have described the arc AM; AP will be its *abscissa*, and MP parallel to the axis AY, will be its *ordinate*. Call the first of these lines x and the second y, we shall have $x=\frac{gt^2}{2}$ and $y=vt$; in this latter equation, taking the value of t, and substituting it in the first, we have $x=\frac{gy^2}{2v^2}$ or $y^2=\frac{2v^2x}{g}$; or, calling h the height due to the velocity v, and recollecting that $\frac{v^2}{2g}=h$, $y^2=4hx$; an equation of a parabola of which $4h$ is the parameter. Hence the theorem, that *a heavy body, impelled by any force of projection, describes a parabola whose parameter is four times the height due to the velocity of projection.*

Fig. 10. What we have just said of a body in general is applicable also to every jet of water issuing from an orifice. If this orifice is in a vertical side, the axis of projection being horizontal, the ordinates will be horizontal; they will be the distances of the different points of the jet from the vertical, let down from the centre of the orifice; and if through any point c of that vertical, we imagine a horizontal plane, the distance CD is called the *reach of the jet* on that plane. According to our theorem, the square of this range, or in general of a distance MP, divided by four times its corresponding perpendicular AP, will give the height due to the velocity of exit $(h=\frac{y^2}{4x})$; and consequently, we shall have for this velocity, $v=\sqrt{2gh}=2.215\,\frac{y}{\sqrt{x}}$ in metres, or $4.0113\,\frac{y}{\sqrt{x}}$ in feet.

By following this mode of determination, Bossut, in two experiments, found 0.974 and 0.980 for the ratio of the actual to the theoretic velocity. Michelotti having caused jets to issue from each of the three stories of the tower of his hydraulic establishment (25), through a vertical orifice, 0.889 feet diameter, obtained the results given in the following table:—

HEAD.	JET.		VELOCITY.		RATIO.
	Abscissa.	Range.	Real.	Theoretic.	
feet.	feet.	feet.	feet.	feet.	
7.513	20.615	24.706	21.819	21.983	.993
12.894	15.289	27.724	28.446	28.807	.988
23.590	4.626	20.506	38.289	38.978	.983

The difference between the two velocities increases with the head. It should be so, since the cause of this difference, the resistance of the air, increases as the square of the velocity, and consequently, nearly as the head. Without this cause, the difference would have been almost nothing. Consequently, there are grounds for concluding that, in the flowing (*of water*) *through orifices in a thin side, the velocity of exit is nearly that due to the height of the reservoir, and it is not sensibly diminished by contraction.*

Case of the fluid arriving with an acquired velocity.

38. If the water contained in the reservoir, instead of being at rest, were animated with a velocity which carried it towards the orifice; for example, if the basin having a small section, were fed by a course of water which came directly to the side on which the orifice is open, the fluid particles would go out, not only in virtue of the pressure exerted by the fluid mass above, but also in virtue of the velocity which they had at the moment of entering the sphere of activity of the orifice; we should thus have to add to the head measuring the pressure, a new force, which will be the head generating that velocity. Thus, if u represent that velocity, we shall have

$$Q=mS\sqrt{2g\left(h+\frac{u^2}{2g}\right)}=mS\sqrt{2gh+u^2}.$$

Example. There is a basin 65.62 feet long, 6.562 feet broad, and 3.281 feet depth of water; at one extremity is a dam of plank, with a rectangular opening 1.804 feet wide by 1.181 feet high; its sill or lower edge is 2.986 feet below the level at which the water is constantly kept in the basin; it is supplied by a stream arriving at the other extremity. What is the discharge?

We have $S=1.804\times1.181=2.131$ square feet; $h=2.986-\frac{1.181}{2}=2.396$; m, according to the table at No. 26, supposed to be prolonged, will be about 0.600; as to u, it will be given by one of the means to be indicated hereafter (147 to 154). In a great number of cases, we can regard it as being the mean velocity of the water in the basin, a velocity to be determined as follows: the discharge Q, taken at first by neglecting u will be $0.600\times2.131\sqrt{64.364\times2.396}=15.878$ cubic feet. When the water runs in a canal, we have $Q=S\,u$ (108); dividing then the value of Q found, by the section (of the basin) 21.53, we find $u=.73748$, the square of which is .54389. Putting this value into the general expression of the discharge, we have $0.600\times 2.131\sqrt{64.364\times 2.396+.5439}=15.906$ cubic feet.*

The difference between these two results may be entirely neglected. The effect of the velocity u has been almost nothing; in most cases, it will be so.

Orifices in the additional canals.

39. Very often, the water at the exit of the orifices made in the side of a reservoir, is taken and conducted by canals or channels, uncovered on the upper part, the bottom of which, as well as the sides, agree with the lower edge and sides of the orifice, which are thus in the planes of the bottom and sides respectively. MM. Poncelet and Lesbros determined, by a great number of experiments, the coefficients of the discharge for such canals, which they fitted to orifices on which they had already made the fine observations whose results we have recorded in No. 26; the canals varied in form, inclination and position. The last of these philosophers had the kindness to communicate to me a part of the results given by a rectangular canal $3^{m}=9.843$ ft. long and $0^{m}20=.656$ ft. broad, like all its orifices. The reservoir in whose side the orifices were, was $3^{m}68=12.074$ ft. broad. The canal was first placed at an equal distance from the two sides of the reservoir and $0^{m}54=1.772$ ft. above the bottom; it was kept horizontal; it is canal No. 1 of the following table. I here give the coefficients m of the formula $mS\sqrt{2gH}$, which MM. Poncelet and Lesbros obtained, and I place them opposite those which they had obtained previ-

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* D'Aubuisson's book has an error in taking the section of the orifice, instead of the section of the basin, and also another error in solving the example. What is here given is supposed to be what D'Aubuisson intended. TRANSLATOR.

ously with the same orifices, when the water flowed freely into the atmosphere. (26)

Height of orifice.	Head on orifice.	COEFFICIENT.		
		Without canal.	WITH CANAL	
			No. 1.	No. 2.
feet.	feet.			
.6562	4.2850	0.604	0.601	0.601
	3.1235	0.605	0.602	0.599
	1.3124	0.600	0.591	0.580
	.7940	0.596	0.559	0.552
	.4003	0.572	0.483	0.482
.3281	4.4490	0.643	0.614	
	3.3040	0.615	0.614	
	1.5814	0.617	0.615	
	.5282	0.611	0.590	
	.3740	0.608	0.562	
	.2887	0.602	0.523	
	.1969	0.590	0.459	
.1640	4.7935	0.621	0.624	0.627
	3.5468	0.627	0.626	0.628
	1.6350	0.631	0.625	0.624
	.6956	0.634	0.631	0.615
	.3478	0.629	0.614	0.597
	.1542	0.617	0.495	0.493
	.1181	0.612	0.452	0.443
.0984	4.4261	0.622	0.622	
	1.5289	0.630	0.629	
	.6792	0.634	0.632	
	.2658	0.639	0.633	
	.2067	0.640	0.627	
	.1870	0.640	0.610	
	.1214	0.639	0.511	
.0328	4.449	0.620	0.621	0.660
	3.2580	0.627	0.631	0.665
	1.6307	0.643	0.648	0.671
	.6398	0.655	0.665	
	.4167	0.664	0.669	
	.2494	0.671	0.671	0.680
	.1378	0.684	0.640	

By comparing the coefficients of the third and fourth columns, allowing for the inevitable errors in observation, and excepting the orifice of 0.328 ft., we see that so long as the heads taken above the centre of the orifice were from 2 to 2½ times greater than the height of that orifice, the canal had no marked difference in the discharge; the discharge was the same as if no canal were there. But in small heads, the discharge diminished perceptibly, and as much more so as the head was less; the diminution has reached a quarter, and even more.

This difference in great and small heads appears to proceed from the fact, that with the former, the fluid, rushing forth as into the air, is not influenced by the resistance of the sides. "The canal," says Lesbros, "has no influence, except when the head is not great enough to detach the fluid jet at its exit from the orifice entirely from the bottom (and sides) of this canal."

The same canal was then placed, as is often done in practice, in such a manner that its floor was at the level of the bottom of the reservoir, and was, in fact, a prolonging of it. It was natural to suppose, that the contraction being then suppressed on the lower edge of the orifice, the coefficient of discharge would be greater (32); but generally, and the orifice of .0328 feet still excepted, it was less, particularly with small heads, as was seen in the above table, where the canal, in its new position, is designated by No. 2. Other circumstances, perhaps the resistance of the bottom of the reservoir, which may have diminished the velocity of arrival, perhaps the less facility which the fluid sheet had in raising itself above the sill at the entrance of the canal, will have more than compensated for the diminution in the contraction.

In withdrawing the canal from the middle of the reservoir, and placing it nearer one of the sides, this diminution took place in part, and a small increase in the discharge was obtained.

The canal was then inclined, leaving it in other respects in the position it last had. When the inclination was $\frac{1}{100}$ or 34′, the coefficients were sensibly the same as when the canal was horizontal. But when the inclination was carried to $\frac{1}{10}$, or 5° 44′, the coefficients were increased from three to four per cent., as seen in the following table:—

Height of orifice.	Head on orifice.	Coefficients, with the Canal	
		Horizontal.	Inclined.
feet.	feet.		
.0443	1.1188	.660	.691
.0666	1.1123	.654	.681
.1555	.6890	.616	.639
.1775	.6660	.612	.636

2. CYLINDRICAL AJUTAGES.

40. Cylindrical ajutages, called also *additional tubes*, as we have seen (19), give a more considerable discharge than orifices in a thin side, the head and area of the opening remaining the same.

But in order to produce this effect, it is necessary that the water entirely fill the mouth of the passage; it is commonly so, when the length of the tube is two or three times its diameter. If it is less, it often happens that the fluid vein, which is contracted at the entrance of the tube, does not again increase and fill the interior; the flowing then takes place in all respects as through a thin side; this is always the case when the length of the tube is less than that of the contracted vein, and consequently, is only half, or less than half, the diameter.

Coefficient of reduction for the discharge.

41. The coefficient of reduction from the theoretic to the actual discharge, through an additional tube, pre-

sents a few variations, as may be seen in the following table:—

Observer.	TUBE.		Head.	Coefficient.
	Diameter.	Length.		
	feet.	feet.	feet.	
Castel,	.0509	.1312	.6562	.827
Do.	.0509	.1312	1.5749	.829
Do.	.0509	.1312	3.2478	.829
Do.	.0509	.1312	6.5620	.829
Do.	.0509	.1312	9.9414	.830
Bossut,	.0755	.1772	2.1326	.788
Do.	.0755	.1772	4.0684	.787
Eytelwein,	.0853	.2559	2.3623	.821
Bossut,	.0886	.0341	12.6318	.804
Do.	.0886	.1772	12.6975	.804
Do.	.0886	.3543	12.8615	.804
Venturi,	.1345	.4200	2.8873	.822
Michelotti,	.2658	.7087	7.1526	.815
Do.	square. .2658	.7087	12.4678	.803
Do.	.2658	.7087	22.0155	.803

Cylindrical tubes being little employed, I shall not extend this table or discuss the experiments. I shall confine myself to remarking, that the mean of the coefficients there given, abstracting the first two of Bossut, manifestly anomalous, is 0.817; .82 is generally taken, and we have

$$Q=.82\,S\sqrt{64.364H}=6.5786\,S\sqrt{H}=5.1668\,d^2\sqrt{H}.$$

Velocity of issue from an Ajutage.

42. Since the jet in a full tube runs out in lines parallel to the axis of the orifice, and consequently, its section is equal to that of the orifice, the diminution of the discharge can arise only from a diminution in the velocity (16); and the ratio of the actual to the theoretic discharge will also be that of the actual to the theoretic velocity, as is seen by the following results of three experiments cited in the above table; one of Venturi and two of M. Castel:—

JET.		VELOCITY.		COEFFICIENT	
Abscissa.	Ordinate.	Real.	Theoretic.	of velocity.	of discharge
feet.	feet.	feet.	feet.		
4.796	6.128	11.204	13.628	.824	.822
1.791	2.208	6.6175	7.959	.832	.827
3.7402	5.803	12.037	14.481	.832	.829

Thus we may admit that the velocity of a jet, at its passage from a cylindrical tube, is only 0.82 of that due to the height of the reservoir; and the height due to the velocity of the jet will be only .67 ($=.82^2$) of that due the height of the reservoir, since the heights or heads are as the squares of the velocities. (12)

In the hypothesis of the parallelism of the sections, the principle of the *vis viva:* that the quality of action developed by the motive force, during a certain time, is equal to half the increase or diminution of the *vis viva* during that time—this principle, I say, gives for the velocity v of the water passing from a short prismatic tube, of which S is the section, and which is terminated by an orifice whose section s is smaller than the preceding, m and m' being the coefficient of contraction for these sections respectively

$$v=\sqrt{\frac{2gH}{1+\left(\frac{m's}{S}\right)^2\left(\frac{1}{m}-1\right)^2}};$$

and for the case of our additional tubes entirely open at their extremity, and consequently, where $s=S$ and $m'=1$

$$v=\sqrt{\frac{2gH}{1+\left(\frac{1}{m}-1\right)^2}}.$$

If it be admitted that the contraction at the entrance of the tube is the same as in the orifices in a thin side, that is to say, if we make $m=.62$, we have $v=0.855\sqrt{2gH}$ and $Q=.855\ S\sqrt{2gH}$; with $m=.65$, it would be $Q=.0885S\sqrt{2gH}$.

Cause of the increase of discharge through Ajutages.

43. The fluid vein, after its contraction at the entrance of the additional tube, tends to take and preserve a cylindrical form, whose section would be that of the contracted vein; and consequently, it tends to pass out without touching the sides of the tube; but some lines of water are carried towards the sides, either by a divergent direction, by an attractive action, or by the two causes united. As soon as they arrive in contact, they are strongly retained by the molecular attraction, that which produces the ascension of water in capillary tubes; by an effect of this same force, they draw the neighboring lines, and by degrees the whole vein, which then rushes out, filling the tube, and passes through the contracted section more rapidly. Such appears to be the physical cause of the increase of discharge due to tubes.

The immediate cause is the contact; and all the circumstances which cause the contact, or which favor it, will produce that increase.

Among these circumstances we will notice:

1st. The length of the tube; the longer it is, the more chances it will present for contact; there will be no contact when the length is less than that of the contracted vein.

2d. A small velocity; the fluid lines will then be less forcibly retained in the direction of the primitive motion; they will deviate and approach the sides with more facility. M. Hachette, in his experiments made on this subject, succeeded, by augmenting the head and consequently the velocity, in detaching a vein from the side it was following. On the contrary, by diminishing the head, allowing it, however, a head of 0.9843 ft., he succeeded in making the tube more full, the length of which was 0.01968 ft., and its diameter 0.03117 ft.

Traité de Machines, edition de 1828, pp. 73-102.

3d. The affinity of the material of the tube, or rather, its disposition to be more readily moistened. Thus, by rubbing tallow or wax on the sides, the water will not follow them as it did before. Hachette, by covering an iron tube with an amalgam of tin, caused mercury to run out with a full tube, which did not take place before the coating. The interposition of air, or its arrival in a tube, is sufficient to detach the fluid vein from it. Venturi, after having fitted to a vessel full of water, a tube of $0^{m}0406$=.1332 ft. diameter and $0^{m}095$=.3117 ft. length, perforated near the middle and quite round its perimeter, with a dozen small holes; when the flowing took place, not a drop of water passed through these holes, nor did the water touch the sides. The holes were then successively stopped, and the same results continued; but when all were closed, the vein filled the tube, and the discharge was increased in the ratio of 31 to 41.* M. Hachette, on repeating the experiments and closing the holes with caution, saw the vein continue to pass out without touching the side; but a slight agitation was then enough to produce contact, and to produce a flow with the full tube.

44. It is more than a century since Poleni made known the singular effects of cylindrical tubes, and the investigation of the cause has been a serious study with philosophers.

It was generally said, since the convergence in the direction of the fluid lines, on their arrival at the orifice, produces a contraction in the fluid vein, there will also be a contraction at the entrance of the tube; but in consequence of the attractive action of the sides, the contraction will be less, and the discharge will consequently be greater. The experiments of Venturi do not allow us to admit of such a cause producing a less contraction.

That ingenious philosopher opened, in a thin side of a reservoir, an orifice, whose diameter AB (Fig. 11), was $0^{m}0406$=.1332 ft.; and under a head of $0^{m}88$=2.8873 feet, he obtained $0^{mmm}137$ =4.8384 cubic feet of water, in 41″. To this orifice he then fitted the tube ABCD, having nearly the form of the contracted vein, (he had CD=$0^{m}0327$=.1073 feet, and AC =$0^{m}025$=.082 feet); under the same head, he obtained the same volume of water, in 42″. To the first tube he fitted the tube CDHGC, in which GH=EF=AB, and the duration of the flowing, all else being equal, was only 31″. Lastly, for all this apparatus, he substituted the simple cylindrical tube ABHG of the same length, and also of the diameter .1332 ft., and the flowing of 4.8384 cubic feet again took place in 31″.

Fig. 11.

Fig. 12.

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* Rechérches Expérimentales sur la communication latérale du mouvement dans les fluides. 1797. 5.e Expérience.

Thus, in this simple tube, in which everything went on as in the compound tube, there was or there may have been an equal contraction; and the contraction which necessarily took place in the latter at CD, is very nearly equal to that of orifices in a thin side. The effect of the cylindrical tube, therefore, was not to lessen the contraction, but to pass the fluid through the contracted section CD, with a velocity increased in the ratio of 31 to 41 or 42. Hence alone the increase of discharge.

Venturi attributed it to an excess in the pressure of the atmosphere on the fluid surface contained in the reservoir, an excess proceeding from a vacuum tending to arise in the part of the tube where the greatest contraction took place. He sought to prove this opinion by several examples, very interesting on other accounts, but he has sometimes generalized the results too much. For example, because in one of them the water ceased to flow with full tube under the receiver of an air pump, he concluded that the phenomena of additional tubes did not take place in the vacuum, and yet Hachette is certain of having produced them there. This single fact would overthrow an hypothesis, against which other peremptory objections are also raised.

Negative pressure of fluid against sides of Ajutages.

45. Among the experiments of Venturi, is one which presents, in a very distinct manner, a very remarkable fact, which Bernoulli had already made known. To a cylindrical tube 0^m0406=.1332 ft. diameter and 0^m122=.4003 ft. long; at E 0^m018=.0591 ft. from its origin, he fitted a curved tube of glass, the other extremity of which was plunged into a vessel M, containing colored water; the flowing was caused by a head of 0^m88=2.8873 feet; and the water was raised in the tube 0^m65=2.1326 feet.

In the hypothesis of Venturi, this elevation, joined to the head, would be the height due to the velocity through the contracted section, as the head alone is the height due when there is no additional tube; if it were so, the ratio of the velocities must be as $\sqrt{2.8873}$; $\sqrt{2.8873+2.1326}$, or as 31 to 40.9, and experiment has actually given a similar result (31 to 41). But from this fact, peculiar perhaps to the case taken for example, a general principle ought not to be deduced. Moreover, the true cause of the ascension of the colored water in the tube was indicated more than a hundred years ago, by Daniel Bernoulli (Hydrodinamica, p. 264). That celebrated geometrician, author of the chief part of the theoretical principles of the flowing of water, established the law, that the pressure which a fluid exerts against the sides of a tube in which it moves, is equal to the head minus the height due to the velocity of the motion. It is necessary to remark, that in speaking of absolute pressure, the weight of the atmosphere should be added to the *head* properly so called; thus, if K represents that weight, that is to say, a column of water equal in weight to that of the column of the barometer, H the head and v the velocity of the fluid at a determined point of the tube, $K+H-.01553\ v^2$ will be the interior pressure at that point. For the exterior pressure, we have K, as on all the other points. In one example, at the place of greatest contraction, where $v=\frac{41}{31}\sqrt{2gH}$ and $H=2.887$ feet, the interior pressure is $K+2.887-5.050=K-2.163$ in feet, it is less by 2.163 feet than the exterior pressure; the exterior pressure will therefore prevail, and will cause the water to ascend 2.163 feet, and in general, a quantity equal to its excess over the other.

By neglecting K, which is found both in the value of the interior and exterior pressures, the interior pressure on the same point compared to the other is, $H-.01553v^2$; it will be negative, whenever the height due the velocity is greater than the head.

Venturi having placed the same tube $0^m054=.177$ ft. from the reservoir, the colored water was not raised; the height due, 0^m594 or $0.051v^2-0.051\ (0.82)^2gH$ in metres, or, $.01553v^2-.01553\ (0.82)^2gH$ in feet, was then smaller than the head 2.8873 feet; the interior pressure was positive, and consequently there was no ascension.*

3. Conical Converging Tubes.

46. Conical tubes, properly so called—that is to say, those which slightly converge towards the exterior of the reservoir—increase the discharge still more than the preceding; they afford very regular jets, and

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* Should the reader find difficulty as to the formation of this formula, it will vanish in remembering that the velocity from cylindrical pipes is but $\frac{82}{100}$ of that due to height of reservoir, (or $v=.82\sqrt{2gH}$) and by substituting this value in the equation $H=\frac{v^2}{2g}$. Translator.

throw them to a greater distance or height. They are also almost exclusively employed in practice. However, their effects as to the discharge and velocity of projection are much more varied; they change with the *angle of convergence*, that is, with the angle which the opposite sides of the truncated cone constituting the tube, form by their extension.

They are, however, the tubes on which we have the fewest documents. In reference to them, I know of only four experiments of Poleni, published at Florence in 1718, and which Bossut gives in his *Hydrodynamique* (§ 530); notwithstanding the merit of their author, and although made on a great scale, I have very strong reasons for doubting their accuracy, and shall not bring them forward again. Struck by the gap which hydraulics presents in this important part, I projected a series of experiments suitable to fill it; but before reporting those that have been made, I state briefly the condition of the question.

Coefficients of discharge and velocity.

47. According to what was said (16 and 19), there are, or there may be, two contractions of the fluid vein, in running through conical tubes: one interior, or at the entrance of the tube, which diminishes the velocity produced by the head; the other exterior, or at the exit, by which the section of the vein a little below the exterior mouth of the orifice is smaller than the mouth itself. Consequently, if s is the section of the orifice and V the velocity due to the head, the real discharge will be $ns \times n'V = nn'SV$ (16); n and n' being two coefficients to be found by experiment; n is the ratio of the fluid section to the section of the orifice, or the coefficient of the exterior contraction; n' is the ratio of the actual to the theoretic velocity, or the *coefficient of the velocity;* and nn' is the ratio of the

actual to the theoretic discharge, or the *coefficient of discharge.*

The knowledge of the two latter, for the different cases which may present themselves, is sometimes useful in practice, as we shall see in treating of *jets of water;* it is this utility, or rather necessity, of having their value, that is, of knowing the discharge and force of projection of different tubes, which has induced me to make researches on this subject.

48. To determine properly the different coefficients in question, and, above all, to fix the angle of convergence giving the greatest discharge, I thought it necessary to subject many series of tubes to experiment; in each, the diameter of the orifice of exit and the length of the tube remaining constantly the same; but the diameter of the entrance, and consequently the angle of convergence, was gradually increased. The water flowed through each under different heads. At each experiment, the actual discharge was determined by direct measurement, and the velocity of exit by the mode indicated above (37); the discharge, divided by SV, would give nn', and the velocity, divided by v, ($v=\sqrt{2gH}$), would give n'. The series of nn' would show the discharge corresponding to each angle of convergence, and consequently, the angle of greatest discharge; and the series of n' would indicate the progression according to which the velocity increased.

The water-works of Toulouse offered all the desirable facilities for executing such a plan, which I give in some detail. M. Castel, the hydraulic engineer of that city, a thorough experimenter, who introduces the most scrupulous accuracy in all his operations, was pleased, on the invitation of the Academy of Sciences, to undertake the execution.

Experiments of M. Castel.

49. Already, in 1831, with a very small apparatus, and under small heads, he had made a series of experiments, the details and results of which were published in the *Annales des Mines* of 1833.

In 1837, he resumed and considerably extended his works, by the aid of the fine experimental apparatus established at the water-works (see No. 72).

This apparatus consisted principally of a rectangular cast iron box 0^{m}41=1.345 feet long, 1.345 feet wide, and 0^{m}82=2.69 feet high; it received at its lower part, and by means of a great tube, the water coming from a reservoir established more than 29.529 feet above it and kept constantly full; on the front face of the box is a rectangular opening, .459 ft. high by .328 ft. wide, it was closed by a well finished copper plate, to which were fitted additional tubes, in such a manner that their axes were horizontal. When the box was opened at top, the fluid surface could rise there to about .689 ft. above that axis. The upper opening is commonly surmounted with short tubes of .656 ft. diameter, the first of which is .984 ft. high, and the rest 1.64 feet high, so that heads of about .656 ft. 1.64 feet, 3.281 feet, 4.921 feet, 6.562 feet, &c., above the tube subjected to experiment, could be obtained.

By means of two cocks, placed, one at the entrance of the water into the box, and the other on the upper part of the tubes which surmount it, a perfectly constant level was obtained.

The tubes which M. Castel used were of brass, as well turned and polished as possible. He had two series of them; in one, the diameter of the exit was .05086 ft. and the length about .1312 ft.; in the other, the diameter was .06562 ft. and the length .164 ft.

The two diameters of each were measured and re-measured with much care, but the want of an instrument proper to operate accurately with such measures, did not permit of a measurement nearer than 0^{m}00005=0.002 inch ($\frac{2}{1000}$), and such an error might give an error of half a hundredth in the discharges and coefficients.

M. Castel rarely had them so large. He operated under heads of .6562 ft. 1.64 feet, 3.281 feet, 4.921 feet, 6.562 feet, and about 9.843 feet; he measured them with very great exactness.

He then gives, as very exact, the volumes of water obtained in a certain time.

To determine the velocities with which the water passed from the tubes, he erected, 3.74 feet below their axis, a horizontal flooring, in the middle of which was a longitudinal groove .328 ft. broad, into which the jet passed; its range was measured by means of a graduated rule fixed on the flooring and quite near. This range was the ordinate of the curve described by the jet; .374 ft. was its abscissa, and from these two ordinates was deduced the velocity of projection (37). Finally, these velocities could only be taken for heads of 6.562 feet and less; beyond that, the jets were broken, and passed beyond the plane where they could be measured.

I refer, for all the details of the apparatus and the experiments, to the paper inserted in the *Annales des Mines* of 1838, and I confine myself here to communicating the principal results obtained.

50. The same tube, under heads which varied from 0.689 ft. to 9.941 feet, gave discharges always proportional to $\sqrt{H}$, and consequently, the coefficients were sensibly the same. Perhaps they experienced a very slight increase under the head of 9.941 feet. We here give those which were obtained with the pipe of each of the two series which furnished the greatest discharge.

TUBE OF .05085 FEET DIAMETER.			TUBE OF .0656 FEET DIAMETER.		
	COEFFICIENT			COEFFICIENT	
Head, in ft.	of discharge	of velocity.	Head, in ft.	of discharge	of velocity.
.7054	.946	.963	.6923	.956	.966
1.5847	.946	.966	1.5847	.957	.968
3.2547	.946	.963	3.2646	·955	.965
4.8952	.947	.966	4.9149	.956	.962
6.5817	.946	.956	6.5782	.956	.959
9.9414	.947		9.9414	.957	

As to the coefficients of the velocity, it seemed that they would have been sensibly constant, were it not for the resistance of the atmosphere. But this resistance diminishing the range of the jet, and as much more so as the head was greater, there must be, in the calculated coefficients, a diminution varying with the head, although, in reality, there was none in the velocity with which the fluid passed out or tended to pass out. We will now compare together the coefficients, both those of the discharge and of the velocity, obtained with the different tubes of the same series; tubes which, in other respects, differed only in the angle of convergence; for each of them, the mean term was taken between the six or five coefficients which were given under the six or five heads nearly equal to those which are noted in the preceding table.

AJUTAGE .05085 FT. IN DIAMETER.				AJUTAGE .0656 FT. IN DIAMETER.			
Angle of Convergence.		Coefficient of		Angle of Convergence.		Coefficient of	
		Discharge.	Velocity.			Discharge.	Velocity.
0°	0′	0.829	0.830				
1	36	0.866	0.866				
3	10	0.895	0.894	2°	50′	0.914	0.906
4	10	0.912	0.910				
5	26	0.924	0.920	5	26	0.930	0.928
7	52	0.929	0.931	6	54	0.938	0.938
8	58	0.934	0.942				
10	20	0.938	0.950	10	30	0.945	0.953
12	4	0.942	0.955	12	10	0.949	0.957
13	24	0.946	0.962	13	40	0.956	0.964
14	28	0.941	0.966	15	2	0.949	0.967
16	36	0.938	0.971				
19	28	0.924	0.970	18	10	0.939	0.970
21	0	0.918	0.971				
23	0	0.913	0.974	23	4	0.930	0.973
29	58	0.896	0.975	33	52	0.920	0.979
40	20	0.896	0.980				
48	50	0.847	0.984				

It follows, from the facts set down in these columns:

1st. That for the same orifice of exit, and under the same head, starting from 0.83 of the theoretic discharge, the actual discharge gradually increases, in proportion as the angle of convergence increases up to $13\frac{1}{2}°$ only, where the coefficient is 0.95.

Beyond this angle, it diminishes, feebly, at first, as do all variables about the maximum; at 20°, the coefficient is again from 0.92 to 0.93. But afterward, the diminution becomes more and more rapid; and the coefficient would end by being only 0.65, the coefficient of small orifices in a thin side (26), these orifices being the extreme term of converging tubes, that in which the angle of convergence has attained its greatest value, 180°. The angle of greatest discharge will then be from 13° to 14°.

What can be the reason of this? In the conical tubes, the theoretic discharge is altered by two causes, the attraction of the sides, which tends to augment it (43), and the contraction, which tends to diminish it, by diminishing the section of the vein a little below the exit. From the experiments of Venturi (43), it would seem that the fluid vein, at its entrance into a tube, preserved its natural form, that of a conoid of 18° to 20°; so that the nearer the angle of the tube approached such a value, the nearer its sides will be to the vein, at the moment when, after having experienced its greatest contraction, it tends to dilate, and when it is, as it were, left to their attractive action; this action then being stronger, the discharge will be greater. But, on the other hand, already at 10° of convergence, the exterior contraction begins to be sensible and to reduce the discharge; it has reduced it 5 per cent. at 18°; and, after that, it will not be extraordinary that the angle of greatest discharge is found between these two values, about 14°.

The tubes of .0656 ft. diameter at the exit, gave coefficients from one to two hundredths greater than those of the tubes of .0509 ft. An error of 0.004 inch in the estimate of the diameter of the first set, would afford reason, to a great extent, for that difference; and I was inclined to admit a cause of that kind. The tubes of .0509 ft., examined several times since 1831, inspired me with more confidence.

2d. In following the coefficients of the velocity, they are seen, again starting from the angle 0°, to increase like those of the discharge up to near the convergence of 10°; then they increase more rapidly; and beyond the angle of the greatest discharge; while the others diminish, these continue to increase and approach their limit, 1; they are quite near it at the angle of 50°, and even at 40°. The conical tubes, by their different convergence, form a progression of which the first term is the cylindrical tube, and the last is the orifice in a thin side; their velocity of projection, increasing with the convergence, will therefore vary from that of the additional tube to that of the simple orifice, that is to say, from 0.82 $\sqrt{2gH}$ to $\sqrt{2gH}$.

3d. In comparing the coefficients of the discharge with those of the velocity, or their successive values nn' and n', and dividing the first by the second, we shall have the series of n, or the coefficients of the exterior contraction. From the angle 0° to that of 10°, we have sensibly $n=1$, and consequently, there is no contraction; notwithstanding the convergence of the sides, the fluid particles pass out very nearly parallel to the axis. But beyond 10°, contraction is manifested: it reduces the section of the vein more and more, and it would end by rendering it equal to that which passes from orifices in a thin side, as is seen in this table: —

ANGLE.	n
8°	1.00
15°	0.98
20°	0.95
30°	0.92
40°	0.89
50°	0.85
100°	0.65

Experience having taught that cylindrical tubes certainly produce all their effect, as to the discharge, when their length equals at least two and a half times their diameter; by analogy, and for the sake of not complicating our results with the action of the friction of the water against the sides, I have fixed the length of conical tubes at about 2½ times the diameter of exit; thus it was .1312 ft. for those of .0509 ft. diameter, and .164 ft. for those of .0656 ft. diameter. However, to be able to determine the effect of their length, I proposed for the tubes of .0509 ft. diameter, two other series; in one, the common length would have been .0984 ft., which I regarded as the *minimum;* for the other, it would have been .3281 ft., a dimension quite common in practice.

But this work is yet to be done; still, M. Castel has already made some primary trials. For the tubes of .0509 ft. diameter, he took five .1148 ft. long, and, taken together, they gave as the coefficient of discharge, 0.938; next, with a length of .1312 ft., he had as coefficient 0.936; another tube, .0984 ft. long, gave 0.941 instead of 0.938; and one of .0787 ft. indicated 0.931 instead of 0.926; so that here the diminution of length would have a little increased the discharge. But with the tubes of .0650 ft. diameter, the discharge, on the contrary, was increased with the length; the length passing from .1640 ft. to 0.3281 ft., the coefficient under the angle of 11° 52′ was 0.965; under that of 14° 12′, 0.958; and under 16° 34′, 0.950. Thus the effect of the length of tubes is far from being established; its determination demands other series of experiments.

While waiting for M. Castel to perform such experiments, we will assume, for each of the tubes to be employed, provided extraordinary lengths are not taken, the coefficient in the above tables corresponding to the angle of convergence, without fear of introducing any error of moment.

Discharge of great troughs.

51. As to very great conical tubes, or rather, to pyramidal *troughs*, which in mills throw the water on to hydraulic wheels, we have three valuable experiments made by the engineer Lespinasse (*), on the mills of the canal of Languedoc. The troughs there are truncated rectangular pyramids, having a length of 9.5904 ft.; at the greater base, 2.3984 ft. by 3.199 ft.; at the smaller base, .4429 ft. by .6234 ft.

The opposite faces make angles of 11° 38′ and 15° 18′. The head was 9.5904 ft.

The first two of the three experiments, the results of which are here given, were made on a mill of two stones, each having its wheel; in the first experiment, the water was let on to only a single wheel; in the second, it was let on to two at a time.

Discharge.	Coefficient.
cubic ft.	
6.7667	0.987
6.6926	0.976
6.7138	0.979

We see how little such tubes diminish the discharge; the discharge given is only one or two hundredths less than the theoretic discharge.

4. Conical Diverging Tubes.

Increase of discharge due to these Ajutages.

52. Of all tubes, those which give the greatest discharge are truncated cones, fitted to a reservoir by their smaller base, and of which the opening for exit is consequently greater than that of entrance. Although

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* Anciens Mémoires de l'Académie de Toulouse. Tom. II. 1784.

very little used, they present phenomena of too much interest to be passed by.

Their property of increasing the discharge was known to the ancient Romans; some of the citizens, to whom was granted a certain quantity of water from the public reservoirs, found by the employment of these tubes, means of increasing the product of their grant; and the fraud became such, that a law prohibited their use; at least, they could not be placed within 52½ feet from the reservoir.

Bernoulli had studied and subjected to calculation their effects; in one of his experiments, he found the real velocity at the entrance of the tube greater than the theoretic velocity, in the ratio of 100 to 108; but to Venturi is principally due our knowledge of the products they can give.

Experiments of Venturi.

Fig. 13.

53. The tubes which he used had a mouth-piece ABCD presenting nearly the form of the contracted vein; AB=.1332 ft., and CD=.1109 ft.; the body of the tube CDFE varied in length and flare, the flare being measured by the angle comprised between the sides EC and FD sufficiently prolonged. These tubes were fitted to a reservoir kept constantly full of water; the flowing took place under a constant head of 2.8873 ft., and the time necessary to fill a vessel of 4.8384 cubic ft. was counted as in the experiments of the same author which we have already mentioned.

I give, in the following table, the result of the principal observations, after having remarked that the time corresponding to the theoretic velocity was 25″49:—

AJUTAGE.		Time of running.	Coefficient.	OBSERVATIONS.
Flare.	Length.			
	feet.			
3° 30′	.3642	27″5	0.93	
4 38	1.0959	21	1.21	Jet very irregular.
4 38	1.5093	21	1.21	Jet did not fill the ajutage.
4 38	1.5093	19	1.34	To fill ajutage a projecting body introduced.
5 44	.5775	25	1.02	
5 44	.1936	31	0.82	Exit mouth=that of entrance.
10 16	.8662	28	0.91	Jet did not fill ajutage.
10 16	.1476	28	0.91	Jet very regular.
14 14	.1476	42	0.61	Jet detached from sides.

Venturi concluded from his experiments, that the tube of the greatest discharge ought to have a length nine times the diameter of the smaller base, and a flare of 5° 6′; figure 13 represents it; it would give, adds the author, a discharge 2.4 times greater than the orifice in a thin side, and 1.46 times greater than the theoretic discharge. Moreover, he observes, that the dimensions of the tube should vary with the head.

54. Of all the experiments which he made on diverging tubes, and for which I refer to his *Recherches Expérimentales*, I shall cite only the following:

To one of the above-mentioned tubes, that which gave 4.8384 cubic feet in 25″, he fitted three tubes, and plunged them into a small bucket filled with mercury; the first at the origin D of the tube; the second at one third of its length, and the third at two thirds. The mercury was raised respectively .3937 ft., .1509 ft., and .0518 ft.; this would be equivalent to columns of water 5.348 feet, 2.067 feet, and .7054. According to the theory of Bernoulli, the pressure at the point of greatest contraction D, where the velocity is $\frac{42}{25}\sqrt{2g\times 2.8873}$ ought to have been $2.8873 - 2.8873\,(\frac{42}{25})^2 = -5.2618$ ft.; the experiment of Venturi gave -5.348 feet.

Fig. 14.

Experiments of Eytelwein.

55. Eytelwein also used diverging tubes in experiments, the results of which are directly interesting in practice.

He took a series of cylindrical tubes .0853 ft. diameter, and of different lengths, which he successively fitted to a vessel full of water; at first separate; then applying to the front extremity the mouth piece M, which had nearly the form of the contracted vein; then applying to the other extremity the tube M, of the form recommended by Venturi; lastly applying at the same time the mouth-piece and the tube.

Fig. 15.

The flowing took place under a mean head of 2.3642 feet.* The principal results obtained are given in the following table:

Length of Tube.	Coefficient of discharge of the tube, only according to		Discharge of the tube alone being 1, ' Discharge	
	Experiment	Formula of Conduits.	With mouth piece.	With Ajutage.
feet.				
.0033	0.62	0.99		
.0853	0.62	0.97	1.56	
.2559	0.82	0.95	1.15	1.35
1.0302	0.77	0.86	1.13	1.27
2.0605	0.73	0.77	1.10	1.24
3.0907	0.68	0.70	1.09	1.23
4.1176	0.63	0.65	1.09	1.21
5.1479	0.60	0.61	1.08	1.17

These experiments show:

1st. The rate according to which the length of the tubes diminishes the discharge; and this, up to a point where the formula for the motion of water in conduit pipes may be applied. The numbers of the third column indicate that this application can take place, for small tubes, those under .0984 ft. diameter, when their length exceeds 6.562 feet. These experiments thus in part fill up the void which existed in our knowledge of additional tubes and conduit pipes.

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* Here the head was not constant. At each experiment, the vessel was filled up to 3.0841 feet above the orifice, and the fluid was suffered to fall until the surface was only 1.7389 feet above the orifice; the constant head, which would have given the same discharge in the same time, would have been 2.3642 feet. Let, generally, H′ be that constant head; H the head of the reservoir at the commencement of the flowing, and h that at the end, we shall have $H'=\left(\frac{H-h}{2(\sqrt{H}-\sqrt{h})}\right)^2$.

The occasion to make use of this formula will be presented quite often in practice.

2d. That the increase of the discharge proceeding from the flare given to the mouth of entrance of pipes, diminishes in proportion as their length is greater. It were desirable that these experiments had been carried further, for the purpose of knowing what would have been the result of this diminution in large conduits; until this is done, and however small may be the good effect of the flaring at the entrance, it is proper not to neglect it.

3d. The effect of the flaring at the exit also diminishes in a ratio more rapid still, in proportion as the pipes increase in length. Eytelwein having taken one 20.6 feet long and of .0853 ft. diameter throughout, found no difference in the discharge, whether he did or did not use the tube with flaring end.

On fitting this tube immediately to the reservoir, the discharge was 1.18, the theoretic discharge being 1.

On fitting it to the mouth-piece, but without the intermediate tube, it rose up to 1.55. The mouth-piece alone gave only 0.92; so that the effect of the tube N added to the mouth-piece M, was to augment the discharge in the ratio of 0.92 to 1.55, or of 1 to 1.69.

Measure of the force of the Ajutage.

56. Venturi had that of 19″ to 42″, or 1 to 2.21. In the two experiments which furnished the terms of this last ratio, the velocities of the water at the passage through the section CD (Fig. 13) were therefore as 1 to 2.21; and consequently, the heights due as 1 to 4.89, since they follow the ratio of the squares of the velocities.

In the experiment which gave the term 1, that where the mouth-piece M alone was used, the actual velocity, which was obtained by dividing the discharge by the section, was 11.9297 feet; it corresponds to a generating head of 2.2114 feet. The head corresponding to the velocity in the second experiment will then be $2.2114 \times 4.89 = 10.8137$ feet; whence it follows, that the discharge was equal to what would have occurred, if, instead of adding the tube N to the mouth-piece M, the water had been raised in the reservoir, above the level which it had during the flowing, $10.8137 - 2.2114 = 8.6023$ feet. Thus, the accelerating effect of the velocity due to the diverging tube is measured by a column of water 8.6023 feet; this is more than a quarter of the weight of the atmosphere. This is a very considerable effect for a force which seems quite small; for we see no other physical cause of the augmentation in the discharge produced by the tube, than the action of the sides, and, in short, the molecular attraction.

ARTICLE THIRD.

On flowing under very small heads.

57. When the head over the centre of the orifice is very small compared to the height (vertical dimension) of that orifice, the *mean* velocity of the different lines of the fluid vein, that is to say, the velocity which, being multiplied by the area of the orifice, gives the discharge, is no longer that of the central line. It differs from the velocity of the central line as much more as the head is smaller; it will be about a hundredth less if the head is equal to the height, and a thousandth less if the head is three times (3.2) greater than the height. Let us see what theory teaches us in this respect: and first, the law which it indicates for the velocity of the fluid lines, in proportion as the point from which they issue is lower than the level of the reservoir.

58. Let a vessel be filled with water up to A; upon its face AB, which we will suppose vertical for greater simplicity, imagine below each other, a series of small holes, of which B will be the lowest. Designate by H the height AB; the velocity of the line passing out at B will be $\sqrt{2gH}$ (8); and if BC be made equal to that quantity, it will represent that velocity. For every other point P, below the level of the reservoir, the distance AP or x, the line PM, which would represent the velocity of the fluid at its exit from that point, would

Velocity of any filet.

Fig. 16.

be $\sqrt{2gx}$, and calling it y, we should have $y=\sqrt{2gx}$. If through the extremity of all these lines PM, a curve be made to pass, they will be its ordinates, and the heights AP or x will be its abscissas; and since $y^2=2gx$, this curve will be a parabola having $2g$ or 64.364 feet for its parameter.

Thus the velocity of a fluid line passing from a reservoir at any point, is equal to the ordinate of a parabola, of which twice the action of gravity is the parameter, the distance of this point below the level of the reservoir being the abscissa.

Discharges. 59. Suppose now, that instead of opening a series of small holes on the face AB, there had been perforated in it, from top to bottom, a rectangular slit, of the breadth l; let us find the expression of the discharge.

Divide this opening, in thought, by means of horizontal lines very near each other, into a series of small rectangles. The volume of water which will pass from each of these in a second, or its discharge, will evidently be equal to the volume of a prism which shall have for its base the small rectangle, and for its height the corresponding ordinate. The sum of all these little prisms, or the total discharge, will evidently be equal to another prism, having for its base the parabolic segment ABCMA, and for its height or thickness, the width of the slit. Now, according to a property of the parabola, this segment is two thirds of the rectangle ABCK, whose surface is $AB \times BC = H \times \sqrt{2gH}$. Thus, the discharge through the rectangular opening of which H expresses the height and l the breadth, is

$$\tfrac{2}{3}\, l\, H \sqrt{2gH}.$$

60. We now seek the discharge through a rectangular orifice open on the same side, but from B to D only, and having the same breadth l; call h the head AD, on

the upper edge of the orifice; the discharge of the slit which we suppose from A to D would also be $\frac{2}{3}\,l\,h\,\sqrt{2gh}$. Now, it is evident that the discharge through the rectangular orifice of which BD is the height, will be equal to the difference of the discharges through the two slits, and which consequently will be

$$\tfrac{2}{3}\,l\sqrt{2g}\,(\mathrm{H}\sqrt{\mathrm{H}}-h\sqrt{h}).$$

The first elements of the integral calculus lead in an extremely simple manner to this expression. But I repeat, this treatise is not a work of mathematics; and from its nature, it appeared to me, that synthetic demonstrations, keeping constantly before the eye the object in question, were to be preferred.

Mean Velocity.

61. Let us revert to the mean velocity; and first to that which we have when the slit is quite open.

Let G be the point from which the fluid line animated with this velocity proceeds; if we make $\mathrm{AG}=z$, it will be $\sqrt{2gz}$; being multiplied by the area of the slit $l\times\mathrm{H}$, it must give the discharge. But we have seen that this discharge was also expressed by $\frac{2}{3}\,l\,\mathrm{H}\sqrt{2g\mathrm{H}}$; we shall then have, $l\,\mathrm{H}\sqrt{2gz}=\frac{2}{3}\,l\,\mathrm{H}\sqrt{2g\mathrm{H}}$; whence, $z=\frac{4}{9}\,\mathrm{H}$, and consequently, $v=\sqrt{2g\frac{4}{9}\mathrm{H}}=\frac{2}{3}\sqrt{2g\mathrm{H}}$.

Thus, the mean velocity will be two thirds of the velocity of the lower line. In fact, GH, which represents the first, is, according to the above-mentioned property of the parabola, two thirds of BC, which represents the second.

For the rectangular orifice of which BD or $\mathrm{H}-h$, is the height, z' being the height due to its mean velocity, we should in like manner have

$$(\mathrm{H}-h)\;l\sqrt{2gz'}=\tfrac{2}{3}\;l\sqrt{2g}\;(\mathrm{H}\sqrt{\mathrm{H}}-h\sqrt{h});$$

whence,

$$z'=\tfrac{4}{9}\left(\frac{\mathrm{H}\sqrt{\mathrm{H}}-h\sqrt{h}}{\mathrm{H}-h}\right)^2.$$

Example. There is a prismatic basin, at the bottom of which is a rectangular orifice .82 ft. base, and .3937 ft. height; and during the flowing, the fluid surface is constantly .7218 ft., above the lower edge of the orifice. We then have, H=.7218; h=.7218 —.3937=.3281; thus

$$z'=\frac{4}{9}\left(\frac{.7218\sqrt{.7218}-.3281\sqrt{.3281}}{.7218-.3281}\right)^2=.48 \text{ ft.};$$

consequently, the mean velocity will be $\sqrt{2g\times.48}$=5.558 feet.

The head should be measured from full Reservoir.

62. I make here an observation which applies more particularly to the case of small heads.

During the flow through an orifice, the surface of the fluid in the reservoir, starting from certain points, is curved, and inclines towards the side in which the orifice is pierced; so that the height or vertical distance of the surface, above any part of the orifice, is greater on the up-stream side of the points where the inflection begins, than near to and touching the side. It is the first of these heights or heads which must always be introduced into the formulas of flowing; we shall see reasons for it hereafter (68 and following). The distance between the orifice and the line where the fluid surface joins the side is very often introduced (into the formulas); from this, there results an error in deficiency, in estimating the discharges which, in some cases, very rare, to be sure, may extend even to a tenth of the discharge.

Such errors diminish when the head increases; and according to the experiments of MM. Poncelet and Lesbros, who have also fully explored this question, they will be insensible when the heads exceed .4921 or 6562 ft., say six or eight inches. Yet, in very great orifices, the depression of the surface is still perceptible; I have seen it from 1½ to 2 inches against the sluice gates of the canal of Languedoc, when the two paddle gates were open.

Orifices not rectangular.

63. If the orifice had a figure different from the rectangle, the expression of the mean velocity, and consequently of the discharge, would be more complicated; its determination would become a problem of analysis of little utility in practice, where great orifices are almost always rectangular. The solution of these problems can be seen in the *Architecture Hydraulique* of Belidor, and in the *Hydrodynamique* of Bossut. I will now limit myself to that which concerns the circle. Designating by d the diameter, by h the head above the centre, we have for the expression of the discharge, $\pi' d^2 \sqrt{2gh}\,(1-\frac{d^2}{128h^2}-\frac{d^4}{3277h^4}-$ &c.); this discharge is that which corresponds to the velocity of the central line diminished in the ratio indicated by the complex factor.

Coefficient of Reduction.

64. The discharges, of which we have just given the expression, are theoretic discharges; for reducing them to actual discharges, it is necessary to multiply them by the coefficients deduced from experiment.

These, also, will be furnished us by MM. Poncelet and Lesbros. I indicate them in the following table:

Head upon the centre.	HEIGHT OF ORIFICES.					
	.6562 ft.	.3281 ft.	.1640 ft.	.0984 ft.	.0646 ft.	.0328 ft.
feet.						
.03281						0.712
.0656				0.644	0.667	0.700
.0984				0.644	0.663	0.693
.1312			0.624	0.643	0.661	
.1640			0.625	0.643	0.660	
.1968		0.611	0.627	0.642		
.2625		0.612	0.628	0.640		
.3281		0.613	0.630	0.638		
.3937	0.592	0.614	0.631			
.4921	0.597	0.615	0.631			
.6562	0.599	0.616	0.631			
.9843	0.601	0.617				
1.6404	0.603	0.617				
3.2809	0.605					

65. The numbers above are the true coefficients of the contraction of the fluid vein, or the coefficients of the reduction of the theoretic discharge to the actual discharge; for theory gives no other general formula for flowing through orifices than $\frac{2}{3} l \sqrt{2g} (H\sqrt{H}-h\sqrt{h})$.

That which was established (15) $S\sqrt{2gh}$; where $h'=\frac{1}{2}(H+h)$ applies only to particular cases, very frequent, to be sure, where h' is three or four times greater than $H-h$. In the other cases, it is erroneous, and the coefficients which are adapted to it and which it has served to determine, are erroneous also; they are the coefficients found above the transverse lines which divide the columns. (The coefficients below the lines, although determined by the aid of that formula, are accurate, coinciding with those obtained by the general formula). Finally, in the first, $mS\sqrt{2gh'}$, the error of the coefficient m is compensated by the error of the formula, and the discharges which it gives are sensibly identical with those of the other; and as it is, besides, more simple, it is commonly employed in all cases.

Examples. 66. Example. What would be the discharge of a rectangular orifice .9843 ft. wide and .49215 ft. high, under a head of only .16405 ft. on its upper edge? Here $H=.16405+.49215=.6562$ ft. and $l=.9843$ ft. The head on the centre, therefore, is .410125 ft.; the coefficient which corresponds to this head, according to the above table, is nearly .603; a mean term between .593 and .614. Thus the discharge will be $\frac{2}{3}\times.603\times.9843\times.8.02052\,(.6562\sqrt{.6562}-.16405\sqrt{.16405})=1.476$ cubic feet. The ordinary formula, with its coefficient .592, taken from the ordinary table in section 26, would have given $.592\times.9843\times.49215\times8.02052\sqrt{.410125}=.1473$ cubic feet.

67. We have a circular vertical orifice of .0888 ft. diameter, with a head of .0592 ft. above the centre. What will be the discharge? Here $d=.0888$ ft., $h=.0592$ ft.; so that the expression of No. 63 becomes $.012086\left(1-\frac{1}{56.89}-\frac{1}{647.3}\right)=.011863$ cubic feet. This is the theoretic discharge; and to have the actual discharge, it is necessary to multiply it by the coefficient indicated in the table of No. 64. We there find 0.667 for an orifice of .6562 ft. diameter, under a head .0656 ft. (or of .0592); under this same head, we then also have 0.644 for an orifice of .0984 ft. from which we shall take 0.650 for the orifice of .0888 ft. The actual discharge will then be $0.65\times.011863=.00771$ cubic feet.

Experiment gave Mariotte .008077 cubic feet, and Bossut .007332 (with one line=.0888 in. head measured directly above the summit of the orifice); the result of calculation would thus be a mean term between the results of experiment.*

The discharge just determined, that obtained through an orifice of one French inch diameter, under the head of one line ($=\frac{1}{12}$ in′) taken immediately above the summit of that orifice, is the *pouce d'eau* of water-works agents, a measure to be investigated hereafter (206). Mariotte, in the work which he wrote more than one hundred and fifty years ago, to fix its value, observed, that to obtain a height of water of one line immediately over the orifice, there must be a height of two lines in the full reservoir, and consequently, eight over the centre.† Thus the phenomenon of the inflexion of the fluid surface toward the orifice, and its influence upon the discharge, were well known to him.

THE FLOWAGE OF WATER OVER WEIRS.

Nature and formulæ of Flowage.

68. If, at the upper part of the sides of a basin, a rectangular opening be made, with a horizontal base, the water of the basin, which we suppose kept constantly full, will flow out in the form of a sheet, over this base or sill. To such an opening is given the name of *weir*; and we also extend the name to dams which entirely close up the bed of a stream of water, in such a manner that the water, on meeting with them, is obliged to rise up and pass over the top or crown.

Fig. 17.

The surface of the water, before arriving at the weir, and in starting from a point C, which is at a small distance from it, is inclined along the arc CD; so that its height immediately above the sill is no longer AB, but only BD.

69. Conformably to the ordinary theory, it is first admitted that the particles which follow the curve CD

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* The Pouce d'eau of France was determined by Prony to be 19.1953 cubic metres of water in the twenty-four hours; this is equivalent to .0078463 cubic feet per second.

† Traité du mouvement des eaux. III.e partie, &c.

have, on arriving at D, the same velocity as if they had fallen freely from the height AD, and that the particles beneath go out also with a velocity due to their vertical distance from the point A. We find, then, that for the velocity of issue of the different fluid threads, for their number dependant on the height BD, and consequently for their discharge, exactly the same case as if we had a rectangular orifice closed by an upper edge which might be at D, and as if the fluid were extended without inflexion up to A. Therefore, representing by Q the discharge or volume of water flowing in one second, by l the breadth of the weir, by H and h the heads, one of the lower edge and the other of the upper edge, and by m the coefficient of reduction of the results of theory to those of experiment, we have established (as at No. 60),

$$Q=\tfrac{2}{3}\sqrt{2g}\, m\, l\, (H\sqrt{H}-h\sqrt{h}).$$

70. However natural this mode of treating the subject may appear, yet facts have shown that the discharges were more exactly given by a calculation based on the supposition that the flowing occurred under the whole height AB, the fluid always extending, without inflexion, up to A. We then find ourselves in the case explained at No. 59: $h=0$ and

$$Q=\tfrac{2}{3}\sqrt{2g}\, m\, l\, H\sqrt{H}=5.3484\, m\, l\, H\sqrt{H}.$$

The flowing over weirs would be therefore only a particular case of flowing through orifices in general, that in which the head upon the upper edge is nothing. MM. Bidone and Poncelet had already shown that it was so, and that the coefficient m, which answered for ordinary orifices, was suited for weirs also, when the flowing occurred under analogous circumstances.

71. In establishing the above two formulæ, we have

implicitly admitted that the fluid was at rest above the weir, or rather, above the point where the surface begins to incline towards the sill; but very often, the water comes to this point with a certain velocity.

In this case, proceeding as we have already done in the case of orifices, properly so called (38), we must add the generating height of the velocity of arrival, to the height due to the velocity of flowing for a fluid at rest, which is in this instance $\frac{4}{9}$ H only (61). Let u be that velocity, $.0155u^2$ will be the generating height, and we shall have for the real velocity at the exit, $\sqrt{2g\ (\frac{4}{9}\mathrm{H}+.015536u^2)}$, which is reduced to 5.3484 $\sqrt{\mathrm{H}+.034956u^2}$, and consequently,

$$\mathrm{Q}=5.3484\ ml\mathrm{H}\ \sqrt{\mathrm{H}+.034956u^2}.$$

The quantity u represents the mean velocity of the section of water which goes to the weir; its exact determination is nearly impossible, but as its value will differ but little from that of the velocity at the surface, a velocity which we obtain quite easily by means to be investigated hereafter, we shall admit the equality, and then, modifying the value of the coefficient to be determined by observation, if we designate by m' this new coefficient, and by w the velocity at the surface, we shall have

$$\mathrm{Q}=5.3484\ m'l\mathrm{H}\ \sqrt{\mathrm{H}+.03495w^2}.$$

Experiments of M. Castel.

72. Let us put these formulas to the test of experiment.

The expression of the discharge includes two variables, the breadth of the weir, and a function of the velocity or of the head. In order that these formulæ be well established, it will be necessary that the discharge be exactly proportional to each of the variables; then only the coefficient would be constant. The degree of its constancy will thus be the mark, as it were, the measure, of its being well established. The numerous experiments which M. Castel, engineer of the Toulouse water-works, made in 1835 and 1836, at the water-works of that city, with extraordinary care and exactness, inform us with regard to this constancy and these proportionals. (1)

The water-works of Toulouse, or building enclosing the hydraulic machines which raise the waters destined for a hundred and more fountains of that city, was 61.027 feet in diameter, and 49.215 feet in height, of which 26.25 feet were beneath the pavement surrounding it.

In the middle is raised a tower 26.25 feet in diameter, and 45.93 feet high; in the upper part is a cistern, into which all the water is conveyed; the quantity of which is at a mean of 45 litres=9.9 gallons per second, and it can easily be raised to 60=13.2 gallons. At the foot of the tower, and on the body of the building, extends a terrace 15.75 feet broad, which presents a very commodious place for observations; and consequently, they permanently established here the great apparatus for hydraulic experiments, already mentioned (49).

To this apparatus, M. Castel added a second for weirs. It was a wooden box or canal, rectangular, 19.686 ft. long, 2.428 ft. broad, and 1.805 ft. deep; at one end it receives the water of the first apparatus, and to the other are fitted thin plates of copper, in which the weirs were opened.

The breadth of these varied gradually from .03281 ft. to 2.428 ft.; the sill was constantly at .558 ft. above the bottom of the canal. The water that flowed from it was received at pleasure, and for a certain time, in a second box lined with zinc, with a capacity of 113.024 cubic feet; this was the gauging basin; it had been measured with the greatest care. The time occupied by the water in arriving at a certain height was measured by a time-piece, marking quarter seconds.

The heads or heights of water in the canal above the sill of the weirs were increased gradually, from .09843 ft. to .3281 ft., and even to .78744 ft. for narrow weirs. The most important and difficult point in the experiment, was to measure the heads exactly. To accomplish this, M. Castel fixed upon the top and middle of the canal, parallel to its length, a ruler, which he kept quite horizontal, and which bore, at intervals of .16405 ft., ten vertical rods of brass divided into millimetres, and each capable of being raised and lowered in a groove, on which was a vernier indicating tenths of millimetres. When he wished to make an experiment, after having admitted a suitable quantity of water into the canal, and satisfied that the regime was properly established, he lowered the rods and placed their points as exactly as possible in contact with the fluid surface. Then subtracting their length from the vertical distance between the ruler and sill, he had the ordinates of the curve described by the fluid particles passing directly to the middle of the weir. These ordinates increased in proportion as they were distant from the weir; but soon, at .6562 ft., or .9843 ft., or 1.3124 ft., the increase became sensible, and they had the greatest of the ordinates, or the head properly so called, H; the smallest, that raised vertically above the sill, was $H-h$, or the thickness of the fluid sheet at the moment of its passage over the sill. After having made all the observations he could upon the canal of 2.428 ft. broad, M. Castel provided himself with one 1.1844 ft. broad, by narrowing the first by means of two plank partitions, only 7.35 ft. long. At the entrance of this small canal, which was placed in the middle of the large one, there was, during great discharges of water, a slight fall, which could have produced some small modifications upon the results which might have been obtained, if the partitions had been prolonged to the extremity of the large canal.

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(1) For the details of these experiments, the reader is referred to Mémoires de l'Académié des Sciences de Toulouse, t. IV. 1837.

Upon both, M. Castel effected a long series of experiments. Each observation was repeated once or twice; in all, there were 494. For each, the values of Q l and H being immediately given by experiment, it was easy to deduce from them the value of the coefficient m of the formula

$$Q=5.348\, m\, l\, H \sqrt{H}.$$

The mean values obtained for each head and breadth of weir are given in the following tables. There were no observations for the cases corresponding to the gaps which most of the columns present. The heads and breadths which are there noted in an exact number of hundredths, are not entirely those of the experiments. It was not possible to obtain from the workman breadths of a precise number of hundredths; they differed but very slightly from the truth. As to the heads, it would have required too many adjustments, and too much time, to get rigorously at a given value; but a close approximation was made. Hence, the differences between the values of the coefficients, with the heads and breadths really employed, and with those which have been admitted, are so small, that by means of the mode of interpolation used, we have the coefficients of the tables as exact as though they had been directly given by experiment. We shall, however, find them in the memoir of M. Castel, with the breadths and heads really observed.

CANAL 2.428ft. BROAD.

COEFFICIENTS,
BREADTH OF WEIR BEING IN FEET.

Head upon the sill	2.428 ft.	2.231 ft.	1.969 ft.	1.640 ft.	1.312 ft.	.984 ft	.656 ft	.328 ft	.164 ft	.098 ft	.065 ft	.033 ft
feet.												
.787								.595	.615		.639	
.722								.594	.614		.639	
.656							.596	.594	.614	.629	.640	.670
.590							.595	.594	.613	.628	.641	.672
.525							.595	.592	.613	.628	.642	.674
.454						.603	.593	.592	.612	.628	.643	.675
.394					.621	.604	.592	.591	.612	.628	.645	.678
.328		.657	.644	.631	.621	.604	.593	.591	.612	.627	.648	.687
.262	.662	.656	.644	.632	.620	.606	.595	.592	.612	.627	.652	.698
.197	.662	.656	.645	.632	.622	.610	.604	.595	.612	.628	.658	.713
.164	.662	.656	.644	.633	.626	.616	.611	.597	.613	.629	.663	
.131	.662	.656	.645	.636	.632	.623	.619	.604	.614		.669	
.098	.663	.660	.651	.642	.636	.631	.624	.618				

CANAL 1.1811ft. BROAD.

COEFFICIENTS,
BREADTH OF WEIR BEING IN FEET.

Head upon the sill	1.184 ft	.984 ft	.656 ft	.328 ft	.302 ft	.260 ft	.164 ft	.098 ft	.066 ft	.033 ft
feet.										
.787				.619			.624	.629	.647	.666
.722				.615	.613	.617	.620	.627	.646	
.656				.611	.608	.614	.618	.626	.645	.667
.590			.633	.608	.606	.610	.616	.626	.644	
.525			.628	.605	.603	.608	.615	.625	.644	.668
.454		.678	.624	.603	.601	.605	.614	.624	.644	
.394	.700	.666	.620	.600	.599	.603	.614	.623	.646	.674
.328	.684	.656	.617	.598	.598	.600	.614	.624	.648	
.262	.672	.652	.616	.599	.597	.599	.613	.624	.654	
.197	.669	.652	.617	.600	.597	.600	.613	.626		
.164	.667	.653	.620	.605	.604		.614			
.131	.668	.653	.624	.613	.611		.613			
.098	.670	.665	.632	.628	.625					

Usual Formula.

73. Let us analyze, first, the most simple and most frequently employed of the formulæ, $Q=5.3484\, l\, H \sqrt{H}$.

Ratio of discharge to heads.

Let us examine, in the first place, up to what point the discharges Q are proportional to the function $H\sqrt{H}$ of the head. For this purpose, take the twenty-two series of discharges obtained, each with the same breadth of weir, but under different heads, (recollecting that the discharges were directly given by experiment, and that we can, besides, reproduce them by means of the above formula, by assigning to each their respective coefficients noted in the tables). Reduce the discharges of each series to what they would have been, if one of them, that obtained under the head of .2625 ft., for example, had been taken for unity. Reduce, in like manner, the series of values of $H \sqrt{H}$, and bring together all these series, as has been done for the three concerning the discharges; the first two have been given on the canal of 2.428 ft., through weirs 1.969 ft. and .328 ft. broad; the third belongs to the canal 1.1844 ft. broad, with a weir of .164 ft.

Heads, in feet.	Series of Discharges.			Series of	
	1	2	3	$H \sqrt{H}$	$H \sqrt{H}$ $-h \sqrt{h}$.
.6562		3.96	3.98	3.95	4.01
.5906		3.38	3.39	3.38	3.42
.525		2.83	2.84	2.83	2.87
.459		2.31	2.32	2.31	2.34
.394		1.83	1.84	1.84	1.86
.328	1.40	1.39	1.40	1.40	1.41
.262	1.00	1.00	1.00	1.00	1.00
.197	0.650	0.652	0.650	0.650	0.643
.164	0.494	0.498	0.495	0.494	0.486
.131	0.354	0.381	0.354	0.354	0.345

There results from the comparison of the twenty-two series of discharges among themselves, and with the series of $H\sqrt{H}$, 1st. That, above the head of .1969 ft., or even of .164 ft., leaving out some great heads, the differences between numbers of the same horizontal line are very small, they do not exceed a hundredth; thus, confining ourselves to all the exactness which is required in practice, they may be regarded as nothing; and the ratio between the discharges is the same as that between the correspondent values of $H\sqrt{H}$.

2d. That, for heads of .164 ft. and lower, the discharges decrease in a less ratio than $H\sqrt{H}$, and as much less as the head is smaller, but only in medium breadths; for when they are very small or approach that of the canal, the equality recurs. Such irregularities, and some other reasons, should cause us to avoid these small heads in practice.

3d. In some great heads, especially with broad weirs, we still see the discharges increase in a less ratio. This fact, which was almost insensible in the canal of 2.428 ft., became prominent in that of 1.1844 ft., when the water with those heads and those breadths came to the weir with a great velocity. Now, in these cases—and they present themselves always when the fluid section ($l \times H$) at the passage of the weir exceeds the fifth part of the section of the current in the canal—the discharges should not increase as $H\sqrt{H}$, but as $H\sqrt{H+.03495w^2}$; and it is no longer the ordinary formula, but that given in No. 71, which we must then use.

Hence it results, that so long as we have the case of dams, properly so called, those where the water in the upper level experiences a retardation which destroys or remarkably lessens the velocity of arrival, Q will be very sensibly proportional to $H\sqrt{H}$; and in this respect, the formula is well established.

Ratio of discharge to width of weir.

74. The formula will not be quite so well established in what concerns the breadth of weirs; the discharges in this case will no longer be so near the ratio of breadths, however natural it may appear to suppose so. Starting from the breadth of the basin, they will diminish with the breadth of the weir, but with greater rapidity up to a certain point; beyond which, they will, on the contrary, diminish less rapidly. The opposite columns will fix our ideas on this subject. On the canal of 2.428 ft., we have twelve breadths, which are to each other as the numbers placed in the first column; in the second, we see the progression which the corresponding discharges follow — discharges obtained under heads of from .1968 ft. to .3281 ft. For the canals of 1.1811 ft., where we have ten breadths, we have here noted those only which have something analogous to those in the other canal. These series of ratios show that, in the two canals, the discharges follow the same law comparatively to the breadths of the weirs, but to the breadths relative to that of their respective canal, and not to the absolute breadths.

CANAL OF			
$2.428^{\text{ft.}}$		$1.1811^{\text{ft.}}$	
Breadth.	Discharge.	Breadth.	Discharge.
1000	1000	1000	1000
919	911		
811	788	831	807
676	645		
540	507	554	507
405	371		
270	243	277	246
135	121	138	125
68	62		
40	40		
27	27		
13	14		

Coefficients.

75. Since, extremes being omitted, the discharges are sensibly proportional to $H\sqrt{H}$, for the same breadth of weir, the coefficients ought to be nearly equal, and they are so in fact, as we see in the tables which we have given (72). In strict rigor, and taking the coefficients of the same vertical column in the

tables, we shall see them, starting from high heads, decreasing, very slightly, to be sure, in most cases, down to a certain head, beyond which they will augment rapidly; there will then be at this head, which generally will be near .3281 ft., a *minimum.*

Since, the heads remaining the same, the discharges decrease, at first more and then less rapidly than the breadths of the weirs, it follows, that under the same head, reckoning from the breadth of the basin, the coefficients will go on diminishing up to a certain point, beyond which they will increase. Here will then still be a *minimum*, and it will take place when the breadth of the weir shall be nearly a quarter of that of the basin.

76. Thus, in the horizontal lines, as in the vertical lines of the tables of coefficients, we have a *minimum;* in each table there will then be a common *minimum.* Near this, and up to a certain point, according to the general law, as according to the result of experiments, the variations are very small; the coefficients will vary very little from each other, and they may be regarded as constants. But beyond that distance, it is no longer so, and the differences may be quite considerable; they exceed one eighth in the tables, so that the discharge by weirs would not be exactly given, with a constant numerical coefficient, by an expression of the form $l H \sqrt{H}$; in mathematical rigor, such an expression would not be admissible. In practice, we could not make use of it, except by aid of tables of coefficients very extended, the reduction of which would require many hundreds of experiments.

Yet the study of the progress which the coefficients follow, affords the means of contracting this great field, and of reducing to a small number of quite simple rules, the determination of those which agree with the different cases which generally occur in practice.

(See further on details of this progress of the coefficients, in the papers of M. Castel, and in the notes which I have added there.)

Coefficients and formulas to be used.

77. We have seen (73), that the expression $l\,H\sqrt{H}$ must not be applied, on the one hand, when the heads are below .1968 ft.; on the other, when the heads multiplied by the breadth of the weir exceed the fifth of the section of the water in the canal. Between these limits, the above expression can be employed, with a coefficient, variable indeed, but which will vary only with the breadth of the weir.

To reckon from that of the canal, the coefficients diminish with the width of the weir, until it be about one quarter of the first, and then they increase, although the widths continue to diminish (75); and, what is very remarkable, the diminution of the coefficients follows that of the *relative* breadths of the weir compared to that of the canal, whilst the increase which follows depends only on the *absolute* breadths.

We have, consequently, four cases to be distinguished relatively to the coefficients to be employed.

1st. Near the *minimum*, which we have just indicated, their variations are inconsiderable; according to the experiments made at the water-works of Toulouse, from a breadth of weir almost equal to a third of that of the canal supposed to exceed .984 ft., to an absolute breadth of .1640 ft., the coefficients will vary only from .59 to .61. Taking the mean term, remarking that $5.3485 \times .60 = 3.209$, we shall have, between the limits which we have just indicated, $Q = 3.209\,l\,H\sqrt{H}$. This formula furnishes the best mode of gauging small courses of water; we shall recur to it in treating of this gauging (159).

2d. When the breadth of weir is at its *maximum*, i. e., equal to that of the canal, and it is thus in case of a dam properly so called, the coefficients present a remarkable constancy. M. Castel, in his experiments on the canal of 2.428 ft., with a dam .5576 ft. high, had no difference between the coefficients obtained under heads which varied from .0984 ft. to .2624 ft. (72); and with a dam of .738 ft., the coefficients varied only from .664 to .666, for heads from .10168 ft. to .2428 ft. Taking a mean, he had .665; and since $5.3485 \times .665 = 3.5567$, designating by L the breadth of the canal or length of the dam, we shall have,

$$Q = 3.5567\ L\ H\ \sqrt{H}.$$

This formula will also be employed with advantage in certain cases, even on great water courses, and with heads of from .1312 ft. to .0984 ft. But to ensure full security, it will be necessary that the head be less than the third of the height of the dam.

3d. For breadths of weirs comprised between that of the basin and that which would be a third of it, the coefficient of the expression $5.348\ l\ H\ \sqrt{H}$ will vary with the relative breadth, i. e., with the ratio of the breadth of the weir to that of the canal, and it will be given in the following columns. We formed them by taking proportional parts between the coefficients deduced directly from experiment, and what is seen in the tables of No. 72; this mode of interpolation would here give no error. We have noted separately the coefficients deduced from the observations made on each of our two canals, to show that for the same relative breadth, the same coefficients sensibly correspond, although the real value of the breadth be, in one of the canals, more than double the other; evident proof that, above .8202 ft., or a quarter of the breadth of the basin, the coefficients depend on the relative breadth, and not on the absolute breadth of the weir.

Relative breadths.	COEFFICIENTS FOR CANAL of 2.428 ft	of 1.181 ft
1.00	.662	.667
.90	.656	.659
.80	.644	.648
.70	.635	.635
.60	.626	.623
.50	.617	.613
.40	.607	.609
.30	.598	.600
.25	.595	.598

4th. It is quite different when that breadth descends below one quarter that of the canal. Then, and when, at the same time, it is less than .2624 ft. or .1968 ft., that of the canal has no influence, and each absolute breadth of weir has its own coefficient; thus, on the canal of 1.184 ft., as well as on that of 2.428 ft., the breadths .1640 ft., .0984 ft., .0656 ft., and .0328 ft., have equally for their respective coefficients .61, .63, .65 and .67.

Observations upon the Formulæ of No. 69.

78. After having explained, in detail, what relates to the most simple of the formulæ of the discharge in weirs, we pass to two others; and first to

$$Q=5.348\, ml\, (H\sqrt{H}-h\sqrt{h}),$$

in which h represents the quantity AD (Fig. 17), by which the fluid surface is already depressed on its arrival at the weir.

A simple glance at the last column of the table given at No. 73, shows that, although the series of quantities $H\sqrt{H}-h\sqrt{h}$ is not very remote from those which belong to the corresponding discharges, it follows them, however, less exactly than the series of values of $H\sqrt{H}$. Thus, in this principal point, this second formula is not so well founded as the first.

Besides, it is much more difficult of application; it contains one term more, $h\sqrt{h}$, a term whose exact determination is a matter of great difficulty, as we shall soon see (82). So that, although reasoning first led to this formula, we make no use of it.

Observations on the formula of No. 71.

79. It is not entirely so with that which includes a term which is a function of the velocity with which the water running in the canal arrives at the weir.

At the time of the experiments made at the water-works of Toulouse, we had frequent occasion to observe the effect of this velocity. As soon as it became sensible, the greater it was, (and it became greater the greater the head, and especially as the weir was made broader,) the more the expression of the discharges $5.348\ lH\sqrt{H}$, in which the running is supposed to take place only in virtue of the pressure or head H, failed through deficiency, and its coefficient of contraction m became greater. Such is, in part, but in part only, the cause of the increase of the coefficients, in proportion as the breadth of the weir, starting from .1968 ft., increases. It is evident, that in the case of a notable velocity, when the running is effected in virtue both of the head and of a previously acquired velocity, it is necessary to add to the head a term dependent on that velocity; which leads to the equation (71)

$$Q=5.3484\ m'lH\sqrt{H+.03495w^2}.$$

The experiments of M. Castel will give the values of the coefficient m'. In these experiments, the velocity w of the surface of the current in the canal was not measured, it is true; but we can determine it from the mean velocity (108), which is equal to the discharge Q divided by the section of the current, which is here L $(H+a)$; L being the breadth of the rectangular canal, and a the elevation of the sill of the weir above the bottom of this canal. In fact, according to the experiments of Dubuat, which we shall by and by investigate (109), the velocity of the surface is, as a mean term, a quarter greater than the mean velocity; so that we should have $w=\frac{1.25\ Q}{L\ (H+a)}$.

Even with this value of w, which, however, is the greatest we can admit, the coefficient m' will differ from the coefficient m of the ordinary formula, only as the velocity in the canal will be sufficiently great for the term $.035w^2$, which makes the difference between the two formulas to have a value comparable to H. As it will generally be very small, and as it is under the radical, it will scarcely influence the value of m' by half its own relatively to H; if it be two, four or six hundredths of H, the coefficients, all things else being equal, will only differ one, two or three hundredths. In these three cases, the section of the fluid sheet at the weir, or $l\times H$, is respectively 5.8, 4.1 and 3.35 times smaller than the section in the canal, or $L\ (H+a)$; whence we draw the conclusion, which we have already used, that when the first of these sections is less than the fifth part of the second, the coefficients m and m' will be the same, to a hundredth, nearly. Such was the case for the weirs of M. Castel, as long as their breadth was below half of that of the canal. When it was considerably more, the term $.035w^2$ had greater influence, and the differences became greater. But the employment of this term is far from reducing to equality the coefficients m' for different breadths of weirs; it did not even reduce to half, the differences which the values of m present; and the expression

$$5.348\ m'lH\ \sqrt{H+.035w^2},$$

hardly more than $5.348\ mlH\ \sqrt{H}$, can be employed with a constant coefficient, only in cases of a breadth of weir equal to that of the canal. For this case, it will exact less restrictions, and if it is less simple, and even if it is not more exact, it will be more general and more rational.

To obtain his coefficient, M. Castel barred the canal of 2.428 ft. by dikes of copper, the heights of which were successively dropped from .738 ft. to .105 ft., and he obtained the coefficients placed opposite. Those of the first five dikes are generally the same, although, however, they do not present the regularity which there was in those of ordinary weirs; their mean term is .650. As to the coefficients of the dikes of .134 ft. and .105 ft., they belong to a peculiar class. These dikes were very low, and the heads much surpassed their heights, so that we were at least as much in the case of a water course running in an ordinary bed, as in that of weirs; moreover, the close approximation to equality between the coefficients for the same dike testifies in favor of the formula which gave them. The experiments on the canal of 1.184 ft., with its dam of .558 ft. height, indicated coefficients of which the mean was .654. Admitting the mean term between this number and .650, observing that 5.3485×.652=3.4872, we shall finally have

Height of the dam, in feet.	COEFFICIENTS m', THE HEAD BEING IN FEET.			
	.2624	.1968	.1640	.1312
.738	.651	.655	.657	.660
.558	.640	.647	.650	.654
.426	.650	.649	.652	.656
.305	.635	.642	.646	.650
.246	.647	.652	.655	.660
.134	.667	.664	.665	.668
.105	.676	.676	.676	.680

$$Q=3.4872\ LH\sqrt{H+.03495w^2}.$$

The velocity w in this will be directly determined by observation.

In rectangular canals, such a determination is superfluous, and, giving to *w* its value, as above, we have, as long as H is smaller than $\frac{1}{4}$ *a*,

$$Q = \frac{3.4872\ LH\sqrt{H}}{\sqrt{1 - .664\left(\frac{H}{H+a}\right)^2}}$$

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Weirs with additional canals.

80. Very often we apply to weirs, canals which are, as it were, exterior extensions of the sides of the weir. The water, constrained to follow them, experiences from their sides a resistance which retards the motion; and this retardation being communicated to the fluid which arrives at the weir, diminishes the discharge. Experiments alone can make known this diminution for the different cases which present themselves, and we have but very few. MM. Poncelet and Lesbros have, it is true, made a great number of them, but they are not yet published. However, the latter savant, in communicating to me some of those which he made upon the canals adapted to orifices closed on all their periphery, and of which we already have the results (39), had the kindness to send me a series of those which he also made with orifices open on their upper part, that is, with weirs. The additional canal was always that of 9.84 ft. in length and .656 ft. broad, like the weir, and it was kept horizontal. I here give the results obtained, as well as those previously had from the same weirs and with the same heads. The diminution of the product with the canal was as much less as the head was greater. From this fact, as well as from those which were obtained with closed orifices, might it not be inferred, for heads of 3.28 ft. and more, such as we

Head, in ft.	COEFFICIENTS.		Loss in 100.
	Without canal.	With canal.	
.676	.582	.479	18
.476	.590	.471	20
.338	.591	.457	23
.197	.599	.425	29
.148	.609	.407	33
.092	.622	.340	45

often have at the head of great canals and raceways, that the diminution of the discharge due to the presence of the canal would be but very small? After all, we await the publication of the work of MM. Poncelet and Lesbros before drawing, and especially before generalizing, such a conclusion.

M. Castel also made some experiments, on a kind of canals, of a peculiar interest. It was required to ascertain what was the discharge through channels of navigation opened in the dikes of rivers. To answer this question, he added to the weir of .656 ft. broad, on the basin of 2.428 ft., a small canal, 0.67 ft. long, and inclined 4° 18′, or $\frac{1}{13.3}$.

Here are the coefficients obtained with the formula $Q=5.348\,mlH\sqrt{H}$. They varied but very little, although the heads were more than doubled; and the mode of experimenting pursued is a guaranty that there was no error. The mean coefficient was .527; it would probably have been raised to .53, if, as in our ordinary channels, the inclination had been $\frac{1}{12}$. For the weir alone, the coefficient was .60; so that the additional canal would not have diminished the discharge as much as twelve per cent.

Head.	Coefficient.
.364	.526
.312	.527
.250	.527
.197	.528
.164	.530

Demi-Weirs.

81. Let us say a few words concerning a kind of weir to which Dubuat gave the name of *demi-weirs*, or *incomplete weirs*. They are those in which the level of the water in the lower reach is above the sill, or the crest of the dam, as is seen in Fig. 33. Dubuat, in thought, here divided the height of the water AC above the sill into two parts, A*b* and C*b*. In the first, the flowing takes place as in an ordinary weir, where A*b* (=H) would be the head; so that

the volume of water discharged would be (79)

$$3.4872\ l\mathrm{H}\ \sqrt{\mathrm{H}+.03495w^2}.$$

In the second part, it is admitted that the discharge is the same as through a rectangular orifice, whose height would be *b*C, and where the head would equal the difference of height between the upper and lower level (95); *b*C is the elevation of the latter above the sill of the weir; and it will be *a*—*b*, if we designate by *a* the elevation *b*D of the surface above the bottom of the canal, and by *b* the height of the sill above the same bottom; to the head A*b* or H will be added, as in the case of closed orifices (38), the height due to the velocity *u* of the water of the canal, and the velocity of exit will be found

$$\sqrt{2g\ (\mathrm{H}+.015536u^2)}=\sqrt{2g\ (\mathrm{H}+.01942w^2)},$$

since (79) $w=1.25\ u$, consequently, we shall have for the discharge (16),

$$8.0227\times.62\ l\ (a-b)\ \sqrt{\mathrm{H}+.01942w^2};$$

uniting these two partial discharges, and designating by Q the total discharge, it will become

$$\mathrm{Q}=3.4872\ l\mathrm{H}\ \sqrt{\mathrm{H}+.034957w^2}+4.974\ l\ (a-b)\ \sqrt{\mathrm{H}+.01942w^2}.$$

Inflexion of surface towards the dam.

Let us terminate this article by a succinct examination of a remarkable circumstance presented in the flowage over weirs. The water, on approaching them, and as soon as it has entered into their sphere of activity, precipitates itself in some manner towards the middle of the sill, and its surface is inclined from all sides towards it.

Cross profile.

In the plane of the weir, the inclination commences some centimetres (cent.=.03281 ft.) from the opening, along the wings, (or parts of the partition in which the weir is made comprised between the opening and the lateral sides of the canal). This inclination, at first insensible, increases little by little; it is, at its maximum, at the edges of the orifice; it diminishes, then, towards the middle; sometimes it is nothing there, the fluid remaining horizontal, to a certain extent; at other times, it rises at this middle, to fall anew. Fig. 91 (Pl. V.) shows two examples of the transverse section of the surface of the water at the weir; at *abc* it is simply concave; at *defgh* it presents the swell *f;* sometimes there are two risings, one towards *c* and the other towards *g;* the surface is then, as it were, undulating.

Measure of H. M. Castel, with the view of furnishing for practice an easy method of measuring the heads, took, upon his canal of 1.181 ft. broad, fifteen transverse profiles, under different heads and with different breadths. (See the Memoirs of the Academy of Sciences of Toulouse, tome IV., page 280.) It results from his observations:

1st. That the inflexion does not extend along the wings, at least, in a sensible manner, at more than .2296 ft. or .2624 ft. from the opening.

2d. That beyond this distance, in most cases, deduction being made of the effects of capillary attraction, the water maintains itself against the wings exactly at the same level as in the full basin; but that, with broad weirs, and under great heads—that is to say, in great velocities—the fluid surface rises against the wings, and the rising has even been up to .00984 ft. As there will be none of this in the weirs to which the formula $Q=3.209\, lH\sqrt{H}$ is applicable, to obtain in these cases the head H, it will suffice to take on each of the two wings a point of the water line, to stretch a line from one to the other, and to measure its elevation above the middle of the sill of the weir. In these cases of rising up against the wings, we should seek to ascertain its magnitude, or to be freed from its action, for example, by fixing the two extremities of the line against the lateral sides of the canal, a little above the weir.

We may even disregard altogether the rising on the wings, and treat it as if it did not exist, taking in all cases for H the height of the water line above the sill; for this rising above the level being a consequence of the impulse of the fluid against the wings, and therefore an effect of the velocity of the water in the canal, will represent in part that effect; it will in part take the place of the term $.035w^2$; it will render the formula $Q=3.209\ lH\sqrt{H}$ exact, even for quite great velocities.

Whenever the rising above the level would wholly represent the effect of the velocity, and would be the height due to w, some have thought that it should be added to H throughout, and they establish $Q=5.348\ ml\ (H+.0155w^2)\sqrt{H+.0155w^2}$. I am assured, by experience, that such a formula gives too much influence to the velocity w.

As to the absolute quantity of inflexion h, that is to say, the settling of the middle of the transverse profile below the level of the water in full basin, we shall give the value in the following number. It may suffice to remark, here, that the form and variations of this profile will render its exact determination very difficult, if not impossible; this form is often undulatory, and the summits of the waves are moveable, so that, from one moment to the other, h, or the depression of the fluid at the middle of the weir, may be found .0065 ft. or .0098 ft. greater or less.

Longitudinal profile.

83. Different authors, who have studied the running of water over weirs, have also given attention to the inflexion of the fluid, in proportion as it advances towards the orifice of exit; and they have given longitudinal profiles. But no one has given so many as M.

Castel; each of his determinations of the head, (and he made more than four hundred,) was made by means of such a profile; the depressions below a horizontal plane were taken at intervals of .164 ft., and measured in tenths of millimetres, the millimetre being .00328 ft.

I shall not enter into detail upon these observations and their consequences; they will be found in the memoir of that observer, and I shall confine myself to summing up the principal results.

1st. The appreciable length of the inflexion of that which exceeds .000328 ft. varied only from .492 ft. to 1.3776 ft., and it never attained to 1.64 ft., reckoning from the weir. It was naturally as much greater as the head and breadth of the opening were more considerable.

2d. The absolute quantity of inflexion, h, was about .0164 ft. under the head of .0984 ft., whatever might be the breadth of the weir; then it increased with that breadth and the head. In the canal 1.181 ft. broad, with a simple dam, and under the head of .3937 ft., we had h=.055104 ft.; and .065928 ft. in the canal of 2.428 ft., with a breadth of weir of .656 ft.; and with the head of .656 also, this was the greatest inflexion that was seen.

3d. The inflexion compared to the head, or the ratio $\frac{h}{H}$, was from .16 to .17, under very small heads, and in all the weirs; this expression then diminished in proportion as the head increased, and as much as the weirs were narrowed. Thus, in the canal of 2.428 ft., and under the head of .656 ft., we had .3182 ft. with the weir of .656 ft., and only .0984 ft. with that of .164 ft.

CHAPTER SECOND.

EFFLUX OF WATER, WHEN THE RESERVOIR EMPTIES ITSELF.

When a vessel, instead of being kept constantly full, receives no additional water, or receives less than it discharges through an orifice in its lower part, the fluid surface gradually sinks, and finally the vessel becomes empty.

The laws of efflux are in such circumstances different from those which we have just explained in the preceding chapter, and other questions are to be solved. We will examine these laws and these questions; and first, in the case of prismatic vessels or basins.

Ratio of velocities at the orifice and in the vase.

84. Suppose the fluid to be divided into extremely thin horizontal strata, and that they fall parallel to each other; each of their particles will be animated by the same velocity; this is the *hypothesis of the parallelism of the strata*, admitted, perhaps too extensively, by many geometers.

Let v be the velocity of the particles in the vessel, V the velocity which they have at the orifice, A the horizontal section of the vessel, S, or rather, mS, the section of the orifice, allowing for the contraction; the volume of water which will flow in an infinitely small portion of time τ, will be $s\mathrm{V}\tau$. During this same time, the fluid surface will have fallen the vertical distance $v\tau$, and the corresponding volume of water will be $\mathrm{A}v\tau$. These two volumes necessarily being equal, we have $\mathrm{A}v\tau = m\mathrm{S}\mathrm{V}\tau$, or $v : \mathrm{V} :: m\mathrm{S} : \mathrm{A}$. (A new example, and a new proof that when a fluid mass is in motion, without destroying the continuity of its parts, *the velocities are in the inverse ratio of the sections* (19).

Height due the velocity of issue.

85. The velocity of the issuing fluid is not uniform, and for a given moment is not a simple effect of the pressure or of the height of the reservoir; it is also a consequence of the velocity v, acquired during the descent of the strata; the two actions, operating in the same direction, downwards, their resultant will be equal to their sum. Thus, if H′ is the generating height of the velocity of efflux, H always being the height of the reservoir, we shall have

$$H' = H + \frac{v^2}{2g} = H + \frac{V^2}{2g} \cdot \frac{m^2S^2}{A^2} = H + H' \frac{m^2S^2}{A^2}.$$

Whence

$$H' = \frac{H}{1 - \frac{m^2S^2}{A^2}} = H \frac{A^2}{A^2 - m^2S^2}.$$

Such is the famous rule given by Daniel and John Bernoulli, the same as for the case of vessels always full.

When mS is small compared to A, which is almost always the case, m^2S^2 will be very small compared with A^2, and may be neglected; then $H' = H$, that is to say, the velocity of efflux, at any instant, is that due to the height of the reservoir at the same instant.

We shall admit it to be so in what follows; and the more readily, since the hypothesis of the parallelism of the strata, which led to the above value of H′, although admissable before the strata, in their descent, have arrived within the sphere of activity of the orifice, cannot be admitted after having reached it; the circumstances of the motion of the fluid particles then become very complicated, and are entirely unknown to us.

Fig. 18. Nature of Motion.

86. Let M be a prismatic vessel filled with water up to AB; divide its height, from B to C, the place of the

orifice, into a very great number of equal parts, B*a*, *ab*, *bc*, &c.

Now, suppose that a body P be projected upwards with a velocity such that it rises to the point H, PH being equal to CB, and divide PH also into the same number of equal parts. As the body ascends, its velocity will diminish, in such a manner that when it arrives successively at the points a', b', c', the velocities will be, respectively, as is known in the first elements of mechanics, as $\sqrt{Ha'}$, $\sqrt{Hb'}$, $\sqrt{Hc'}$, ... 0.

As the fluid flows from the vessel M, its surface AB will settle, and when it is successively at the points *a*, *b*, *c*, the respective velocities of the effluent water will be (85) as $\sqrt{aC}$, $\sqrt{bC}$, $\sqrt{cC}$, ... 0, or, according to the construction, as their equivalents $\sqrt{Ha'}$, $\sqrt{Hb'}$, $\sqrt{Hc'}$, ... 0.

So that, as the vessel empties itself, the velocity of efflux will decrease till it becomes nothing, following the same law as the velocity of a body projected upwards, which is the law of *uniformly retarded motion;* the efflux, therefore, will take place with such a motion. The same will hold respecting the descent of the fluid surface, the velocity of the descent being to that of the efflux, in the constant ratio of the section of the orifice to the transverse section of the vessel.

Volume of efflux.

87. According to the laws of uniformly retarded motion, when a body starting with a certain velocity gradually loses it till it is reduced to zero, it passes through half the space it would have passed through, in the same time, if it had constantly preserved the velocity of departure. Moreover, the volume of water which flows from a vessel, until it is quite empty, may be regarded as a prism having for its base the orifice in the vessel, and for its height the space which the first

effluent particles would pass through, with a retarded motion equal to that with which the efflux is made; but if these same particles had always preserved their initial velocity, that due to the first head, the space passed through in the same time, or the height of the prism, and consequently the volume of water discharged, would have been double.

Hence the theorem: *the volume of water discharged through an orifice, from a prismatic vessel, which entirely empties itself, is only half of what it would have been, during the time of emptying, if the efflux had taken place constantly under the same head as at the commencement.*

Time required to empty the basin.

88. Let H be the head, A the horizontal section of the basin supposed to be always prismatic, T the time necessary to empty it. The volume of water discharged during that time, that is, all the water contained in the vessel (above the orifice) is $A \times H$. The volume which would have been discharged, under the head H, according to the above theorem, would be 2AH; this same volume, or the discharge during the time T, is also (16) $mST\sqrt{2gH}$.

Equating these two values, we have

$$T = \frac{2AH}{mS\sqrt{2gH}} = 2\frac{A\sqrt{H}}{mS\sqrt{2g}}.$$

If we represent by T′ the time which the volume AH would have required to flow under the constant head H, we should also have

$$AH = mST'\sqrt{2gH} \text{ or } T' = \frac{A\sqrt{H}}{mS\sqrt{2g}}.$$

Thus

$$T = 2T':$$

that is to say, *the time in which a prismatic vessel* *empties itself is double the time in which all its water would have run out, if the head had remained what it was at the commencement of the efflux.*

Time required to lower the fluid a given quantity.

89. To obtain the time t, in which the level of such a vessel descends a given quantity a, take the time required to empty the vessel entirely, which is $\frac{2A\sqrt{H}}{mS\sqrt{2g}}$: then take the time required to empty it, starting not from the first level, but from that to which it will have descended, after having passed down the quantity a, the head will then be $H-a$; call it h, and we shall have $\frac{2A\sqrt{h}}{mS\sqrt{2g}}$. The time required being evidently only the difference between those of which we have thus given the expression, there results

$$t=\frac{2A}{mS\sqrt{2g}}(\sqrt{H}-\sqrt{h}).$$

Example. There is a prismatic vessel, whose horizontal section is a square of 3.199 ft. at its side, and which has in its bottom an orifice .0889 ft. diameter; it is filled with water up to a height of 12.435 ft. above the centre of the orifice. What is the time required to draw down the level 4.265 ft., reckoning from the moment of opening the orifice?

We have $A=3.199\times3.199=10.2336$ square feet; $S=\pi'(.0889)^2$ $=.00621$ square feet; $H=12.435$ ft.; $h=12.435-4.265=8.17$ ft., and m, according to the table of No. 26, will be 0.61; so that

$$t=\frac{2\times10.2336}{0.61\times.00621\sqrt{2g}}(\sqrt{12.435}-\sqrt{8.17})=450''=7'\,30''.$$

Bossut, operating with the above data, found $t=7'\,25''.5$.

This author also made, with the same apparatus, the three experiments presented in this table:

Diameter of orifice, in feet.	Falling of level, in feet.	Time of falling, according to experiments.	Time of falling, according to the formula.
.0889	9.5805	20′ 25″	20′ 41″
.1775	9.5805	5′ 6″	5′ 10″
.1775	4.2653	1′ 52″	1′ 52″

Although the times deduced from calculation are generally a little greater, the excess is so small that it may be neglected: it is probably the effect of some small error of observation.

I would remark, that the time which a vessel requires to empty itself entirely, could not be exactly determined by the formulæ; when the water descending is near the bottom, it assumes the form of a funnel, the middle of which is occupied by air, and it thus diminishes the orifice of efflux. Besides, when the water is only about $\frac{1}{4}$ inch from the bottom, the molecular attraction retains its particles, and the flow is checked, or rather, it proceeds only drop by drop.

Volume of water passed in a given time.

90. The expression of the time required by a fluid to fall a certain quantity, gives, by a simple transformation, the extent of the fall, as well as the volume of water discharged during that time.

For the extent of the fall, $H-h$, we have

$$\frac{tmS\sqrt{2g}}{A}\left(\sqrt{H}-\frac{tmS\sqrt{2g}}{4A}\right).$$

Multiplying this expression by A, (which merely removes A from the incomplex factor,) we obtain the volume of water discharged in the time t.*

Take, for example, a basin, the upper part of which is sensibly prismatic, and having a surface of 10764.3 sq. ft.; the water issues from it through a gate 2.133 ft. broad by 0.27888 ft. high. How much will the surface fall in one hour? Here $A=10764.3$ square ft.; $S=2.1326\times.27888=.59474$, $H=8.8587$ ft., $t=1^h=$

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* Should the reader find any trouble in transforming this equation, he will readily understand the following:

$$\sqrt{H}-\sqrt{h}=\frac{tmS\sqrt{2g}}{2A};\ \therefore\ \sqrt{h}=\sqrt{H}-tmS\sqrt{2g}.$$

Multiply first member by $\sqrt{H}+\sqrt{h}$, and second member by its equivalent $\sqrt{H}+\sqrt{H}-tmS\sqrt{2g}$, and the result given in (90) is produced. TRANSLATOR.

3600″, and m for the openings of the gate is about .70; consequently, the fall demanded will be

$$\frac{3600\times .70\times .59474\times 8.02052}{10764.3}\left(\sqrt{8.8587}-\frac{3600\times .70\times 5.9474\times 8.02052}{4\times 10764.3}\right)=3.0119^{\text{ft.}}$$

For the volume of water discharged in the time of this fall, we should have 10764.3×3.0119=32421 cubic ft.

Basin receiving water while being emptied.

91. Admit that the prismatic basin, while emptying itself, receives a current (furnishing less water than flows from the basin), and let us determine, in this new case, the time required for the surface to fall a given quantity. Preserving the above denomination, call q the volume of water coming to the basin in one second x, and the descent in the time t; dx will be the height through which the fluid will fall during the infinitely small instant of time dt, at the end of the time t. $\text{A}dx$ will express the volume of water discharged during that instant, if the basin receive no water flowing in; but as it receives q in one second, and consequently qdt in dt, the volume of water really discharged will be $\text{A}dx+qdt$. This same volume, according to the formula of the discharge through orifices (16), is also expressed by $m\text{S}dt\sqrt{2g\,(\text{H}-x)}$. We shall therefore have,

$$\text{A}dx+qdt=m\text{S}dt\sqrt{2g\,(\text{H}-x)};$$

making $\text{H}-x=h$, whence $-dx=dh$,

$$qdt-\text{A}dh=m\text{S}dt\sqrt{2g}\sqrt{h};$$

which gives $dt=\dfrac{-\text{A}dh}{m\text{S}\sqrt{2g}\sqrt{h}-q}$.

To integrate this equation, I make $m\text{S}\sqrt{2g}\sqrt{h}-q=y$, and it becomes

$$dt=\frac{-\text{A}}{m^2\text{S}^2g}\left(dy+q\frac{dy}{y}\right),$$

of which the integral is

$$t=-\frac{\text{A}}{m^2\text{S}^2g}\,(y+q\text{ hyp log }y)+\text{C}.$$

Giving to y its value above, determining the constant for the origin of motion, where $t=0$, $x=0$, and $h=\mathrm{H}$, substituting the ordinary logarithms for the hyperbolic logarithms, by multiplying the latter by 2.303, we shall finally have

$$t=\frac{2\mathrm{A}}{(m\mathrm{S}\sqrt{2g})^2}\left\{m\mathrm{S}\sqrt{2g}(\sqrt{\mathrm{H}}-\sqrt{h})+2.303\,q\log\frac{m\mathrm{S}\sqrt{2g\mathrm{H}}-q}{m\mathrm{S}\sqrt{2gh}-q}\right\}$$

When no water flows into the basin, $q=0$, and we have the equation of No. 89.*

A pond, reduced to the prismatic form, has a surface of 38751.48 square ft., and a depth of 11.483 ft.; it is fed by a stream affording 33.55 cubic ft. of water per second; when the gate is wholly raised, the opening is 3.609 ft. wide, and 1.969 ft. high. In what time will the pond draw down to .328 ft. above the upper edge of the opening? (According to what was said in No. 89, the formula would not give the time of descent, when the level of the fluid is only at a small height above the orifice of efflux). We have here, for the head above the centre of that orifice, at the moment of raising the gate; $\mathrm{H}=10.4988=(11.483-\frac{1.969}{2})$ ft.; and for the head at the end of the time, $h=1.312(=\frac{1.969}{2}+.328)$ ft.; $\mathrm{S}=3.609$ ft.$\times 1.9685$ ft.$=7.10612$ square ft.; $\mathrm{A}=38751.51$ square ft.; $q=33.558$ cubic ft.; $m=.70$; consequently, $m\mathrm{S}\sqrt{2g}=39.907$ and $\log\frac{m\mathrm{S}\sqrt{2g\mathrm{H}}-q}{m\mathrm{S}\sqrt{2gh}-q}=\log\frac{95.821}{12.160}=\log 7\dot{}.8792=.89648$.

From this the equation becomes

$$t=\frac{2\times 38751.51}{(39.907)^2}\left\{39.907\,(\sqrt{10.4988}-\sqrt{1.312})+23.03\times 33.558\times .89648\right\}=$$

$7440''=2^{\mathrm{h}}\,4'$.

This is the time required.

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* I give the method by which D'Aubuisson has got this result. Putting in $\mathrm{S}\sqrt{2g}\sqrt{h}-q=y$, the equation stands thus: $dt=\frac{-\mathrm{A}dh}{y}$. We have $dh=\frac{2\sqrt{h}\,dy}{m\mathrm{S}\sqrt{2g}}=\frac{2dy\,(y+q)}{m^2\mathrm{S}^2\,2g}$.

Substituting in above, we have

$$dt=\frac{-2\mathrm{A}ydy-2\mathrm{A}qdy}{m^2\mathrm{S}^2 2gy}=\frac{-2\mathrm{A}}{2gm^2\mathrm{S}^2}\left(dy+q\frac{dy}{y}\right).$$

TRANSLATOR.

92. If it were required to determine the descent of the level in a given time, the question would be reduced to determining the head h at the end of that time, and we should subtract it from the head H at the commencement of the discharge. To obtain h, we will put successively, into the equation of the preceding number, several values, until one is found to satisfy its conditions.

Take the pond just investigated, and let us ascertain how much the surface will be lowered in one hour, H being always 10.4988; we have also $t=3600''$, $A=38751.51$ square ft.; $q=33.558$ cubic ft., and $mS\sqrt{2g}=39.907$.

Putting these numerical values into the equation, and assuming different values for h, we shall, after a few trials, find the value of h, which nearly satisfied the equation to be 3.99745; the reduction in this case gives $+1.27$ ft.$=0$.

Consequently, the fall required will be, $10.4988-3.99745=6.5013$ feet.

When water passes over weirs.

93. In case the water passes from a basin over a weir, admitting that the basin received no fresh supplies, we should have, from what has just been said, and what has been explained elsewhere (70),

$$A\,dx=\tfrac{2}{3}ml\,(H-x)\,dt\,\sqrt{2g}\,\sqrt{H-x};$$

whence, by a method analogous to that before used, we deduce

$$t=\frac{3A}{ml\sqrt{2g}}\left(\frac{1}{\sqrt{h}}-\frac{1}{\sqrt{H}}\right).$$

Take, for example, a basin with a surface of 1076.43 square ft., on one side of which is a weir 1.6411 ft. wide; the level of the water is 2.6251 ft. above the sill. In how long a time will the surface fall 1.97?

Here $h=2.625-1.97=.655$; $H=2.625$; $A=1076.43$; $l=1.64$ and $m=.61$; so that

$$t=\frac{3\times1076.43}{.61\times1.64\times8.02}\left(\frac{1}{\sqrt{.655}}-\frac{1}{\sqrt{2.625}}\right)=248.81''=4'8.81''.$$

Basins not prismatic.

94. Thus far, we have considered only prismatic vessels or basins; the determination of the time of efflux for every other form would be much more complicated; it is even impossible, in most cases which present themselves.

The fundamental equation is still

$$\mathrm{A}dx = m\mathrm{S}dt\sqrt{2g\,(\mathrm{H}-x)};\ \text{whence}\ dt = \frac{\mathrm{A}dx}{m\mathrm{S}\sqrt{2g(\mathrm{H}-x)}}.$$

But here A is variable, and the integration can be effected only after expressing A as a function of x; this could be done only so far as the law of its decrease was known, and consequently, only as the basin would be a solid of revolution, the equation of the generating curve of which was known. In all other cases, it would be necessary to proceed by approximation and by parts. For this purpose, we should divide the basin into horizontal strata of small thickness; each should be regarded as a prism, and we would determine by the above formula the time required to empty it. The sum of these partial times will be the time which the water will have required to fall a quantity equal to the sum of the height of the prisms.

For example, let there be a pond, with a fluid surface of 339075 square ft., which is kept at the level of 7.87 ft. above the centre of an opening at the bottom, provided with a pyramidal shaped mouth-piece of 1.48 ft. square. In what time will the surface fall 3.2809 ft., when that mouth is open?

Suppose the stratum of water 3.2809 ft. thick to be divided into two strata, each 1.6404 ft. thick; according to the plans and profiles of the pond, to be made with care and detail, we should determine the mean section of the first quantity; admit that it is 287407 square ft., and that of the second, 181917 square ft.

For the first, we have h=6.2337 ft. (=7.8741—1.6404); and for the other, h=(6.2337 ft.—1.6404 ft.)=4.5933 ft.

Consequently, the time required to discharge the first stratum,

.98 being the coefficient of discharge through the pyramidal trough (51), will be (89)

$$t=\frac{2\times 287407}{0.98\times(1.4764)^2\sqrt{2g}}(\sqrt{7.8741}-\sqrt{6.2337})=10375.7''=$$

$2^h\ 52'\ 55.7''$

For the second stratum, we have

$$t=\frac{2\times 181917}{0.98\times(1.4764)^2\sqrt{2g}}(\sqrt{6.2337}-\sqrt{4.5933})=7505.6''=$$

$2^h\ 05'\ 5.6''$.

Thus, the time of the descent of surface required will be $4^h\ 58'\ 01''$.

CHAPTER THIRD.

EFFLUX, WHEN THE FLUID PASSES FROM ONE RESERVOIR INTO ANOTHER.

If a reservoir, containing a fluid, instead of discharging it into the atmosphere through an orifice in its lower part, should discharge it into a reservoir already containing a certain quantity of the same fluid, in such a manner that the orifice of communication be covered by the fluid, we shall have to distinguish three cases.

Fig. 19. The level being constant in each of the reservoirs.

95. First, that where each of the two reservoirs sensibly preserves its own level. This occurs when one reach of a canal furnishes water to the reach immediately below, through a sluice-way placed below the surface of the lower level. The water is retained in the upper reach by a sluice-gate AB, at the bottom of which is the opening of which BD represents the height.

To determine the quantity of water which will pass out in the unit of time.

Let m be a fluid particle situated at any point of the opening, the pressure or force which tends to make it pass is represented by Am; but on the other side is a force Cm, tending to hinder its exit, and acting in the opposite direction; so that the resultant or force in virtue of which m will pass out is $Am-Cm=AC$.

For another particle, n, we should in like manner have $An-Cn=AC$. Thus, all the particles will pass out with equal velocities, those due to the difference of level AC.

In general, *when a fluid passes from one reservoir to another, through an orifice covered by the fluid in* *the latter, the effective head on each point of the orifice, and consequently, the head due to the velocity of exit, at any instant, is the difference of level of the two reservoirs at that instant.*

Such is the fundamental principle of flowage considered in this chapter.

96. If h' be the difference of level of the two reservoirs, S being the section of the orifice, m the coefficient of contraction for that orifice, and Q the discharge in one second, we shall have $Q = mS\sqrt{2gh'}$.

But, in this case, has m the same value as when the efflux is made into the atmosphere? In other words, is the fluid vein equally contracted in air and in water? A hundred years ago, Daniel Bernoulli answered this question in the affirmative. Having taken a cylindrical vessel, with an orifice in the bottom, and filling it with water, he remarked that the fluid surface descended the same distance in the same time, whether the orifice were in open air, or plunged slightly in still water. (Hydro. p. 129.) Many experiments made on the orifices of sluice gates indicate a discharge nearly equal, whether these orifices were under water or out of water. They give, in the two circumstances, 0.625 (28) for the coefficient of reduction from the theoretic to the actual discharge. Thus, $Q = 0.625S\sqrt{2gh'}$.

A constant level in the upper reservoir and variable in the lower.

97. Pass to the case where the lower reservoir, that which receives the water, is limited, as a basin of less size would be, whilst the upper reservoir is regarded of indefinite extent, or rather, as kept constantly at the same level. When the orifice of communication is open, the surface of the water in the lower basin will rise; it is required to determine the time it will take to attain a given height.

Fig. 20.

Let M be the basin, the water entering the orifice B has already arrived at C, what is the time requisite for it to arrive at D?

This problem is exactly the inverse of that (89) where, for a vessel which discharges freely into the atmosphere, it is required to assign the time in which the fluid will ascend a given quantity. In that case, as the flowing takes place, the surface of the water above the orifice will fall with a uniformly retarded motion. In the present problem, the surface of the basin M, driven upwards by a force, (the difference of level of the two basins), which decreases in the same progression as the height of the reservoir in the other case decreased, will rise with a motion equally retarded, and will require the same time to pass through the same space, under equal pressures.

If H is always the head AC at the commencement of the time t, h the head AD at the end, A the area of the section of the basin to be filled, S the section of the orifice B, m the coefficient of contraction, we shall here also have

$$t=\frac{zA}{mS\sqrt{2g}}(\sqrt{H}-\sqrt{h}),$$

and for the time of filling up to A,

$$T=\frac{2A}{mS\sqrt{2g}}\sqrt{H}.$$

These formulæ are of great use; they serve to determine the time necessary to fill a lock chamber.

Let us make an application to the canal of Languedoc, or Southern canal. Admitting the middle chamber, such as given in the history of that canal, by Gen. Andréossy, (tom. I., pp. 158 and 251,) we have,

Length of chamber from one gate to the other,	115.1 ft.
Breadth of chamber (swelled in middle) from 21.33 to	36.22 ft.
Fall from one level to the other,	7.46 ft.

Horizontal section of chamber,	3504.86 sq. ft.
Section of an opening,	6.766 sq. ft.
Height of upper level above centre of opening,	6.395 ft.
From the same centre to the lower level,	1.066 ft.
The coefficient of contraction, the two openings of the upper gate being open at a time, is (29)	.548

Consider, first, the part of the chamber below the centre of the orifice. The time of filling it, determined by the rule of efflux into the open air (16), will be, observing that two orifices are open,

$$\frac{3504.86\times1.066}{.548\times2\times6.766\sqrt{2g}\sqrt{6.395}}=24.''84.$$

For the part which is above the centre of the orifice, we have, by the formula just established,

$$\frac{2\times3504.86\sqrt{6.395}}{.548\times2\times6.766\sqrt{2g}}=298'';$$

thus, the time for filling the whole chamber will be about 323″, or 5′ 23″.

The Historian of the canal gives for the time from 5′ to 6′; his mean term, 5′ 30″, scarcely differs from the result of the formulæ.

98. Some experiments made in Germany, on a sluice of the canal of Bromberg, and reported by Eytelwein, (Handbuch, § 120,) will make us still better able to compare the results of calculation with those of experiment.

The chamber was 162.7 ft. long, its breadth from 21.62 ft. to 29.86 ft., and its section 4542.5 square ft.; the orifices were 2.059 ft. (2 ft. of the Rhine) broad; the height of one was 1.373 ft. and of the other 1.845 ft. The water was admitted first through the former and then through the latter; thus there were two series of experiments. In each, the water was previously suffered to ascend in the chamber up to .197 ft. above the upper edge of the orifice, then the edge of the first orifice was 7.294 below the upper level, and the edge of the second was 7.208 ft. The number of seconds which the water required to rise a certain quantity, (1 or 2 inches,) until the chamber was full, was counted. The results obtained are here given:

Number of openings.	Height thro' which the water rose.	TIME OF RISING.	
		By calculation.	By experiment.
	feet.	seconds.	seconds.
1	2.0595	260	263
	2.0595	319	327
	2.0595	458	491
	1.1152	667	682
	7.2937	1704	1763
2	1.0297	93	90
	1.0297	102	102
	1.0297	112	114
	1.0297	128	128
	1.0297	151	149
	1.0297	197	197
	1.0297	476	454
	7.2079	1259	1234

The times of the partial elevations were also calculated, by the formula

$$t=\frac{2A}{mS\sqrt{2g}}(\sqrt{H}-\sqrt{h}),$$

in which m was taken $=.625$.

The value of H in each of these partial experiments, is the sum of the elevations noted in the second column, and taken by starting from the bottom of the column, and comprising the elevation corresponding to the time indicated opposite; h is the same sum, but not comprising that elevation; thus, for the second experiment in the table, we have

$H=1.1152+2.0595+2.0595=5.2342$ and

$h=1.1152+2.0595=3.1747$.

We see, by comparison of the last two columns, that the results of calculation agree pretty well with those of experiment; if in the latter observation we find a great difference, it probably proceeds from the extreme difficulty of taking the exact moment when the water ceased to ascend in the chamber, the elevation in the last moments increasing only by infinitely small degrees.

The level of the two reservoirs varying.

99. We come to the third case, presented by two reservoirs communicating with each other; that where both being limited, and neither of them receiving new

water, the surface of one descends while that of the other ascends. Such is the case of the two basins K and L, communicating by a great tube EF, having a Fig. 21.
cock at G. Before the cock is opened, the level of the water is at AB in the first basin, and at CO in the second; at the end of a certain time, after the opening of the communication, it descends to MN in the first, and rises to PQ in the second; it is required to find the relation between these two elevations; or, *vice versa*, from the relation between the elevations, it is required to ascertain the time of flowing.

Let t=the time, BE=H, CF=h, NE=x, PF=y, A=the section of the first receiver, B=the section of the second, s=the section of the pipe of communication; m will comprise the effect of the resistance of the pipe. While the fluid is rising in the second basin, the quantity dy, during the instant dt, it will fall in the other dx, and, observing that x diminishes when y and t increase, we shall have

$$\mathrm{A}dx = -\mathrm{B}dy,$$

and (16) $\mathrm{A}dx = -ms\sqrt{2g(x-y)}.\ dt$, or

$$dt = -\frac{\mathrm{A}dx}{ms\sqrt{2g}\sqrt{x-y}}$$

The first equation being integrated, (observing that when x=H, y=h,) becomes $\mathrm{A}x+\mathrm{B}y=\mathrm{AH}+\mathrm{B}h$.

Taking from this the value of y, and putting it into the preceding, integrating, and observing that x=H when t=0, we have

$$t = \frac{2\mathrm{A}\sqrt{\mathrm{B}}}{m\mathrm{S}\sqrt{2g}\,(\mathrm{A}+\mathrm{B})}\left\{\sqrt{\mathrm{B}(\mathrm{H}-h)} - \sqrt{(\mathrm{A}+\mathrm{B})x - \mathrm{AH} - \mathrm{B}h}\right\}$$

If it be desired to know the time which the fluid will take to arrive at a certain level, in the two basins, we should make $x=y=\frac{\mathrm{AH}+\mathrm{B}h}{\mathrm{A}+\mathrm{B}}$, and this value, put into the above equation, would give

$$t=\frac{2\mathrm{AB}\sqrt{\mathrm{H}-h}}{ms\sqrt{2g}\,(\mathrm{A}+\mathrm{B})}.$$

Take, for example, in the double lock of a canal of navigation, two contiguous lock chambers. When a boat ascending the canal, has entered the lower chamber by its lower gate, the gate is closed; we then raise the paddle gates of the gate which separates the lower from the upper chamber, (the upper gate of which is closed); the water descends in one chamber and rises in the other, until they have a common level; then the gate of separation is opened, and the boat is introduced into the upper chamber. We require the time which elapsed from the moment of raising the paddle gates, until the water stands at the same level in the two chambers. Suppose the question to apply to the double lock of Bayard, near Toulouse.* Count the time, from the moment when the water, arriving into the lower chamber, has attained the centre of the orifice of the paddle gates; then $\mathrm{H}=13.583$ ft., $h=.787$ ft.; also, $\mathrm{A}=2206.68$ square ft.; $\mathrm{B}=2314.32$ square ft.; $s=13.445$ square ft. (for the two orifices), and $m=.548$; we shall have

$$t=\frac{2\times2206.68\times2314.32\times\sqrt{13.583-.787}}{.548\times13.445\sqrt{2g}\times4521}=137''=2'\,17''.$$

Experiment gave 2′ 29″. This excess of 12″ proceeds from the fact, that the paddle gates were not yet quite raised, when the water attained the centre of their openings; and the formula supposes that they were so.

Note. Vessels divided into different compartments, by partitions or diaphrams pierced with orifices, present, during the flow of the fluids which they contain, diverse phenomena, which have given rise to interesting mathematical considerations. But as these questions are of greater interest as it regards analysis than in respect to immediate application to practice, we shall not dwell on them, but refer to the works specially treating of them, and particularly to the *Hydrodynamique* of Daniel Bernoulli, sec. VIII, and to that of Bossut, tome I., second part, chap. VII.

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* Histoire du Canal du Midi. Tome I., page 251; Tome II., Pl. III.

SECTION SECOND.

ON RUNNING WATERS.

100. Water running naturally on the surface of the globe, forms rivulets and streams, which here will be comprised under the general name of rivers.

Water also runs in canals dug by the hand of man. Both canals and rivers are uncovered; but water is sometimes inclosed in *conduit pipes*, for the purpose of conducting it conveniently to a given point. It also passes from these pipes under the form of *jets d'eau*.

The consideration of the different circumstances of motion in these four states, will be the object of the four chapters of the second section.

CHAPTER FIRST.

CANALS.

Definitions.

101. Canals differ in this regard from rivers, that they have a regular bed, having throughout the same inclination and the same profile; and they carry down the same volume of water throughout their length. In case one of these conditions is not fulfilled, where, for instance, after a certain slope, another is assumed, there will result two canals, the one succeeding the other.

Fig. 22. If from the point o, at the bottom of the canal, a horizontal line op is drawn, its corresponding vertical qp will be the slope of the canal for the length oq. It is called the *absolute slope*, if o and q are at the extremities of the bed of the canal; and the *relative slope*, or the slope per foot, if oq is one foot long. Calling the slope p, if L is taken for any length of a canal, D being the difference of level between the extremities of this portion, we have $p=\frac{D}{L}$; or, if e represents the angle of inclination, $p=\sin. e$.

The *section* of a canal, or any water-course, is the area of the section made by a plane perpendicular to the axis of the current; in a rectangular canal, if $l=$ breadth and $h=$depth, s or area of section is $s=lh$; if it is trapezoidal, $l=$breadth at bottom, and n the slope of the sides, or the ratio of the base to the height, then $s=(l+n\ h)\ h$ or $s=(l+\cos.\ f.\ h)\ h$, where f is the inclination of the sides to the horizon.

That part of the contour of the fluid section, in contact with the bed or bottom, as well as sides or berms, is called the *wetted perimeter* of the section. Designating it by c, for rectangular canals, we have $c=l+2h$; for trapezoidal, $c=l+2h\sqrt{n^2+1}=l+\frac{2h}{\sin. f}$.

Dubuat gives the name of *mean radius* of the section, for the ratio of the area to that of the *wetted perimeter*, or $\frac{s}{c}$.

Let us now examine the nature of the motion of water in canals, that is to say, the nature and expression of the forces which produce it; thus establish the formulæ of this motion, with their various applications; and finally, ascertain the quantity of water which canals can receive at their heads or inlets.

ARTICLE FIRST.

Nature of Motion in Canals.

Cause of Motion.

102. Gravity is the sole force that acts upon a mass of water left to itself, in a bed of any form; it produces all the motion which takes place.

Whenever its action upon each fluid particle (whether it be that which it exerts directly downwards, or that indirectly produced by the lateral pressure of the adjoining particles) is destroyed, so that the fluid mass is brought to a state of rest, its surface will be horizontal. Reciprocally, when the surface of a fluid is horizontal, exception being made for any impulse before impressed upon it, all action of gravity will be destroyed, and no motion can take place. But as soon as this surface is inclined, motion takes place, and continues, even if the bottom of the bed is horizontal, and even if it should have a counter slope for some distance. Whence, the principle, admitted in Hydraulics, and of which we shall give a geometrical demonstration, that "*the motion of particles in a water course, is due wholly to the slope at the surface;*" *this* slope it is, which is the immediate cause of motion, and enables gravity to act.

Mode of action of gravity.

103. Let us examine, now, the mode of action of this force, and what is its measure in the different cases that may occur, which are represented in Fig. 22.

Suppose then a canal, in which the surface of water is parallel to the bottom of the bed, and consider the very small section A. The fluid particles which are on the bottom *a′ b′*, will descend by the direct action of gravity, as down an inclined plane. Those which are above, up to the surface *a b*, forming, as it were, threads laid upon the first, will descend in the same manner. The effective portion of gravity, that which is not destroyed by the resistance of the bed, and which causes the motion, will be represented by the height *a c*, and this height will be *g* sin. *i; i* being the inclination of the surface *a b* to the horizontal *b c*. The indirect action of gravity, or the lateral pressure experienced by each particle, being the same in all directions, by reason of the parallelism of *a b* and *a′ b′*, will not occasion any motion.

Let us admit, now, a current with a surface more inclined than its bed, and represent a small section of it by B. Take, then, into consideration, any particle, *m*, traversing the section in the direction *m n*. This particle, or rather, the linear system of particles *m n*, will experience: 1st. The direct action of gravity, which we represent, as before, by the height *m* of the inclined plane *m n*, or by its equal *c d*, *m d* being taken equal to *n b*. 2d. The indirect action due to the inequalities of pressure upon the two extremes of the system *m* and *n;* at the upper extremity *m*, conformably to the rules of hydrostatics, the pressure is represented by the height of the fluid column *m a*; at the lower extremity, it is represented by *n b*; the resultant of these two pressures, that which produces motion, will equal then $m\ a - n\ b = a\ d$; as for the pressures which each particle of the system experiences at its sides, perpendicular to *m n*, they will be equal to each other, and reciprocally destroy each other, and have no effect. Thus, the system *m n* will be urged downwards by the two forces *a d* and *c d*, or by their sum *a c*, which is *g* sin. *i*, *i* being always the inclination of the surface.

When the bed is horizontal, as in the section C, the direct action of gravity upon the particles in contact with the bottom, will, it is true, be entirely destroyed by the resistance of the bottom; but the indirect action, or the inequalities of pressure, will amount to

$$aa'-bb'=ac=g \text{ sin. } i.$$

For all other particles m, the moving force will be as above

$$mf+(ma-nb)=cd+da=ac=g \text{ sin. } i.$$

Finally, if the bottom has a counter slope, as in D, the particles upon it will be urged back or up stream, by its relative gravity, $k\,a'=c\,d$; but, on the other hand, they will be urged downward by the difference of the pressing columns aa' and bb', or by ad. Hence it follows, that they will be impelled in this last direction by

$$ad-cd=ac=g \text{ sin. } i.$$

104. It follows, from these different facts, that, in a water course of any form, each particle, in traversing a section having an inclination of surface equal to i, receives from gravity an impulse represented by g sin. i; that is to say, that if the impulse continues during one second, it will produce a velocity equal to g sin. i; this, then, is the expression of the accelerating force, and is dependent solely upon the inclination of the surface. Accelerating force.

This slope, so to speak, may vary at every step, or it may be constant for a long space, in which case, a longitudinal section of the surface of the current forms a right line. This is frequently the case in canals, properly so called, of a constant slope and profile; the surface lines and the bottom lines can neither converge nor diverge, and must be parallel; the surface will then have the same inclination as the bottom, and the sin. i will be$=$sin. e or$=p$ (102), and the accelerating force will be$=gp$.

Retarding force.

Resistance of bed.

105. From what has been said, water running in a canal is constantly subject to the action of an accelerating force; so that, if it encounter no other opposing force, it will descend with an accelerated motion, and its velocity would never be uniform. Nevertheless, it often attains this uniformity in a very short space of time, after which, the acceleration is inappreciable. Experience proves this to be a fact; it is to be seen in most canals, even those of great slope. Thus, Bossut, causing water to run in a wooden canal 656 ft. long, with a slope of 1 in 10, and having divided the canal into spaces of 108 ft. each, has found that each division, excepting the first, has been traversed in the same time. There must then be, after a certain period of time, a retarding force, which destroys at each instant the effect of the accelerating force, and which is equal to it. Thus, water will move along with a velocity acquired in the first moments of its running; a phenomenon similar to that produced in nearly all motion; in that of machines, for example.

But in canals, there can be no retarding force but that which comes from the resistance of the bed. This resistance cannot be called in question; from experiments made with a tube 2.06 ft. long, there was a discharge of 5.22 cubic ft. in 100″; and when its length was doubled to 4.12 ft., dimensions in other respects the same, it took 117″ to discharge the same volume. Thus, the velocity in the tube was diminished in the ratio of 117 to 100; and it can only be that the canal, by reason of its increased length, offered a greater resistance to the velocity; it therefore resisted motion.

Nature of Resistance.

106. Let us examine the nature of this resistance.

When water passes over the surface of a body, there being no repulsion, or negative affinity between the two substances, it wets this surface; that is to say, a thin lamina of fluid is applied to it, penetrating its pores, and it is retained there, both by this engagement of its particles, and by the mutual attraction of the particles for each other.

It is over such a revetment or watery covering, fixed against the sides of the canal, that the water which it conducts must pass. The thin sheet of this mass, immediately in contact with this covering, by sliding along and rubbing against it, mingles its particles with those of the covering—it adheres, and its velocity is retarded. In consequence of the mutual adhesion of the particles, this stoppage, gradually diminishing, is communicated from one to another of the adjacent layers, till it is felt by the most distant fillets. The mass, in consequence, receives a mean velocity less than would take place, without the action of the sides and the viscosity of the fluid.

The cause of this diminution of velocity has often been attributed to the friction of the water against the sides of its bed. Such a friction, if it occurs at all, is of a nature entirely different from that of solid bodies against each other; it depends neither upon the pressure, nor the nature of the rubbing surfaces. Dubuat is convinced, by direct experiments, that the *resistance of water is independent of its pressure.* He has never yet found *any variation in the friction of water upon glass, lead, pewter, iron, woods, and different kinds of earth.* (*Principes d'hydraulique,* §§ 34 *and* 36.)

This last fact might be accounted for, by observing, that in all cases, the friction can only take place upon the aqueous layer which covers the sides of the bed. But a friction independent of pressure? It would seem quite natural to admit, that the resistance could proceed from no other source but the adhesion of the particles of water in motion, both among themselves, and with those of the fluid-covering of the sides of the bed.

This adhesion has been measured by weights. Dubuat found, that to detach tin plates from tranquil water with which they had been brought in contact, there was needed, beside their own weight, an effort of .96 lbs. avoirdupois to 1.03 lbs. square ft. of surface.

Venturi, by means of a remarkable experiment, affords a direct evidence of the effect of adhesion, which enables the particles of water in motion to catch up and carry in their train, those which are contiguous to them in a fluid mass at rest. To a reservoir A, kept constantly full, was fastened a box filled with water, in which was placed a trough CD, open at its ends, and its bottom resting on the edge D. A small tube was placed in the reservoir, with its end at O. As soon as this was opened, the jet which issued, passing through the water which had found its way into the trough, drew with it the part adjacent; this was replaced by that immediately next it, which in its turn was replaced by the water in the box; so that, in a short time, the water fell from the level of GD to *gh*. Fig. 23.

Laws of Resistance.

107. Since the resistance is from the action of the sides of the bed, the greater the extent of these sides, that is to say, the greater the *wetted perimeter* for any unit of length, the greater the amount of resistance.

But this resistance of the perimeter will be shared among all the particles of the section, since their motion is connected by a mutual adhesion; thus, the greater the number of particles, or the greater the section, the less will the velocity of each, and consequently their mean velocity, be changed. The effect of resistance will be in the inverse ratio of the section.

On the other hand, the resistance will increase with the velocity. The greater this is, the greater will be the number of particles drawn at the same time from their adhesion to the sides; and, further, it must draw them more promptly, and consequently expend more force; so that the resistance will be in the double ratio of the velocity. The viscosity of the fluid occasions still another resistance, which becomes more sensible, compared to the first, as the velocity is smaller. Dubuat has observed this important fact, and Coulomb, through a series of experiments, made with his characteristic skill and care, has found that it is simply proportional to the velocity. Thus the expression of ratio between the resistance and velocity involves two terms; in one, the velocity is as the second power; in the other, as the first; this last, which is but a small fraction of the velocity, will disappear in great velocities; it is always inferior to the other, when the velocity exceeds .23 ft., but below this, it preponderates. In short, the resistance experienced by water from its motion in a canal, is proportional to the wetted perimeter, to the square of the velocity, plus a fraction of velocity, and is in the inverse ratio of its section. Experience proves that this is very near the truth.

With the symbols already adopted, in calling bv the fraction of the velocity in question, and a' a constant multiplier, the expression of resistance will be

$$a'\frac{c}{s}(v^2 + bv).$$

Mean Velocity.

108. After what has just been said upon the resistance of the bed and its effects, the different fillets of a fluid in motion in a canal will have a velocity the greater as they are more removed from the sides of the bed; thus they will have different velocities. Nevertheless, in estimating the discharge of a canal, we may admit that the whole mass of water in motion is endowed with a mean velocity; which will be such as, being multiplied by the section of the canal, will give

the volume of water passed in one second. So that if Q represents this volume, s being the section and v the mean velocity, we have $Q = sv$.

Ratio of mean velocity to that of surface.

109. From what has been stated above, it follows that the greatest velocity of a current will be at its surface — in its middle, if the transverse profile is regular — if it is not, then in portions very nearly corresponding with the greatest depths; it is there that is generally found the *thread of water*, or fillet of the greatest velocity.

This velocity of the surface, being that most easily determined by experiment, the knowledge of its ratio with the mean velocity is a subject of great interest in practice; it will enable us to determine this last velocity so as easily to calculate the discharge. The investigation of this ratio has been the object of many hydraulic observers, as we shall see in the article on rivers; we confine ourselves here to what concerns canals.

Dubuat is the only one, in my knowledge, who has made precise experiments upon this subject. They are in number thirty-eight. They were made with two wooden canals 141 ft. in length; the one of a rectangular form 1.6 ft. wide — the section of the other a trapezium whose small base was $\frac{1}{5}$ ft., with its sides inclined 36° 20′ to the horizon (making $n = 1.36$); the depth of water varied from .17 ft. to .895 ft., and the velocity from 0.524 ft. to 4.26 ft. Dubuat concludes, from these experiments, that the ratio of velocity at the surface, to that of the bottom, is greater according as the velocity is less, and that this ratio is entirely independent of the depth; that to the same velocity of surface corresponds the same velocity of bottom. He has observed, also, that the mean velocity is a mean proportional between that of the surface and that of the bottom. Calling u the velocity of the bottom, V that of the surface, and v the mean velocity, he gives the results of his observations by the formula·

$$u=(\sqrt{V}-.298868)^2 \text{ and } v=\frac{V+u}{2}=(\sqrt{V}-.149434)^2+.022332.$$

M. D. Prony, after discussing the experiments of Dubuat, has thought this the more convenient formula:

$$v=V\,\frac{V+7.78188}{V+10.34508}.$$

Here is a small table of some values of v corresponding to values of V, as given by this formula. M. D. Prony, taking a mean term, has thought that, in practice, we may take $v=0.8V$; that is to say, in order to have the mean velocity of a current of water, we may diminish that of the surface one fifth.

V	V	v
metres.	feet.	
0.25	.8202	0.77 V
0.50	1.6404	0.79 V
1.	3.2809	0.81 V
1.50	4.9213	0.83 V
2	6.5618	0.85 V

ARTICLE SECOND.

Formulæ of Motion and Applications.

Two kinds of motion.

110. We have two kinds of motion to consider. Most frequently, the surface of a current in a long and regular canal assumes a constant slope, which is the same as that of the bottom of the bed, and this surface becomes parallel to this bed. Then all transverse sections are equal; the mean velocity is the same in each, and the *motion is uniform.*

But it often happens, that the surface varies from point to point, and is not the same with that of the bottom; so that, at different points of the canal, the sections, and consequently their velocities, are no longer equal. Still, the quantity of water admitted in the canal remaining the same, upon each isolated point, the section of the fluid mass will be constantly the same, and the velocity then will always have an equal value: all, then, is constant, and the motion, without being uniform, *will be permanent.*

1. *Uniform Motion.*

111. We have already remarked (105), that when the water in a canal becomes uniform, the retarding force equals the accelerating force; and that the expression for this last, in such kind of motion (104), is gp; so that we have

Fundamental Equation.

$$gp = a' \frac{c}{s} (v^2+bv);$$

or, making $\frac{a'}{g} = a$, we have

$$p = a\frac{c}{s}(v^2+bv).$$

If, at a portion of the canal where the motion is uniform, we take two points upon the surface of the fluid, whose distance apart we represent by L′, and difference of level, or absolute slope by D, we have $p=\frac{D}{L'}$, and $D=a \frac{cL'}{s} (v^2+bv)$. If we take the canal throughout its entire length, which we called L, and H being the difference between the head and foot of the same; from this difference H, we must take a height due to the velocity v of uniform motion, as we shall soon see (127), and we have

$$H-\frac{v^2}{2g} = a \frac{cL}{s} (v^2+bv).$$

112. It remains to determine the two constant coefficients a and b.

M. D. Prony, in combining the results of thirty experiments made by Dubuat, has undertaken and executed this determination. Some years afterwards, Eytelwein following the steps of Prony, but extending his observations upon ninety-one canals or rivers, in which the velocity varied from 0.407 ft. to 7.94 ft., and the fluid section from .151 square ft. to 28.030 square ft., found $a'=.0035855$, or $a=.000111415$ and $b=.217785$, the English foot being the unit.

Thus, putting for g its value $=32.18$ ft., the fundamental equation for the motion of water in canals will be,

$$p=.000111415\,\frac{c}{s}\,v^2+.0000242647\,\frac{cv}{s};$$

or, observing that $v=\frac{Q}{s}$ (108), Q being the discharge,

$$ps^3=.000111415\,c\,Q^2+.0000242647\,cQs.$$

Of the four quantities Q, p, s, and c, or, remembering that $s=(l+nh)\,h$ and $c=l+2h\,(\sqrt{n^2+1})$, (101), of the four quantities Q, p, h and l, three being given, this equation enables us to ascertain the fourth. As for n, the slope to be given to the banks, it will be indicated by the nature of the soil in which the canal is dug.

Expression of velocity.

113. It is seldom that the velocity is found among the list of problems to be resolved; still, for any case where its direct expression is required, the first of the two equations above gives

$$v=-0.1088946+\sqrt{8975.414\frac{ps}{c}+.01185803};$$

or, more simply, and with sufficient accuracy,

$$v=\sqrt{8975.414\frac{ps}{c}}-0.1088946.$$

Expression of discharge.

114. Consequently, we have from $Q=sv$,

$$Q=s\left(-0.1088946+\sqrt{8975.414\,\frac{ps}{c}+.01185803}\right),$$

or,

$$Q=s\left(\sqrt{8975.414\frac{ps}{c}}-.1088946\right).$$

115. In great velocities, those of 3.2809 ft. for instance, or any above this, where the resistance is simply proportional to their square, we have

$$v=94.738\sqrt{\tfrac{ps}{c}},\ \text{and}\ Q=94.738s\sqrt{\tfrac{ps}{c}}.$$

Let there be, for example, a canal, whose section is a trapezium 13.124 ft. wide at top, 3.2809 ft. at bottom, and 4.92 ft. deep; with a slope of 0.001. Required, the quantity of water which it will convey.

We have $p=0.001$; $l=3.2809$ ft.; $h=4.9214$ ft. With regard to u, or ratio of base to height of banks, the height is that of the trapezium, and the base is one half the difference between the two bases: so that $n=\frac{13.124\text{ ft.}-3.2809\text{ ft.}}{2\times 4.9214}=1$. From this, $s=(l+nh)\,h=(3.2809+4.9214)\ 4.9214=40.366$ sq. ft.; and $c=l+2h\sqrt{n^2+1}=17.2$ ft. Consequently, Q, the quantity sought, is $Q=40.366\left(\sqrt{8975.414\frac{40.36\times.001}{17.2}+.01185805}-.108895\right)=180.87$ cubic feet.

If we neglect the term .01185805 under the radical, we have for $Q=180.843$, which only differs from the above by .027.

The formula above for great velocities would give

$$Q=94.738\times 40.366\sqrt{\tfrac{.001\times 40.366}{17.2}}=180.65\ \text{cub. ft.}$$

Slope and observations. 116. The slope is directly given by the fundamental equation which we have already established (112).

The canal de l'Ourcq furnishes both an example of the mode of its determination, and some remarks worthy of attention.

There were 106.61 cubic ft. of water per second to be disposed of; the projected navigation required there a depth of 4.9214 ft.; and in order that the water should always be at hand for the service of the fountains in Paris, it was necessary that it should have at least a velocity of 1.1483 ft.; the soil was such as to admit of a slope of $1\frac{1}{2}$ base to 1 of height.

We have, then, $Q=106.61$ cub. ft.; $v=1.1483$ ft.; $h=4.9214$ ft.; and $n=1.50$. Moreover, from the given terms of the problem, s is known, for $s=\frac{Q}{v}=\frac{106.61\text{ cub. ft.}}{1.1483\text{ ft.}}=92.843$

sq. ft.; l will also be known, since from the expression $s=(l+nh)\,h$ (Sec. 101), we deduce

$$l=\frac{s-nh^2}{h}=\frac{92.843-1.50\times4.9214^2}{4.9214}=11.483 \text{ ft.}:$$

consequently, we have $c=l+2h\sqrt{n^2+1}=29.227$ ft.; whence the general equation,

$$p=.0001114155\,\frac{v^2c}{s}+.000024265\frac{vc}{s};$$

substituting the numerical quantities, gives $p=0.00005502$: such is the slope indicated by the formulæ.

M. Girard, the engineer who planned the canal, arrived at very nearly the same result. But he has observed, with reason, that aquatic plants, growing always upon the bottom and berms of the canal, augment very much the wetted perimeter, and consequently the resistance; he remembered that Dubuat, having measured the velocity of water in the canal (du Jard) before and after the cutting of the reeds with which it was stocked, has found a result much less before the clearing. Consequently, he has nearly doubled the slope given by calculation, and has carried it up to 0.0001056; the length of the canal being 314966 ft., this gives 33.260 ft. of absolute inclination.

To determine the width or depth.

117. If the dimensions l and h were the one unknown, and the other one of the given quantities of the problem to be solved, we take the values of c and s as functions of these two dimensions, and substitute them in the fundamental equation (112); l would then be deduced by the resolution of an equation of the third degree, and h by that of an equation of the fifth degree.

Let us determine, for example, the width to be given at the bottom of a canal, appointed to conduct 123.60 cub. ft. of water, with a depth of 4.9213 ft., the slope being 0.0001; and the soil of such a character as to require for slope the base to be twice the height. Thus, $Q=123.60$ cubic ft.; $p=0.0001$; $h=4.2649$ ft. and $n=2$.

We substitute these two last quantities in the expressions of s and c (No. 101), which in their turn are substituted in the general equation. This will involve, then, only the unknown term

l; and, making all reductions, and arranging according to the powers of l, we have See Appendix.

$$l^3+23.943\,l^2-46.578\,l-3832=0.$$

Substituting for l, we find, on trial, $l=11.138$ ft.

118. Most generally, l and h are not given terms of the problem; we have only Q and p, or the volume of water which the canal ought to conduct, and the slope which it should have, leaving the engineer to determine the width and depth. To obtain these two unknown quantities, there is but one equation; the problem, therefore, is indeterminate. The engineer then supplies the gap, in giving such a figure as he deems best adapted to the profile of the projected canal; this figure, indicating the relation between the two dimensions, furnishes the equation which was hitherto needed.

In the choice of this figure, regard must be had to the object most important to be fulfilled, and that is adopted which fulfils it with least expense of construction and of maintainance. When it is desired to convey the greatest possible quantity of water to the point where the canal empties, according to the formula of discharge (114 and 115), the volume of water brought down is so much the greater, as the section of the fluid mass is greater, and as the wetted perimeter is smaller; consequently, we must take a figure which, with the same perimeter, presents the greatest surface.

Figure of greatest discharge.

119. Geometry informs us that the circle has this property. The semi-circle, and therefore a semi-circular canal, has the same property, the ratio between the semi-circle and semi-circumference being the same as that between the circle and entire circumference.

Then follow the regular demi-polygons, and with the less advantage, as the number of their sides is less; and so among the most practicable forms we have the regular demi-hexagon, the demi-pentagon, and finally, the half-square.

But these figures are not admissable for canals in earth excavations; their berms, not having sufficient slope, would cave in.

In order that they should be sustained without revetment, they should have a slope of from 1.50 to 2 of base to height, as there is more or less consistency in the soil; in the regular semi-hexagon, where the slope is larger than the other named polygons, it is only 0.58. A slope of 1 is only adopted in excavations of small importance or for temporary use; but for canals, the slope of 2 to 1 is usually adopted, and sometimes $2\frac{1}{2}$; such was the slope adopted at the canal of Languedoc.

120. As the usual profiles of canals are trapezoidal, the question of figure of greatest discharge is reduced to taking, among all the trapeziums with sides of a determinate slope, that which yields the greatest section for the same wetted perimeter.

Since the section s, or $(l+nh)\,h$, should be a *maximum*, its differential will be zero, and we have

$$hdl + ldh + 2nhdh = 0.$$

Since the perimeter remains constant, the expression $c=l+2h\sqrt{n^2+1}$ (Art. 101) being differentiated, gives us $0=dl+2dh\sqrt{n^2+1}$. The value of dl, derived from this equation, and substituted in the preceding, gives

$$l = 2h\left(\sqrt{n^2+1} - n\right).$$

With this value of l, we have

$$s = h^2\left(2\sqrt{n^2+1} - n\right) = n'h^2,$$

by making $2\sqrt{n^2+1}-n=n'$; and

$$c = 2h\left(2\sqrt{n^2+1} - n\right) = 2n'h.$$

Putting these equivalents of s and c in the fundamental equation of motion (112), it becomes

$$\frac{pn'^2h^5}{2} = 0.0001114155Q^2 + 0.0000242651Qn'h^2.$$

This, and the preceding equation, give for l and h the *maximum* sought.

Let us take, for example, $Q = 70.6632$ cub. ft., $p = .0012$, $n = 1.75$. The second of the above equations is reduced to

$$h^5 - 1.2522h^2 - 178.04 = 0.$$

Making, by a first approximation,

$h = 2.82$ ft., we have . . . . $-9.6593 = 0.$
$h = 2.85$ ft. . . . . $-0.1821 = 0.$
$h = 2.850567$ ft. . . . . $+0.0002 = 0.$

So that the true value of h will be 2.850567 ft. This will give for l, which is $2h\left(\sqrt{n^2+1}-n\right)$, $=1.5107$ ft.

These dimensions are those of the stream. But the depth of the excavation should be greater. It would be well to increase it to . . . . 3.937 ft.

The breadth at bottom remains the same . . 1.5105 ft.

The breadth at level of earth will be . . . . 15.29 ft.

There will then be, per running foot of cut, an excavation of . . . . 33.1 cub. ft.

In homogenous earth, so long as the depth of excavation does not exceed 6½ ft., and the upper width 16½ ft., the expense of digging will be proportional to the volume of excavation, and the figure of least section will therefore be the most economical.

Rectangular Canals.

121. As for those canals where there is no fear of caving in, such as those excavated in rock, or protected with masonry, which are more particularly termed Aqueducts, as well as those in wood and mill courses, they most always have a rectangular form. Still, as we have seen, the regular demi-hexagon of the same section will conduct more water; but simplicity, facility and economy of construction have prevailed. We must remember, that the dimensions of the rectangle

should have a width nearly double the depth of the fluid mass it is destined to carry, and consequently it should be $\sqrt{\frac{2Q}{v}}$.

2. *Permanent Motion.*

122. We have seen (110) that permanent motion differs essentially from uniform in this, that the mean velocity in each section, remaining constant, is not the same as in the adjacent sections; consequently, the sections of water are no longer equal to each other, their depth is not the same, the surface of the fluid is not parallel to that of the bed of the stream, and its inclination varies from one point to another. We have examples of such motion in canals too short for the velocity to acquire a uniformity, at the head and foot of long canals, and in those whose bottom is horizontal, etc.

It is but lately that the attention of philosophers and engineers has been directed to this subject; among others, we may note MM. Poncelet, Bélanger, Saint-Guilhem, Vauthier and Coriolis. I would refer to their works for details and applications, and here confine myself to establishing the equation of motion and the indication of its uses.

Equation of Motion.

Fig. 24.

123. Let there be a current endowed with permanent motion, and let us regard that part of it comprised between A and M. Through these two points of the surface, and through N infinitely near to M, imagine transverse sections AO, MP and Np, made perpendicular to the axis of the current. From the points A and M, we draw the horizontal lines AE and Mt; EM will be the fall of the surface from A to M, which we designate by p'; tN, or the elementary increment of

the slope, will be dp' or MN sin. i, i being always the angle tMN of inclination of the surface to the horizon. Let us consider upon the section AO, taken up stream for the point of departure, the particle having the mean velocity of the section, whatever else may be its position, and let mm' be the path which it describes as far as MP. Call z the length of this path, t the time employed in traversing it, and v the velocity of the particle on arriving at m. We have, then, $m'n'=dz$; dt will be the time in passing dz, and dv the increment of velocity during this passage (which will be $-dv$, when motion is retarded).

The forces which act upon the particle m, while traversing $mm'n'$ are: first, on one side, gravity, which tends to accelerate its motion, and whose whole action, according to what we have said in Sec. 103, is g sin. i; second, on the other side, the resistance of the bed, which tends to retard its motion, and whose expression is (Sec. 107) $a'\frac{c}{s}(v^2+bv)$.

These two forces acting opposite to each other, their resultant, or the effective accelerating force, will be equal to their difference. But in all variable motion, the accelerating force is also expressed by the increment of the velocity, divided by that of the time, or by $\frac{dv}{dt}$; we have then

$$\frac{dv}{dt}=g \text{ sin. } i-a'\frac{c}{s}(v^2+bv).$$

Multiplying all the terms by dz, (remarking that $\frac{dz}{dt}=v$, the space, divided by the time, equalling the velocity; remarking, further, that dz sin. $i=dp'$, since for dz or $m'n'$ we may take MN, which will not differ from it, save in extreme cases, but by an infinitely small quantity of the second order, and that MN sin. $i = t\text{N} = p'$,) we have

$$vdv = gdp' - a'\frac{c}{s}(v^2 + bv)\ dz.$$

Such is the equation established by M. Poncelet.

Integrating, determining the constant for the section A, when $p'=0$, $z=0$, and $v=v_0$, we have

$$\frac{v^2}{2} - \frac{v_0^2}{2} = gp' - \int a'\frac{c}{s}(v^2 + bv)\ dz.$$

But (Sec. 108) $v = \frac{\text{Q}}{s}$; and if we designate by s_n the area of the section at the final point M, and by s_0 that at the initial point A, which let us divide by g, and remembering that $\frac{a'}{g} = a = 0.000024265$ (112), and that $b = 0.000111415$, we have finally

$$p' = \frac{\text{Q}^2}{2g}\left(\frac{1}{s_n^2} - \frac{1}{s_0^2}\right) + \int\left(0.0001114155\,\frac{c\text{Q}^2}{s^3} + 0.000024265\,\frac{c\text{Q}}{s^3}\right) dz\,;$$

a formula which gives directly the slope of the surface from A to M.

In the application, the quantity under the sign $\int$ may be integrated by approximation. For this purpose, divide the arc AM or z into portions, AB, BC, CD, etc., whose lengths are such that the divisions of the arc may be taken, without sensible error, for right lines. Designate these lengths by z', z'_2, z'_3 . . . . . z'_n, and the areas of the sections at A, B, C . . . . M, by s_0, s_1, s_2, s_3 . . . . . . s_n, and by c_0, c_1, c_2 . . . . c_n, their respective wetted perimeters. We measure or take immediately these lengths, sections and perimeters upon the given stream, and all will be known in the integral, which will become

$$0.0001114155\left(\frac{z'_1 c_1}{s_1^3}+\frac{z'_2 c_2}{s_2^3}+\dots\frac{z'_n c_n}{s_n^3}\right) Q^2+0.000024265$$
$$\left(\frac{z'_1 c_1}{s_1^2}+\frac{z'_2 c_2}{s_2^2}+\dots\frac{z'_n c_n}{s_n^2}\right) Q.$$

Let us represent by M the multiplicator of Q^2, and by N that of Q; let us make also $\frac{1}{2g}\left(\frac{1}{s_n^2}-\frac{1}{s_0^2}\right)=D$, the equation will then be $p'=(D+M)\,Q^2+NQ$.

124. From this we deduce

Discharge.

$$Q=-\frac{N}{2\,(D+M)}+\sqrt{\frac{p'}{D+M}+\left(\frac{N}{2(D+M)}\right)^2}.$$

In the discussion of Rivers, in the following chapter, we shall have occasion to apply this formula, with its details, to streams whose form and delivery were otherwise known, and we shall see that its deductions are not far from the truth.

In canals where the slope of the bed and the profiles are constant, the calculations are much simplified; the depth of water at any one station will be sufficient to know its section and wetted perimeter; moreover, the depths, with the inclination of the bed, will give that of the surface.

As an example, let us determine the volume of water which a rectangular mill course, 8.202 ft. wide, with a horizontal bed, will conduct to a mill. At four points, distant 328.1 ft. apart, we take four depths, noted in column h of table. Since the canal is rectangular, and $l=8.202$ ft., then $s=8.202h$ ft., and $c=8.202+2h$ ft. We calculate these values for the different stations, and then, through these, those of $\frac{z'c}{s^2}$ and $\frac{z'c}{s^3}$. All are in the above table.

No.	z'	h	c	s	$\frac{z'c}{s^2}$	$\frac{z'c}{s^3}$
	feet.	feet.	feet.	sq. ft.		
0	0	5.052	18.306	41.44	0	0
1	328.09	4.901	18.004	40.20	3.655	.0909
2	328.09	4.845	17.892	39.74	3.717	.0935
3	328.09	4.573	17.348	37.51	4.045	.1078
	P	.479			11.417	.2922

The canal being horizontal, $p'=5.052-4.573=.479$ ft.

We have $D=\frac{1}{64.364}\left(\frac{1}{37.51}-\frac{1}{41.44^2}\right)=$ 0.000001995

$M=$ sum of $\frac{z \cdot c}{s^3}\times 0.0001114155=$ 0.00003255

$N=$ sum of $\frac{z \cdot c}{s^3}\times 0.000024265=$ 0.0002770

$\frac{N}{2(D+M)}=4.0092$, $\frac{p'}{D+M}=13866$, $\left(\frac{N}{2(D+M)}\right)^2=16.0742$ ft.

So that $Q=-4.0092+\sqrt{13866+16.0472}=113.81$ cub. ft.

With the formula for uniform motion in taking a mean height between the extreme heights, and for a slope per foot, .479 divided by 984.27 ft., the sum of the $z\cdot$, we have

$$Q=-4.299+\sqrt{15072+18.478}=118.54 \text{ cub. ft.}$$

Slope of Surface.

125. The equation (123) which gives the slope of the surface of the current knowing some of the sections, will further, by the taking of one depth only, enable us to trace in its progress the curve described by a fluid point of the surface of a water course in a canal, whose slope, profile and discharge are otherwise known.

For the place, when the depth of water is given by the aid of the profile, it will be easy to establish its section and wetted perimeter; let us designate them by s_0 and c_0. Take a second station, at a distance $z\cdot$ from the first, so small, that in this distance there shall be but little variation in s_0 and c_0, and so that they may be regarded as constant in the expression of the resistance of the bed, and we have

$$p'=\frac{Q^2}{2g}\left(\frac{1}{s_1^2}-\frac{1}{s_0^2}\right)+ac_0\left(\frac{Q^2}{s_0^3}-\frac{Q}{s_0^2}\right)z\cdot.$$

We may neglect the first part of the second member at the first trial, which amounts to supposing a uniform motion throughout the whole length $z\cdot$, and we shall have the first approximate value of p'. This will enable us, knowing the slope of the bed, to assign very nearly the depth of the stream at the second station, and consequently gives us s_1. All will then be known in the above equation, and we have a second and more approximate value of p' than the first. If it is thought best, we are able from this to calculate a third, which shall be still more exact. In the same manner, we may determine the depth at the third and fourth stations, and so arrive at all the ordinates of the curve required to be constructed.

126. But this method involves much uncertainty, and many suppositions, and often leaves us much embarrassed. We can avoid, in part, these inconveniences, and go directly to the solution of the problem, by introducing the slope of the bed in the problem, according to the method of M. Bélanger.

For this purpose, let us take in hand the first differential equation of Sec. 123; and we remark, that the angle i, or tMN, or MNs (Fig. 24), is composed of two other angles: first, MNr, which measures the inclination of the surface upon Nr, parallel to the bottom of the bed Pp; designate this by j: second, the angle rNs, which this bottom makes with the horizon, and which we have already called e; so that $i=j+e$, and consequently, $\sin. i=\sin. j \cos. e+\sin. e \cos. j$. But $\sin. e=p$ (Sec. 101), $\cos. e=\sqrt{1-p^2}$, and $\cos. j=1$, considering the smallness of the angle j; thus $\sin. i=\sin. j\sqrt{1-p^2}+p$, and the equation becomes

$$\frac{dv}{dt}=g \sin. j\sqrt{1-p^2}+gp-a'\frac{c}{s}(v^2+bv). \qquad \text{(A)}$$

The term $\frac{dv}{dt}$ may take a finite form, which will depend upon the figure of the bed. When the canal is of small extent, we usually consider the slope as uniform, with a mean width l. From this supposition results $s=lh$ and $c=l+2h$; so that $v=\frac{Q}{s}=\frac{Q}{lh}$, and $dv=-\frac{Qldh}{l^2h^2}$; moreover (Sec. 123), $v=\frac{dz}{dt}$ or $dt=\frac{dz}{v}=\frac{lhdz}{Q}$; then $\frac{dv}{dt}=\frac{Q^2ldh}{l^3h^3dz}=\frac{Q^2l}{l^3h^3}\sin. j$, since $\frac{dh}{dz}=\frac{Mr}{MN}=-\text{tang.}\ i$ or $-\sin. j$.

Substituting this value in the equation (A), neglecting p^2, which will always be small compared to 1, substituting for g, a' and b their numerical values (112), and evolving sin. j, we have

$$\sin. j = \frac{Pl^3h^3 - \{0.0001114155(l+2h)Q^2 + 0.0000242651(l+2h)lhQ\}}{.031073\ lQ^2 - l^3h^3}$$

We have taken for the curve of a fluid thread of the surface of the stream, a polygon, each of whose sides has a finite length MN = $z^{\cdot}$, and whose inclination relative to the bed is j: the difference Mr between the depths of the two extremities of a side will be its slope compared to this bottom; designating it by p'', we have sin. $j = -\frac{p''}{z^{\cdot}}$, and consequently,

$$p'' = \frac{pl^3h^3 - \{0.000111415\ (l+2h)\ Q_2 + .0000242651\ (l+2h)\ lhQ\}z^{\cdot}.}{l^3h^3 - .031073\ lQ^2}$$

The series of values of p'' will enable us to trace the polygon, or required curve.

Instead of comparing the slopes to the bed, we might compare them with the horizon, and thus have their value p', in observing that $p' = p'' + p$.

ARTICLE THIRD.

Inlets of Canals.

Canals, with the exception of those for navigation at their points of departure, receive their water from reservoirs or retaining basins placed at their head, and which most frequently are portions of the river whose level has been raised for this purpose by dams.

The head of the canal, at the point for receiving water, is either entirely open, or furnished with gates. Let us examine these two cases.

1. *Canals of open entrance.*

Fall at entrance of Canals—its value.

127. Water, on its entrance in an open canal, forms a fall, its level being lowered for a certain distance; then it is elevated a little by light undulations, beyond which the surface takes and maintains a form very

nearly plane and parallel with the bed, its slope and profile being always considered as constant. The velocity is accelerated from the top to the foot of the fall; it then diminishes during the elevation of its surface, and soon after, its motion continues in a manner sensibly uniform. Dubuat, who has made a particular study of the circumstances of motion at the entrance of canals, and throughout their course, has found such an order of things established, that when the motion has become regular and uniform, the velocity of the surface is very nearly that due to the entire height of the fall, and that the *head due to the mean velocity is equal to the difference between the height of the reservoir and that of the uniform section.* So that if H represent the height of water in the reservoir above the sill of entry into the canal, h the height of the uniform section, that is to say, the constant depth of the current after it has attained a uniform motion, and v the velocity of this motion, we have $H - h = 0.015536v^2$; or rather, $0.015536\frac{v^2}{m^2}$, m being the coefficient of contraction which the fluid mass experiences at its entrance into the canal, a contraction which occasions a greater fall.

Dubuat, from several experiments made with wooden canals (109), with heights of reservoir H from .394 ft. to 2.887 ft., has found that m varies from 0.73 to 0.91; but he remarks, that in great canals, where the height due to the velocity is small compared to the depth, the contraction will be less, and he thinks there would be no sensible error in taking $m=0.97$. Eytelwein assumes 0.95 for large canals, and 0.86 for the narrow, such as is adopted for most mill courses. He, as well as Dubuat, supposes, for these coefficients, that the bottom of the canal is at the same level with the bottom of the reservoir, and that it is but a prolongation of it. If this were not the case, there would be a contraction at the bottom, and the value of m would be a very little smaller (32); however, the experiments reported in Sec. 39 lead me to think it would be but a very slight quantity.

Modes of diminishing the fall.

128. The fall which takes place at the entrance of a canal, by diminishing the depth h, lessens the discharge Q, of which this depth is an element. So that, in order that the canal should receive all the water which it can afterwards convey, we must prevent the fall.

Theoretically, to accomplish this end, we must enlarge the upper part of the canal, for a length somewhat beyond $.015536 \frac{v^2}{p}$ ft., so that the mean widths of the new profile should increase as they approach the reservoir, with an inverse ratio to the velocity of the stream at each of these widths, beginning with 0, its value in the reservoir, till, by the uniform acceleration of its descent, it reaches v ft. at the foot of the enlarged part. According to this law, the width at the reservoir should be infinite, since the velocity is zero. Such a case would be impracticable, and any approach to it would involve much labor and expense.

Consequently, the engineer who, without involving himself in unnecessary expense, desires to obtain for the canal all the water that can reasonably be expected, will be content to widen the approach, and in doing this, must be governed by local circumstances. For instance, if the head is to be laid in masonry, he will give to the approach the form of the contracted vein; that is to say, taking the width of the canal as a unit, we shall have for length of the enlarged part 0.7, and 1.4 for width at the mouth, as comprising the full sweep to be given to the angles. But it is not worth while to exaggerate the advantages from these widenings, as the discharge by them will hardly be increased by more than some hundredths.

Effective slope.

129. Dubuat also concludes, from his observations, "*that the velocity and section are uniformly established at a certain distance from the reservoir, just as if uniformity commenced at the origin of the canal.*" (§ 177.) In this case, we may suppose the fall to be made suddenly on its entrance to the canal, and thence the fluid surface maintains a uniform slope. Its value is obtained (101 and 111) by dividing the difference of level of the two points by their distance apart; one may be taken at the origin of the canal, and according to our supposition, its level will be less than that of the reservoir, by a quantity equal to the height of the fall H—*h*. Consequently, if D is the difference of level between the reservoir and any point of the surface at the distance L from the reservoir, but where the motion has acquired its uniformity, p being always the effective slope, we have

$$p=\frac{D-(H-h)}{L}=\frac{D-0.015536v^2}{L}.$$

Formula of Discharge.

130. With these given quantities, we can resolve the various questions pertaining to a canal from a reservoir, supposing always that the motion becomes uniform, which will not be the case, unless the canal has a certain length, or should it have no inclination, or approach 90°, etc.

Let us resume the equation, $H-h=0.015536\frac{v^2}{m^2}$,

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and in place of v substitute its value, given in Sec. 113, and we have

$$H-h=\frac{0.015536}{m^2}\left(\sqrt{8975.414\frac{ps}{c}}-.108895\right)^2.$$

Moreover, we have

$$Q=s\left(\sqrt{8975.414\frac{ps}{c}}-.108895\right).$$

By means of these two equations, in giving to s and c their expression, as functions of the dimensions of the canal, and substituting the preceding value of p, when p is not directly given, we can determine either the discharge, or the slope, or one of the dimensions; the other quantities being known. I give an example.

Suppose we purchase the site where it is intended to locate the entrance to the canal, with the condition that it shall be rectangular in form, open to the height of the dam, with a width of 13.124 ft., and whose sill is to be 6.562 ft. below the ordinary low-water line. We wish to conduct this water to a mill distant 869.438 ft., so that the surface of the stream, on its arrival there, shall not be over 1.4436 ft. below the low-water mark of the reservoir above. What will be the quantity of water conducted to the mill?

The cutting being made in the dam, the rectangular canal 13.124 ft. by 6.562 ft. deep is fitted in; the clause of the grant forbids any attempt to enlarge the approach; and every alteration within the appointed limits would diminish the discharge.

Since the canal is rectangular, and 13.124 ft. wide, we have $s=13.124h$ ft., and $c=13.124+2h$; moreover, $p=\frac{1.4436-(H-h)}{869.4384}$ $=\frac{h-5.1184}{869.4384}$ ft., H being 6.562 ft. Although the canal is large, so that the coefficient of contraction would probably be above 0.95, yet, to be prudent, we will take a mean between those indicated by Eytelwein, and call it $m=0.905$. With these values, the first of the two equations above will be

$$6.562-h=\frac{0.015536}{.905^2}.\left(\sqrt{8975.414\,\frac{13.124h\,(h-5.1184)}{869.4384\,(13.124+2h)}}-.108895\right)^2.$$

Reducing

$$6.562-h=.018969\left(\sqrt{135.47h\,\frac{(h-5.1184)}{(13.124+2h)}}-.108895\right)^2$$

gives us the value of h. To obtain it, put successively for this unknown quantity in the second member, several numbers; first, 6.234 gives $h=5.889$ ft.; which in its turn gives 6.114. In this manner, we obtain successively 5.968, 6.053, 6.001, 6.040, 6.014, 6.034, 6.020, 6.027, 6.0237 ft. Thus, the true value of h falls between these two last numbers; let us take the smallest, $h=6.0237$ ft. Then $p=\frac{6.0237-5.1184}{869.4384}=0.001041$ ft.

All the quantities required to ascertain the discharge being known, we introduce them into the second equation, and so obtain $Q=417.795$ cub. ft. Such is the volume of water per second which the canal will lead to the mill.

When the velocity of the current is required to be 3.28 feet or more, we substitute the expression for velocity given in Sec. 115, and the two equations to be used will be

Vide Appendix.

$$H-h=\frac{139.44}{m^2}\cdot\frac{ps}{c} \text{ and } Q=94.738s\sqrt{\frac{ps}{c}};$$

or, supposing a mean width l, and taking always $m=.905$,

$$H-h=170\,\frac{plh}{l+2h} \text{ and } Q=94.738lh\sqrt{\frac{plh}{l+2h}}.$$

The slope p will be given either directly, or by the expression

$$p=\frac{D-(H-h)}{L}.$$

In the above example, the values of H, l and p, put in the first of these equations, which is of the second degree, will give readily $h=6.027$ ft.; also, $p=.001045$ and $Q=418.86$ cub. ft.; results nearly identical with the preceding.

The greatest dynamic force of water conducted by a canal.

131. Among the questions relating to the admission of water in canals, there is one of too much interest to millwrights for us to pass it by without a notice in this treatise.

The force of a current to move machinery depends not only upon the quantity of water which it conveys, but also upon the height from which it falls; so that this force will be measured by the product of the quantity with the height of the fall of water. The greater the slope given to the canal, the greater will be the amount of water brought, and this is one of the factors of the product; but, at the same time, the fall (the other factor) is diminished, and it will be found that the product having been at first augmented with the slope, will after that be diminished, and then continue to decrease. There is then a *maximum* of power, which it is essential to determine and put in use. Without employing analytical formulæ, this determination can be arrived at in a simple manner, as will be seen in the following example.

Let us resume that given in the last number, and let us suppose the height of fall there to be 14.764 ft. The water taken by the canal has arrived at the mill with a loss of level of 1.447 ft.; consequently, the effective fall will only be 13.317 ft. In multiplying this by the quantity of water brought down, 418.86 cub. ft., we have for the product 5577.9 cub. ft.; the corresponding slope was 0.001045. Let us increase this slope successively to 0.0015, .002, .0025 and .003; the respective products of the quantity by the fall will be 1859.42, 1931.12, 1939.94 and 1907.45 cub. ft. The slope of .003 has already occasioned a diminution; in trying that of .0026, the product will be 1938.18 cub. ft.; whence we conclude that the *maximum* of effect lies between the slopes of 0.0025 and .0026. Finally, as the variations of the product are very small between 0.002 and 0.003, we adopt, between these limits, those best suited to the locality and nature of the machinery used; there may be some for which a great fall will be preferred.

I will remark that the given solutions of all the problems in question can be regarded only as simple approximations; for in order that they should be exact, the bases on which they rest, that is to say, the conclusions which Dubuat has drawn from experiments, should be explicitly confirmed by observations made upon great canals; and it would moreover be necessary to be quite sure that the water, before it reaches the extremity of the canal, has attained a uniform motion, and we have but limited means of coming to a positive assurance.

If water which is in the reservoir of a river to which a canal has been adapted, should arrive there directly, with an acquired velocity, the height of fall which takes place at the entrance will be less than that indicated (127) by a quantity equal to the height due to this velocity.

2. *Canals with Gates.*

When a canal receives its water through openings of a system of gates, established at its head, which is generally the case with mill courses, either the upper edge of the orifice will be completely and permanently covered by the water, already passed into the canal, or it will not.

Discharge when water does not cover the opening of the gate.

132. If the head above the centre of the orifice is great, so as to exceed two or three times the height of the orifice, its upper edge will not be covered by the water below, and the discharge will be the same as if there had been no canal. Experiments with orifices in thin sides and furnished with additional canals, which have been already reported (39), leave no doubt upon this subject; they justify an assertion, long since made by Bossut, the exactness of which has been questioned.

This hydraulician fitted to an orifice .0886 ft. high and .4429 ft. wide, made at the bottom of a reservoir, a horizontal canal of the same width, and 111.55 ft. in length; he produced in it currents under heads of 12.468 ft., 7.802 ft., and 3.937 ft., and he received "at the extremity of the canal, the same quantity of water that issued from the orifice when the canal was taken away." (*Hydrod.*, § 750.)

The cause of this equality is apparent. When the water is urged by a great head, and consequently issues with great velocity, the contraction it experiences on all sides renders the section smaller immediately beyond the interior plane of the orifice, so that, on issuing, it touches neither the sides nor the bottom of the canal; it acts as if it were projected in air, and the discharge continues the same that it would if this were really the case. Beyond the contracted section, the vein dilates, it is true; it joins the sides of the canal; it meets with resistance, and runs less swift; but then it is too far from the orifice to react against what issues from it, so as to reduce its discharge. This will always be given by the formula $ml'h'\sqrt{2gH}$, l' and h' being the width and depth of the orifice; m will have the same value as for orifices in thin partitions (26).

But if this is true in case of the canal adapted to an orifice with sharp edges, opened in a side of the reservoir, does it follow that it will be equally so for a canal furnished with a common gate, sliding in grooves made in the middle of two posts of considerable thickness, and gates, as is most generally the case, with canals placed somewhat below their inlets? I have my doubts. In experiments which I have elsewhere recorded, (*Annales des Mines*, tome III., p. 376, 1828,) where I believed the circumstances were nearly similar to the case of orifices in thin partitions, and where I expected to have coefficients of 0.65, I have found those of 0.67 to 0.71. Generally, we take 0.70 for the ordinary gates of flumes, but without any precise fact to justify us in so doing. It is principally to procure such facts upon this important point, as well as to afford correct ideas upon every thing pertaining to the admission of water in canals, that MM. Poncelet and Lesbros have undertaken their great work upon the flow of water; it is unfortunate that this undertaking has not yet been completed.

In such a state of things, and without adopting another coefficient for each particular case, the volume of water which enters a canal furnished with large gates, and under a great head, may be had approximately by the formula $0.70lh'\sqrt{2gH}$.

133. When the water, impelled beyond the gates by a great head, falls into the canal, it meets a resistance which diminishes gradually its first velocity, and so increases the section of its current. If the width of the canal is constant and equal to the opening of the gate, it will be the depth which receives the gradual increase, so that the surface of the fluid below the orifice, or rather below the point of greatest contraction, up to that where the increase of depth ceases, will present a counter slope. Frequently, masses of water will be detached from the summit, and will, rolling back, return towards the orifice; usually, they will be retained, being as it were repelled by the velocity of the stream; though sometimes they will return even to the gate, and re-cover the orifice, though but for a moment. Even in this case, the discharge will be the same as if there were no canal, and it will be calculated by the formula of the preceding number.

Case when the orifice is covered again.

134. These phenomena do not occur when the head is small. Water, on issuing from the gates, is in contact with the sides of the canal; it experiences a retarding force, which is communicated to the fluid at the instant of its passage through the orifice; the dis-

charge, and therefore its coefficient, is lessened; but we have no further guide for its determination. There may be some cases where, with a very small head, the gate is without sensible influence; thus Eytelwein has found the same discharge, whether the gate was wholly raised, or slightly dipped in the down-stream side.

But in case it is immersed any considerable depth, and the fluid vein at its issue is entirely covered over with still water, we are brought back to the case mentioned (95), and the height due to the velocity of issue will be the difference between the elevation (above any given point) of the surface above the gate and of that below the gate. For the elevation below the gate, we take the height or depth of water in the canal, when its motion has become regular; as that immediately at the gate would be found too small. Consequently, if h is the height in the canal, H′ the height up stream above the sill of the inlet, the discharge of the orifice of the gate, and consequently that of the canal, will be expressed by

$$mlh' \sqrt{2g(H' - h)}.$$

But the discharge of the canal, the motion having become uniform, is also (114)

$$s\left(\sqrt{8975.414\frac{ps}{c}} - .108895\right).$$

We have, then,

$$mlh' \sqrt{2g(H' - h)} = s\left(\sqrt{8975.414\frac{ps}{c}} - .108895\right),$$

an equation which enables us to solve the various questions relative to canals furnished with gates at their heads.

Suppose, for instance, we would determine the quantity h'; we must raise the gate, at the entrance of a long rectangular canal of 4.265 ft. width and .001 slope, in order that the water may have a depth of 2.625 ft.; the width of the gate is 3.609 ft., and the height of the reservoir 3.937 ft. We take $m=0.70$ (132): we have then $l'=3.069$ ft.; $H'=3.937$; $h=2.625$; $l=4.265$; $p=0.001$; $s=4.265\times 2.625=11.195$ sq. ft.; $c=4.265+2\times 2.625=9.515$ ft. These numerical quantities, substituted in the equation above, give us

$$23.209h'=35.180\text{ ; whence } h'=1.514\text{ ft.}$$

CHAPTER II.

RIVERS.

Man establishes and excavates canals; nature has established and excavated the beds of rivers: she has accomplished this conformably to the laws from which she never swerves, and by which she maintains her work. We can in no wise change them, and but slightly modify them; the engineer who has done all for canals, can accomplish but little with rivers. His role is confined to observing the circumstances of the motion and action of their waters. Consequently, after a few remarks upon the general formation of their beds, we shall examine successively the nature of their motion, its influence upon the form of their surface, the respective velocities in different parts, and the methods of gauging their waters; we shall then discuss the subject of backwater, occasioned by dams and bridges, and conclude with some observations concerning the action of water upon constructions made in their bed.

ARTICLE FIRST.

The Establishment of the Bed.

Formation of Bed.

135. The surface of the globe, at its origin, or immediately after its consolidation, was not entirely smooth; it had elevations and depressions; it presented

undulations of different orders, the principal of which have formed our great mountain chains.

The atmosphere, by its decomposing agency, rain-waters, both by their currents and erosive action, have quickly assailed this surface of rock. They reduced this surface to earth; they abraded, cut through and furrowed out valleys of various magnitudes, directed generally according to the line of greatest slope of those parts of the earth presented to their action. The remains or debris of the elevated portions were borne away and spread over the lower, covering them with alluvial. All this work of nature was anterior to the epochs of the last great flood, from which has resulted the actual state of our continents, and which has reduced our rivers and streams to the quantity they bear this day.

136. The waters which now fall upon the surface of the earth, unite and flow into the hollows, gorges and vales excavated in primitive times. In passing over the alluvial, they there open and shape new channels for themselves.

In mountains with steep sides, they are constrained to follow in ancient courses, and have produced and are producing but slight changes. When running immediately upon rock, which is indeed quite rare, their tendency to excavate or enlarge their beds can have but a scarcely appreciable effect in the lapse of some centuries. Most generally, they flow over the blocks, fragments and debris of rocks, fallen from the steeps and ridges which border the channel. In great freshets, they urge forward and bear these materials further away, whose place is afterwards refilled by others. They move them the more easily, and carry them further, according as the ground slopes more, and according as their volume and specific gravity are less; the effects of slope are barely appreciable, save at the origin of valleys; the specific gravity of rocks and rocky matter varying only from 2.2 to 2.7, will be without marked influence, except in the case of metallic particles, and some peculiar stones: it is, then, the volume which has the greatest influence as to the distance of the transport of rocks and their debris. So, in general, when we descend a great valley, we find at first, at a small distance from its origin, in the bed of the torrent or the river occupying its bed, angular pieces of rock; then, and in succession, we find blocks rounded smaller and smaller, round pebbles, gravel, and finally we meet with little else but sand and earth. Finally, this decreasing progression in the volume of substances forming the bed of a river is not solely the effect of the successive impulses of great currents. There is still another cause, which, though seemingly weak, is not less effectual in its results, when we regard the duration of its agency, often exceeding a long lapse of centuries; it is the decomposing power of the atmosphere, conjoined with the action of running water. The further distant these materials are from their origin, the longer will be the time since they were borne away; and consequently, the longer will time have operated on them to have reduced their primitive volume. But it is only as a general feature, I repeat, that the substances constituting the bed of rivers is ascertained to be of less volume, the further down stream they are found; for we very frequently find sand in the elevated parts of the river, and pebbles in the lower parts. Touching the matter of pebbles found in these lower portions, I would remark that most generally they were already present in the transported earth or "alluvial" through which the stream has opened for itself a channel, and have been exposed by the rivers, in times of freshets.

In regions slightly elevated, but where the river runs between hills, its bed is still limited, and it can be extended but little.

It will only be, then, in plains and large valleys, whose soil is moveable, that rivers less constrained, and finding fewer obstacles in their course, establish in reality a channel whose dimensions bear a certain relation to the nature of the soil and the volume and velocity of its water. If the earth has not tenacity apportioned to this velocity and this volume, it will yield to the action of the water, and its channel will be deepened and enlarged. If otherwise, the depth or the width is too great, the river will be reduced in its dimensions by deposits on its bottom or at its sides of stones and earths brought down in freshets.

Establishment of the Regime.

137. When a proper relation is established, so that the channel contains all the water brought down by the river, in its great freshets, without injury, it is said to have acquired *stability*, and the *regime* of the river is established.

The velocity of the regime is strictly related to the species or rather size of the substances which form its channel.

Dubuat has made some experiments upon this subject of great interest. He has taken different kinds of earths, sands and stones, which he placed in succession upon the bottom of a wooden canal: by inclining it differently, he has varied the velocity of the water passed through it, and has verified how much is necessary to put each substance in motion; he had for

Potters' clay, ...	.2624	ft. per second.
Fine sand, ...	.5249	" "
Gravel from the Seine, size of peas, .	.6233	" "
Pebbles from sea, one in. in diameter,	2.132	" "
Flint stones, size of hen's eggs, . .	3.281	" "

He then spread a bed of sand upon the bottom of the canal, and caused the water to run over it with a velocity of .984 ft. After a while, the surface of this sand presented a series of undulations, or of transverse furrows, .394 ft. wide;—the slope towards the up-stream side was very gentle, that on the down-stream was very steep. The grains of sand, urged by the current, rose upon the first; arrived at the summit, they fell, by virtue of their weight, along the counter slope, up to the foot of the next furrow, when they were again taken up by the current; they were one half an hour in passing one ridge. They consequently would have passed through about nineteen feet in twenty-four hours.

It is in this wise that the sands of Dunes travel onwards, urged by a succession of impulses from the winds.

Cause of greater width.

138. All else being equal, the banks of the channel of a river resist the action of its water less than the bottom; so that it has more width than depth. Independent of this action, the banks are subjected to that of their weight, which tends to produce a caving in of the substances composing it; while this same force, pressing the materials of the channel upon those which are beneath, a pressure which increases the friction, renders their displacement more difficult. Moreover, when the masses of alluvial composing the banks cave in, the water into which they fall dilutes them; it bears away the earthy portion; the stone, gravel and sand, which were mixed with them, remain upon the bottom, and thus augment its stability by their greater resistance. Thus, the channel of rivers will always be wider compared to their depths, as the earth is more moveable, and, at the same time, more pebbly.

Parallelism of surface of rivers to that of ground.

139. The depth of rivers, being always quite small, only a few yards, in a length of a million or more, the bottom of the channel will be very nearly parallel to the surface of the ground through which it

was excavated. If its slope is found to be raised at its sources, it is equally so in the adjoining lands.

Observations on the Reforming of Channels.

140. When a river runs in a vast plain, of small inclination, the fraction of gravity (pg) which moves the fluid mass is small; this mass has less force to overcome the obstacles opposed to its direction, which, of course, is the line of swiftest descent. The least obstacle, a very little more of hardness or tenacity in the earth, it meets, will cause the river to deviate. It will be thrown sometimes on one side, sometimes on the other; its course will be rambling, with continual bends, which augment the length of the channel with the same absolute slope, while the relative slope is diminished, and, of course, its velocity. The fluid mass running less swiftly, its width and depth will increase, and from this cause may proceed inundations and damage, which would not have occurred, had the direction of the channel been a straight line.

Sometimes, when the water-course is small, and the nature and disposition of the locality admit of it, attempts are made to alter the channel. The case is similar to leading a canal from one point to another, a problem which has already been solved in the preceding chapter.

While upon the subjects of these reforms, and upon the general subject of works in rivers, great care must be taken not to produce a greater evil than the one we would avoid, either above or below the locality of the works, or at their site; thus, those who first designed the *Robine*, a canal which goes from the Aude to the Mediterranean, through Narbonne, caused it to take great circuits both above and below this city; they wished, by reducing the velocity of the current, to augment its depth and favor the ascending navigation. At the end of the last century, without any regard to the original design, and supposing the sinuosities of the stream a mere matter of chance, an attempt was made to reform the channel, in order, as it was said, to shorten the time of navigation. When the alignment was made, it was found that there was not a good draught of water; it became necessary to build locks, and to increase the consumption of water.

The questions relating to all the changes of the channel, require a perfect knowledge of the localities, and of the river in its different stages. It is experience, and the genius of the engineer, rather than the rules or general considerations laid down in a short treatise, which is to guide to a suitable solution of them. I refer, consequently, to the works of various savans, Guglielmini, Manfredi, Frisi, Fabre, etc., who have treated upon these subjects, and more particularly to the *Hydraulique* de Dubuat, §§ 127—139.

This last author has offered various considerations touching the bends of rivers, and the modes of easing them. I will confine myself to remark upon this subject, 1st, that the resistance of elbows is generally small: 2d, that the current bearing against a concave bank will have a greater depth, while deposits and alluvions will be formed on the opposite banks.

ARTICLE SECOND.

The motion of water in Rivers.

1. *Kind of motion. Its influence upon the form of the surface fluid.*

Kind of motion. 141. In rivers, from their most remote source to their mouths, the volume of water is continually augmented by the tributaries they receive. But from one tributary to another, the volume remaining sensibly the same, the motion is permanent, and the rules already laid down in the preceding chapter are applicable.

Thus, for each transverse stratum of the fluid mass, the accelerating force will be in the ratio of gravity minus the resistance of the channel (123), or, i being the inclination of the surface of the stratum, $g \sin. i - A'c\left(\frac{Q^2}{s^3}+\frac{BQ}{s^2}\right)$.

So long as this quantity is positive, and continues to have an excess of the first term above the second, the motion will be accelerated. But, if this last predominates, the motion will be retarded. With much greater reason will it be so, if the sin. i should be negative, which is the case when the surface assumes a counter slope.

Longitudinal figure of surface.

142. When the inclination i goes on gradually increasing, the fluid surface is convex; it is concave when this inclination diminishes more and more. If the bed is horizontal, and of a constant profile, to every convexity of surface corresponds an accelerated motion; and for every concavity we have a retarded motion. If the bed is inclined, and of uniform inclination, it will not have an accelerated motion, save when the successive values of i are found to be greater than the inclination of the bottom; if they are not, in spite of the convexity, the motion will be retarded. So that, though, ordinarily, concavity is a sign of retarded motion, still there will be acceleration if the values of i exceed this last inclination. Continual variations in the slope and profile of the channel will increase still more the disagreement between the curvature of surface and the kind of motion.

To sum up all, the longitudinal section of the surface of a river with a smooth bottom will present a series of lines sometimes straight, sometimes convex, sometimes concave, and without the same kind of motion always answering to the same kind of line. Nevertheless, most generally, the right line will be an index of the uniformity of velocity, the convex line that of acceleration, and the concave answers to a retarded motion.

143. Still more, or at least, in a manner much more apparent than the kind of motion, will the inequalities of the bottom affect the form of the surface; they will reäppear in some measure at the surface of the stream. For example, let a shelf of pebbles, narrow and deep, be laid transverse or oblique to the bed of the stream: the fluid will surmount it by virtue of its acquired velocity; on meeting with the shelf, its surface will be considerably raised, after which it will descend, so as to present, in that part, an elevation like that of a great wave; but its elevation above the general surface of the stream will be less than that of the shelf above the general plane of the bottom. Usually, the inequality of the surface will be so much less, compared to that of the bottom, as the depth and velocity of the water is greater; so that in extraordinary freshets, the presence of dykes from six to ten feet in height, is sometimes without any effect upon the surface; and we may see the water pass from the upper reach to the lower, without a sensible elevation or depression.

Let us further remark, that although the inequalities of the surface are produced by those at the bottom, they do not correspond with them vertically, but are generally to be found somewhat more down stream.

Figure across the stream.

144. The transverse section of the surface of a river presents, moreover, a remarkable form; it is a convex curve, whose summit corresponds to the thread of the

current; from this point of greatest velocity, the level is lowered from point to point till it reaches the sides, and it is depressed, sometimes equally, sometimes unequally, towards each of them. The greater the velocity of the different parts of the stream, the more considerable is their respective elevation. Figures 25 and 26 represent this state of things; the first applies to a river, the second to a mill course.

This form of current would be, according to Dubuat, the consequence of a principle, the certainty of which he has established by direct experiments, and which he has enunciated in these terms: "*If, from any cause, a column of water comprised in an indefinite fluid, or contained between solid sides, begins to move with a given velocity, the lateral pressure which it exerts before motion against the surrounding fluid or against the solid walls, will be diminished by all that is due to the velocity of its motion.*[*] Consequently, the particles of the thread of the stream and those adjoining it, moving more swiftly than those at the sides, will exert a less pressure against them; and they will therefore require a greater number of fillets, that is to say, a higher column, to maintain their equilibrium. I should remark, however, that this principle of Dubuat, and the justice of its application to the case in hand, has been contested by different authors.[†] Nevertheless, it may well be considered as an extension of another principle, of which mention has been made (45), and which we shall consider in the following chapter.

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[*] Dubuat, Principes d'Hydraulique, sec. 453.

[†] Bernard, Nouveaux principes d'Hydraulique, p. 172. — Navier, Architecture Hydraulique de Bélidor, p. 342.

2. *The Velocity.*

Its determination.

The knowledge of the velocity of a river is often necessary, whether it be to appreciate the action of the current against its channel, or whether, as is most frequently the case, we wish to deduce from it the volume of water conducted by it. This velocity is usually determined, in a direct manner, by means of instruments called *hydrometers.* We begin with describing the principal of them; and firstly, those which give the velocity of the surface.

Floats.

145. The most simple, direct, and the surest, when it is properly used, is the float, which, placed in the water, partakes of its velocity. In common practice, we employ bits of wood, or other substances of a specific gravity nearly equal to that of water, and count the number of seconds it takes to pass a distance previously measured. When greater exactness is required, we use tin or hollow copper balls, or an apothecaries' vial, ballasted with shot, so as to be nearly submerged in the water. They are put in the strongest part of the current, and far enough above the point where we commence counting the seconds in which it runs through the measured space, so that on their arrival they may have acquired the velocity of the adjoining fluid. In this manner, by repeating the operation two or three times, we expect to obtain the velocity of the swiftest current with sufficient exactness. But for the fillets contained between this and the sides, this mode will not answer; the float will not maintain the necessary direction.

I should observe that floats should not be sensibly elevated above the surface, or their direction and velocity will be subject to the influence of the wind. Further, if they project too much, and the slope is considerable, like bodies placed upon an inclined plane, their velocity would be accelerated, until it shall have acquired uniformity from the resistance of the plane; if the plane itself moves, their absolute velocity will be greater than that of the plane; that is to say, the velocity of the floats will be greater than that of the surface fluid.

146. The velocity in a given part of the surface can be suita-

bly determined by means of a very light wooden wheel, with floats, and with slight friction upon its axis. Placing it in the current so that the floats are sunk in the water, its centre of percussion will partake very nearly of its velocity. Dubuat has used successfully a wheel made of fir, 2.395 ft. in diameter, carrying eight square floats, .262 ft. each side; the axis turned upon two small iron pivots, retained in copper boxes; the whole weighed only 1.52 pounds avoirdupois.

Wheel with floats.

147. The *hydrometric pendulum*, which has been used for the same purpose, consists of a hollow ivory or metallic ball, sustained by a thread, whose end is fixed at the centre of a graduated quadrant. This is to be placed over the point where the velocity is to be taken, so that the ball shall plunge into the water. The current urges it forward, the thread inclines, and the square root of the tangent of inclination, multiplied by some constant number, gives the velocity sought.

Hydrometric Pendulum.

Fig. 27.

Thus, let w be the absolute weight of the ball A; construct the parallelogram ABCD, where AD $= w$, and the angle of inclination EOA $=$ CAD $= i$. In the position of the ball, its effective weight, the force with which it tends to descend, will be $w \cos i$. AB, which is that portion of the weight in equilibrium with the action of the current, which measures its effort, will be $w \sin. i$, and $w \frac{\sin. i}{\cos. i} = w$ tang. i, compared to the effective weight; this effort, then, is proportional to the tangent of the angle of inclination. It is also, as we shall see in the following section, proportional to the square of the velocity of the current. This velocity, then, will be proportional to the square root of the tangent of inclination, and we shall have

$$v = n \sqrt{\text{tang}\; i}.$$

This multiplicator n will be constant for the same ball; and prudence would suggest its direct determination by experiment. For this purpose, the pendulum should be tried in a stream whose velocity has been determined by some other means, as by that of the wheel with floats; and this velocity, divided by the square root of the tangent of inclination obtained in this experiment, will give the value of n.

A more general theory of the simple and compound pendulums may be found in the Hydraulics of Venturoli.

Let us come now to those hydrometers made to measure the velocity below the surface. Many have been devised and used; I cite the three following.

Tube of Pitot.

Fig. 28.

148. The most simple is the *Pitot's tube*, so called from the name of the author who first proposed its use. It is simply a glass tube, bent at its lower end. It is immersed in the stream, so that the orifice of the bent part, turned against the current, shall be at the level of the vein whose velocity is required. This vein, pressing upon the water in the tube, causes it to rise in the vertical branch; and the height of its elevation above the surface of the river is regarded as the height due to the velocity of the current.

But it is not exactly so. This height measures indeed the sum of the pressures exerted against the orifice of the tube; but the pressure against a body plunged in water is dependent upon the form of the body, as we shall see hereafter; moreover, that of the different fluid veins is diminished from their centre to their circumference; so that we must isolate, by some means, a fillet, (the central one, for example,) and, moreover, we must consult experience as to the effects of the form of the tube. Dubuat, the author of these observations, found that in giving to the orifice the form of a tunnel, with its entrance closed by a plate pierced with a small hole at its centre, that two thirds only of the elevation in the tube was the height due to the velocity, and that consequently we have $v=\sqrt{2g\frac{2}{3}h}=6.55\sqrt{h}$ ft.; h being the height of water in tube above the surface of the current.

M. Mallet, engineer, terminated the horizontal branch of the tube with a cone having no where above two millimetres or .078 inches of opening at the summit; the tube was made of iron, nearly 0.13 ft. in diameter; in it was placed a float, surmounted by a stem; this tube was fastened to a pole, as is frequently done with other hydrometers, of which mention will be made in future. When the instrument is in position, and at the point of required velocity, the cone being exactly in the direction of the current, and turned up stream, the height of the stem is observed; then the instrument is reversed down stream, and note is made of the height of the stem. The difference of the two heights, multiplied by the particular coefficient of the tube, given by previous experiments, will be the height due to the velocity of that part of the current adjoining the cone.

Notwithstanding the simplicity of the instrument and of the method, it is but seldom used, as we cannot measure the height of the water with sufficient accuracy to deduce the precise velocity, especially when this velocity is small.

149. Trials for more delicate indicators have been made, in exposing plates directly against the shock of that part of the stream whose velocity is required; the necessary weights used to maintain them against the action of the current are the measurers of its force, and the velocity will be determined by rules which will be given in the following section. The form of these balances, or Roman hydrometers, is much varied. I shall confine myself to a description of one used by Brünings in numerous experiments, which he has called the *Tachometer* (measurer of velocity).

Tachometer of Brünings.

Fig. 29.

It consists of a plate A, fixed to the extremity of a stem AB, (which moves in a socket *m*,) perpendicular to the bar DE, whose foot rests upon the bottom of the channel, and on which the instrument is fastened, at the desired height. A cord is fastened to B, which passes under the pully C, and reaches to the short arm of a balance, whose other arm bears the weight P. When the Tachometer is suitably placed for accomplishing its object, the current, acting upon the disc, drives it from A towards B; and the weight P is drawn back, till it holds it in equilibrium. From its position, we arrive at the effort of the current, and so determine its velocity.

Woltmann's Mill.

Figs. 30 and 31.

150. Preference is given above all these machines to the hydrometric mill of Woltmann, especially in Germany; a description of it and its use was published by that philosopher in 1790. It is simply a revolving axle, carrying four small wings, like those of a windmill. The current causes them to turn, and the number of revolutions made in a certain time, and recorded by the instrument itself, furnishes us directly the velocity.

In reality, saving the slight resistance due to the friction of the axle upon its bearings, the velocity of the current is proportional to that of the wings, and the last is proportional to the number n of turns made in a unit of time, or, what comes to the same thing, to the number N made in a time T. and divided by this time; so that we have $v = an = a\frac{N}{T}$; a being a constant coefficient for the same mill, to be determined by experiment.

For this purpose, the mill is placed in a current whose velocity has been ascertained by other means: the number of turns it makes in a given time is recorded, and this number is divided by the time; we divide the velocity by the quotient thus obtained, and thus have a. More simply still, admitting (and I believe it to be the fact in this case) that the pressure exerted by a fluid at rest upon a small plate in motion, is equal to that exerted by the fluid in motion against the plate at rest, the velocity being the same in both cases, we run the mill through a certain space of stagnant water, a pond, for example, and we divide the space run by the number of turns of the axle; the quotient is the value of a; for $v=\frac{E}{T}$, also, $E=aN$ or $a=\frac{E}{N}$.

The usefulness of this instrument leads me to make known the disposition and dimensions of its principal parts, represented by Fig. 31 at half its full size. The wings, four in number, are square thin copper plates, .082 ft. each side; their middle is .164 ft. from the axis of rotation; their plane is at an angle of 45° with this axis. For small velocities, where greater delicacy of instrument is needed, we double the size of the wings and their distance from the axle. We have thus two sets of wings, and place upon the axle those best suited for the purpose in hand. The wheels have each fifty teeth; the pinion which transmits the motion of one to the other has but five, so that they can indicate five hundred turns. They are supported on a frame moveable about one of their extremities, which is kept clear of the revolving axle by a spiral spring. Upon the axis is a short spiral screw, in which the teeth of the wheels are engaged, by pulling up the cord fastened at the moveable extremity of the frame.

In operating, the instrument should be free of all obstruction to motion; and the teeth of each wheel marked zero are placed opposite their respective index, fixed upon the limb. Then, putting a stick of wood or an iron stem into the socket, the machine is secured at the desired depth. If this depth is small, we place and secure the iron arm some yards in front of the upper end of a skiff, moored to the place of operations. For great depths, we use two boats, joined by strong planks; and upon this the instrument is secured at the desired point; then the bar carrying the mill is lowered, with its extremity in the bottom of the river. All being ready, at a given signal from the time-keeper, we draw by a string the frame bearing the toothed wheels, and have them thus pressed against the revolving axle, which communicates its motion to them. At a second signal, the cord is dropped; the spiral spring repels the frame, the teeth are disengaged, and the wheels stop. The instrument is taken from the water, and the index gives us the number of turns made between the intervals of the signals: this number, divided by the time and multiplied by the proper coefficient of the mill, gives us the required velocity.

Diminution of velocities at different depths.

151. It is by means of such instruments that we have discovered the diminution of the velocity of the current towards the bottom or the sides of the channel, and that we have searched for the law of this diminution.

Previous to the eighteenth century, it was admitted, that in rivers, the respective velocities of the different fluid threads of a stream followed the same law with that of fillets issuing from a reservoir through an orifice made in the vertical sides, the circumstances of which we have already discussed (58), where it is seen that the velocity increases as the square root of the depth of fillets below the surface of the stream; so that the velocity in a river would have increased with its depth, and very nearly as its square root. This doctrine was admitted by Guglielmini, and other philosophers of Italy, at that time the most profound in Europe in all that pertains to running water. But towards 1730, Pitot, by means of the hydrometrical tube which he invented, and in experiments made upon the Seine, found that the velocity diminished, instead of being increased, with its depth. He published this important fact, which a multitude of observations have since confirmed and generalized, and whose cause and effects have already been indicated (106 and 109).

We have there found the velocity of the different fillets of the current to be greater according to the amount of removal from the bed of the channel, and that consequently, the thread of the stream, or that of greatest velocity, is found in that part of the surface answering to the greatest depth. This fillet is sometimes designated under the German name of Thalweg (path of the valley). In reality, the *Thalweg* would be the intersection of two slopes enclosing the valley; in nature, the thread of the stream will be found above this intersection, and will indicate its position; so that we sometimes use this as the boundary lines of estates or territories separated by rivers; it is that which is usually followed by the descending navigation.

Law of diminution.

152. Some observers have thought that the greatest velocity of a river is not exactly at its surface, but a little below it; nevertheless, M. Defontaine, engineer, has concluded, from his observations upon the Rhine, that, allowance being made for the wind, it is found exactly at the surface of the stream.

What is the law of its diminution, as we descend downward? In the second half of the last century, Ximenes, and other Italian hydraulicians, devoted themselves to its investigation. In 1789 and 1790, Brünings, for the same purpose, made eighteen series of experiments upon different branches of the Rhine which traverse Holland; at each of his stations, and for every foot in the same vertical, he measured the velocity of the river, by means of his *tachometer* (149). From these observations, and some others, Woltmann felt authorized to conclude that in descending from the surface, the velocities decrease as the ordinates of a reversed parabola. For example, if in Fig. 16, where AMC is the common parabola, BC represents the velocity at the surface, and GH that at the bottom, DE will be the velocity at the depth BD. Funk assumes a logarithmic function; that is to say, while the depths increase in arithmetical progression, the velocities diminish in a geometrical progression. M. Raucort, after a series of observations made by him on the Neva, at Petersburg, thought that these velocities might be represented, upon the same vertical, by the ordinates of an ellipse, whose lower summit is below the bottom of the river, and whose minor axis is a little below the surface of the same.*

Notwithstanding these scientific trials, the results of observations present and will present too many anomalies and contradictory facts, for any attempt at a mathematical deduction of the decrease of the velocity. The only inference which can be drawn from known observations, and particularly from those of M. Defontaine, made upon the Rhine, with Woltmann's instrument, is that, generally, *in proportion to the depth below the surface of a river, there is a gradual diminution of its velocity; at first nearly insensible, then more marked, and increasing rapidly on approaching the bottom, where the velocity is nearly always greater than one half that of the surface.* Fig. 49, which represents the curve indicated by the mean of two observations, in a part of the Rhine 4.92 ft. deep, will give an idea of the manner of decrease; we have opposite the coördinates of this curve, which approach nearly the arc of a parabola, whose ordinates are the velocities diminished by a constant quantity.†

Depth.	Velocity.
ft.	ft.
0.00	4.023
.66	3.997
1.31	3.931
1.97	3.829
2.62	3.691
3.28	3.468
3.94	3.117
4.59	2.887

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* Annales des ponts et chaussées. Tome IV., p. 1, 1832. It is hoped that the experiments of M. Raucort may be published. At one of the points of observation, the depth of stream was about 62 feet. This engineer, moreover, represents, by the ordinates of an ellipse, the velocities of the surface, from the thread of the stream even to the shores of the same.

† Vide, in the Annales des ponts et chaussées, Tome VI., 1833, the excellent work of M. Defontaine upon the régime of the Rhine, and upon constructions for the protection of its banks.

Mean velocity of a vertical.

153. The mean velocity, in the same vertical, will be the sum of the observed velocities, divided by the number of observations; the greater the number, the nearer the approximation to the truth.

It is in this manner that Brünings has determined the mean velocity of each of his verticals. He sought, moreover, for the ratio of the mean velocity, with the corresponding velocity at the surface, or rather, at 1.03 ft. beneath it; he found that this ratio varied from 0.89 to 0.96; the velocities were from 2.19 ft. to 4.856 ft., and the depths from 5.15 ft. to 14.40 ft. Ximenes, upon the Arno, for a velocity of the surface of 3.294 ft. and a depth of 15 ft., has 0.92 for the ratio of mean velocity of a vertical to that at surface. M. Defontaine, in his observations upon the Rhine, obtained only from 0.85 to 0.89. Nevertheless, for great rivers, observations give oftener above than below 0.90.

The fillet endowed with the mean velocity has usually been found a little below one half and towards three fifths of the depth.

Mean velocity of section compared to that of thread of current.

154. But the mean velocity of the particles of the same vertical is not the mean velocity of the component elements of the section. Since the velocity at the surface decreases from the thread of the current up to its sides, and the mean velocity of the verticals are nearly in the same ratio, the mean combined — that is to say, the mean of the section — will be less than the greatest of them, which corresponds to the thread of the stream; and consequently, its ratio with the velocity of this thread will be smaller than that given in the preceding number, or than 0.90, the mean term. Brünings has found it to be 0.85; but he has seen it go as low as 0.72, and again as high as 0.98. Ximenes found it to be 0.83.

Dubuat, in his experiments, made in small canals, of which mention has been made (109), has obtained a result nearly similar, though by a very different process. A direct gauging gave him the discharge of the canal, and dividing it by the section, he had exactly the mean velocity (108); he then determined readily, and with sufficient correctness, the greatest velocity of the surface. The ratio of one to the other varied from 0.71 to 0.88 (and even in two experiments, which it was thought best to withdraw, it was raised from 0.95 to 0.96). Moreover, this ratio was increased with the velocity, and in designating by V that of the surface, and *v* for mean velocity, we can express it

$$v=\frac{V\ (V+7.78188}{V+10.34508}.$$

But can we admit a ratio entirely independent of the depth? Can we extend the results of observations made in small wooden canals, regular throughout their length, with a depth of water not exceeding a foot, to rivers whose channels are a series of great inequalities, and with a depth often exceeding ten or fifteen feet? We should doubt it, if the experiments made directly upon great streams did not seem to indicate the same results.*

3. *Gauging of Streams.*

The estimate of velocity, whether of each part or of the mean, which has been the subject of discussion, has chiefly for its object the *gauging of water courses;* that is to say, the determination of the quantity of water which they bear, the knowledge of which is often a matter of great interest to the government, as enabling it to decide with exactness how much water can be spared from a river for canals, irrigation, etc., without injury to the navigation; and to divide, with justice and fairness, between many mills or other service, any amount of disposable water.

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* The translator, while employed under the United States Government, in some observations made upon velocities at different depths of the Mississippi River, has seen results entirely at variance with the law here laid down. At present, he is not authorised to publish.

The gauging is effected in different ways.

Gauging by Hydrometers.

155. The best method, for great rivers, is to take a station at any point, to measure the area of its transverse section as well as the mean velocity of this section, by means of the hydrometer, and to multiply these two quantities into each other.

To operate in a suitable manner upon the whole width of the stream, at the appointed station we take many soundings, which divide the section into trapeziums, and we calculate the area of each of them. Then, at equal distances between the points of sounding, we secure the boat or pontoon, bearing Woltmann's mill, or other instrument (150); by means of this, we determine five, six, seven velocities upon the same vertical; we take the mean of them, and multiply it by the area of the respective trapezium. The sum of all these products is evidently the discharge of the river, and is equivalent to the total area of the section, multiplied by the general mean. As every thing is at the disposal of the observer, so that he can multiply at will the soundings and the determination of the velocity, and may take all necessary pains in the work, he is enabled to give whatever exactitude may be wished for the measurement, and thus obtain very nearly the real discharge.

156. This mode, it is true, requires time and expense, and if approximation only is desired, we are content with the following. We take a station near the

middle of any reach, or portion of the stream whose channel, for an extent of several hundreds of yards, is sufficiently regular. By sounding, we have the area of its transverse section. Then, by means of floats (145), we determine the velocity of the thread of the stream, corresponding to the measured section; by means of the formula above given (154), we shall have the mean velocity, which, multiplied by the area already found, will give us the discharge sought.

Gauging by calculation.

157. The formulæ of permanent motion (123 and 124) will furnish still another method of obtaining the delivery of rivers.

For this purpose, we choose a locality where, for a considerable length, the channel presents no marked or abrupt inequalities. We take, then, from four to six stations; at each, we determine, first, the area of the section ($s_0, s_1, s_2, \ldots s_n$); second, the perimeter, or that part of section of bed in contact with water ($c_0, c_1, c_2, \ldots c_n$); third, the distance from one station to the other ($z^{\cdot}_1, z^{\cdot}_2, z^{\cdot}_3 \ldots z^{\cdot}_n$); fourth, the slope of the surface from one to the other.

By means of these given quantities, we have the delivery by the formula

$$Q = -\frac{N}{2(D+M)} + \sqrt{\frac{p'}{D+M} + \left(\frac{N}{2(D+M)}\right)^2}$$

or,

$$D = \frac{1}{64.364}\left(\frac{1}{s_n^2} - \frac{1}{s_0^2}\right)$$

$$M = .0001114155\left(\frac{z^{\cdot}_1 c_1}{s_1^3} + \frac{z^{\cdot}_2 c_2}{s_2^3} + \ldots . \frac{z^{\cdot}_n c_n}{s_n^3}\right)$$

$$N = .0000242651\left(\frac{z^{\cdot}_1 c_1}{s_1^2} + \frac{z^{\cdot}_2 c_2}{s_2^2} + \cdots \frac{z^{\cdot}_n c_n}{s_n^2}\right)$$

p' = amount of slope between the first and last stations.

We must remember that the integration which led us to this formula requires implicitly that the quantities to be integrated, especially the velocities, and so their sections, should be subject to a law of continuity; now, this could never be the case, if there are irregular variations in the width and slope of the bed—and they are to be found in nearly all parts of rivers. The formula is not, therefore, rigorously applicable to them, and the results given by it should only be regarded as approximate. The following example serves to show how we should regard it.

From among a series of one hundred and five observations or levelling stations made on the Weser, near Minden, in Westphalia, and reported in the Hydrotechny of Funk, I select six consecutive ones, in a part of the river presenting the least irregularity; they give the distances, slopes, the wet perimeters, and sections, found in the following table. For each of the respective sections, I add the values of $\frac{z \cdot c}{s^2}$ and $\frac{z \cdot c}{s^3}$.

No.	$z \cdot$	p'	c	s	$\frac{z \cdot c}{s^2}$	$\frac{z \cdot c}{s^3}$
	feet.	feet.	feet.	sq. feet.		
0	0.	.000000	324.819	825.41	.0000	0.0
1	522.98	.564332	363.534	794.84	.3009	.00037860
2	215.23	.232623	325.147	489.88	.2916	.00059524
3	200.14	.216218	308.742	689.45	.1300	.00018850
4	261.49	.279541	309.726	489.88	.3376	.00068888
5	161.42	.174211	386.501	674.71	.1371	.00020311
	1361.26	1.466925	.336411	660.70	1.1971	.00205433

With these data we find

$$D = \frac{1}{64.364}\left(\frac{1}{674.71^2} - \frac{1}{825.41^2}\right) = .0000003716;$$

$$M = .0001114155 \times .00205433 = .000000228884;$$

$$N = .0000242651 \times 1.1971 = .0000290478.$$

These numerical quantities substituted in the above equation give for the discharge sought Q = 2426.83 cubic feet. A measurement made with a hydrometer gave 2652.28.

So the formula has shown a deficit of about one tenth.

	cubic feet.
The first five stations alone would give	2233
“ “ four “ “ “ “	2633
“ “ three “ “ “ “	2254
The last four “ “ “ “	2657

We see from this example, where the bed was as regular as could be expected in large rivers, how great is the respective influence of the areas of the sections.

The formula of uniform motion, in taking the mean of the six sections, and the six wetted perimeters noted in the above table, would give 2813 cub. ft.; a quantity six hundredths greater than the results of the gauging by the hydrometer.

Gauging by Dams.

158. Dams which bar the course of rivers, and over which all the water flows, will sometimes afford us the means of determining this quantity. But for this purpose, the crest of the dam should have a projecting edge, so that the water, in passing over, may fall freely and suffer no reaction from the part already passed; it is seldom that we meet with this arrangement. We may supply its place, by putting upon the crest a plank with the upper edge made thin and horizontal, with sharp corners, and high enough for a free flowage of the water; the height of the water H, above this weir, should be over 0.197 ft., but less than one quarter of the depth of the stream behind the dam. Then L, being the length of the dam, the discharge will be given by the formula (77)

$$Q=3.5567\ LH\sqrt{H}.$$

In case H exceeds one quarter part of the depth, we use the expression (79), as a function of the velocity w, at the surface of the stream,

$$Q=3.4872\ LH\sqrt{H+0.035051w^2}.$$

159. If the method of gauging by weirs is seldom applicable to great streams, it will be found better suit-

ed than any other for small streams. There are two cases to be noted.

That where the current is small, and carries only from thirty-five to seventy cubic feet of water per second. We look for a place where we can easily construct a weir with a width over 0.295 ft., but less than one third of the width of the bed, and in such a manner as to have a head upon the weir greater than 0.196 ft., but not so great that its product into the width of the dam, or lH, shall exceed the fifth part of the section of the stream immediately above the dam; then, without the chance of one per cent. of error, we may apply the formula (77)

$$Q=3.209\, l H \sqrt{H}.$$

If the operation be found more easy, or if the quantity of water exceeds seventy cubic feet, we might dam up the entire bed of the stream; at each of its extremeties we raise a small vertical partition, so that the opening through which the water passes may be rectangular, and we should then use one of the two formulæ referred to in the preceding number, after complying with all the conditions to make them applicable.

Two examples will serve to show the method of proceeding, and will afford an opportunity to add some practical details to what has already been said upon weirs (68—83).

I. It is required to gauge a small stream of water. A suitable place for the construction of a weir is sought; this, for example, will be at a narrow part of the bed, with steep banks, immediately below a wide portion of the stream. Let the width of the stream at the surface in this place be 11.8 ft., and its greatest depth 2.6 ft. After a preliminary examination of the section, and of the velocity, measured by some light bodies thrown into the current, we find that it carries about 36 cubic feet of water per second. Since the breadth is 11.8 ft., the weir can be made 4 ft. in length; the head on it will then be about 1.988 ft. (for the formula $Q=3.209\, lH\sqrt{H}$ gives $H=\sqrt[3]{\left(\frac{Q}{3.209l}\right)^2}=1.988$ feet.) After this approximate estimate, we should make a plank partition, about 15 ft. long at top, 5¼ ft. high, and say from 1¼ to 1½ in. thick, and with a shape conforming to the bed of the stream; fit it so as entirely to dam the stream. For this purpose, insert its ends and bottom into the sides and bottom of the bed; by means of moss, sods and clods of earth, we make the joints as tight as possible, especially a short time before the gauging commences; it must be supported with cross pieces and struts. In the upper half, we cut a rectangular notch, four feet wide by two feet deep; so that the sill of the weir shall be .50 ft above the natural level of the stream, and that the water may fall freely over it. The section of the fluid sheet at the weir (4 ft. × 1.988 ft. = 7.95) not being one fifth nor even one seventh part of the section of the stream, which exceeds sixty square feet, all the conditions for the application of the formula $Q=3.209\, lH\sqrt{H}$ will be satisfied.

When all is ready, and there is but little leakage, and the new regime of the current is well established, we take two points on the partition, one on each side of the opening, and at a foot or more from the vertical edges, and at the level of the water line (making deductions for capillary attraction); then stretch a thread between these points, and measure directly its elevation above the centre of the sill. It was found to be 2.008 ft., and the length of the weir, from careful measurement during the flow, was only 3.986 ft.; thus

$$Q=3.209\times 3.986\times 2.008\times\sqrt{2.008}=36.395 \text{ cub. ft.}$$

II. A suit at law requires the exact determination of the volume of water conveyed by a small river, when its level is at the height of a given bench-mark. It is decided that the gauging shall be made by means of a dam.

At 170 ft. above the mark, at a point where the river is a little embanked, and presents a regular bed, where the current is 65 ft. wide at the surface, and 4.10 ft. mean depth, when the water is at the height of the mark, we establish the temporary dam. It is capped with a well squared piece of wood, 1½ in. in width at the top, the upper face of which is quite smooth and horizontal, and fixed at .66 ft. above the bench-mark. At each of its extremities, we raise a small vertical partition, so that the interval between them, or the length of the dam, shall be 64 ft. Adjoining these two partitions, and at right angles to the same, we place two others, which are five feet wide; at 3.25 ft. from the common intersection, we place a scale against the interior face of each, whose zero point stands exactly at the level of the crest of the dam.

These dispositions being made, wait till the water in the lower reach is at the level of the mark, and then take, by the scales, the height of the upper reach. It was found to be 2.339 ft. As this height is nearly half that of the dam $(4.10+.66=4.76$ ft.), we cannot use with confidence the formula

$$3.5567 \text{ LH} \sqrt{\text{H}},$$

but must have recourse to that of

$$Q = 3.4872 \text{ LH} \sqrt{\text{H}+.035051w^2}.$$

To obtain the velocity w of the surface on its arrival at the dam, we should take, starting from a point where the water begins sensibly to incline towards the dam, a distance of 164 ft. up stream on each bank, and mark the extremities by stakes. At 65 ft. above this, cast into the strongest part of the current a suitable float, and, with a good watch, determine the time occupied in its passing the 164 ft.; a mean of six observations gave 48½ seconds, whence we conclude $w=3.38$ ft., and $.035051w^2=.4004$.

$$Q = 3.4872\times64\times2.339 \sqrt{2.339+.4004} = 864.00 \text{ cub. ft.}$$

The formula $3.5567 \text{ LH} \sqrt{\text{H}}$ would have given 814.28 cub. ft. Thus we may safely affirm, that at the given height, the river furnished at least 850 cub. ft. per second.

Velocities and absolute discharge of Rivers.

160. Before closing our remarks upon the velocity and discharge of rivers, let us say a few words as to the absolute magnitude of this velocity and discharge.

From the smallest brook of the plains, to the impetuous mountain torrents, even to the great river Amazon, we have such a continued series of velocities and discharges, that it is impossible to take them as a basis for the classification of rivers. Moreover, the different regions of the surface of the globe, being unequally divided, in a hydrographic view, what would be large for one region would not be so for another.

We give an approximate idea of the difference in the size of rivers, citing from geographers the developed length of some of them.

	Miles.
The Amazon,	4281
Mississippi,	4213
Nile,	3107
Volga,	2485
Euphrates,	2374
Danube,	2206
Ganges,	1932
St. Lawrence,	1796
The Senegal,	1211
Rhine,	956
Elbe and Vistula,	826
Loire and Tagus,	643
Rhone,	553
Seine and Po,	497
Garonne and Ebro,	466
Thames,	217

These lengths give no true measure of the size of the rivers, or of the volume of water which they bear to the sea: thus, the Rhone conveys more water than the Loire, though it is not so long; the Garonne empties into the ocean nearly a third more than the Seine, and its length is less.

We confine ourselves exclusively to what concerns France, and we shall call the velocity of any river small when it falls short of 1⅓ ft.; that of the Seine is about 2 feet in the vicinity of Paris; an ordinary velocity will be from 2 to 3½ ft.; above that it is great, and very great if it exceeds 6⅓ ft., which is nearly that of the Rhone and of the Rhine; it is even double, in time of great freshets.

As to the volume of water conveyed, or the size properly so called, a water-course ranks among rivers, when, in its ordinary state, it carries from 350 to 450 cubic ft. per second. With from 1000 to 1500 cubic ft., it will be a navigable river, at least under some particular circumstances. The rivers of France bear 3500 cubic ft.; thus, the Seine, with a mean width of 430 ft. and a mean depth of 5 ft., carries about 4600 cubic ft.; the Garonne, at Toulouse, has about 5300 ft. in its ordinary state; and the Rhone, at Lyons, has more than 21000 cubic ft.

The quantity of water conveyed by rivers undergoes great variations; thus, in Lyons, we have noticed the quantity as low as 9000 cubic ft., and even 7000 cubic ft., and on the 12th of February, 1815, it rose as high as 203770 cubic ft. The Registrar of the States of the Rhine, opposite Strasbourg, where the slope was .00061, gave M. Defontaine, even excepting extraordinary cases,

	In low stages	mean	and high.
For the discharge of the river,	13400 ft.	33700	164000
For the velocity,	5 ft.	7 ft.	9.35 ft.

At Nimègue, before its junction with the Meuse, and in its ordinary stage, it carries about 60000 cubic ft.

ARTICLE THIRD.

Backwater, *Eddies*, *&c.* (*Remous*).

161. A *remou*, or eddy, in the strict acceptation of the word, is water without progressive motion, in the bed of a river, near one of its sides, which turns upon itself, in consequence of the impulse of the adjacent part of the current, or from some other cause. This name is also given to every return of water against the direction of the river. Dubuat, extending this last acceptation, has called every elevation of the surface of the stream above its natural level a *remou;* an elevation due to the meeting with some obstacle, and which, extending up stream, seems to be a running back of the

fluid or a true *remou;* it is in this sense that engineers now use the word, and we shall adopt it here.

Such a *remou* or backwater, is produced either by a dam, which bars up entirely the course of a river, or by a construction, which, occupying only a portion of the bed, contracts the passage of the water, as is the case with bridges, dikes, &c.

1. *Backwater produced by a Dam.*

Fig. 32.

162. Let AB be the longitudinal section of the surface of the stream of water, of which HD is the bottom. The dam DE being raised, the course of the water is intercepted throughout its whole breadth. The water will rise up to flow over the crest of the dam; the fluid mass C*aa*AFC thus raised, constitutes the *remou,* and its upper surface will generally take the form represented by Fig. 32. (In this figure, the scale of heights is 840 times greater than that of the lengths.)

We have now to consider, 1st, the rise or elevation of level CF, near the dam; it is the height of the *remou,* properly so called; 2d, the elevation or height *ab*, at a given distance from the dam; 3d, the distance CA to which the swell extends; this is the *amplitude of the flow.*

Height near the dam.

163. The greatest elevation CF, which takes place at the dam, depends principally upon the height of the dam itself; it is composed of that height, minus the primitive depth of the current FG, plus the elevation C*g* (H′) of the water at C above the crest E of the dam. This last quantity, according to the experiments of M. Castel, which give

$$Q = 3.5567\, LH' \sqrt{H'},$$

will be $H' = .42917 \sqrt[3]{\left(\frac{Q}{L}\right)^2}$; an expression in which

Q is the discharge of the stream, and L the length of the dam.

Sometimes the water, instead of flowing over the dam, runs through openings made in the lower part of it. In this case, the greatest depth of the water will be equal to the distance between the centre of the orifice and the bottom of the channel, plus the distance of this same centre from the upper level, which is

$$Q=.039774\frac{Q^2}{a^2},$$

a being the area of the orifice; this follows from the equation $Q=0.625a\sqrt{2gH}$ (29). Subtracting from this depth that of the primitive current, we shall have the height of the *remou* or flow. Although the raising of the water is occasioned by the dam, it is not immediately at the dam that the greatest elevation will be found; it takes place a certain distance above the dam. We know that when water runs over a weir, the fluid surface inclines before it reaches the same; in great back flowage, the inclination, or a marked increase of the slope at the surface, will sometimes commence at quite a distance back.

Height at a given distance.

Nature of the curve.

164. The height of the flow, at a given distance, is a consequence of the curve which the surface fluid takes above the dam. Dubuat, who was the first hydraulician to investigate this subject, has endeavored to ascertain the nature of this curve. Observing that the depth of the water continued to increase with its departure from the extremity A of the swell, and consequently that the velocity of the strata and the inclination of the surface diminished *pari passu*, he concluded that this curve was concave. He also supposed that it would differ but little from the arc of a circle, which would be tangent at one of its extremities to the natural sur-

face, immediately above the end A of the flowage, and at the other, to its origin at C; and the length would be $\frac{1.9\text{H}}{p-p_1}$, H (=CF) being the height of the flow at C, p_1 the slope of the surface at the same point, (it is given by the formula of Sec. 112), and p the slope of the natural stream, or very nearly that of the bottom of the bed. The quantity $p-p_1$ expresses also the length of the arc in degrees; so it will be easy to calculate its radius. Its versed-sines at different distances x from the dam, will be very nearly the elevations of the surface fluid, above a horizontal drawn through the point C; and these elevations, increased by $\text{H}-px$, will give the heights of the flowage. We shall dwell no longer on this hazardous method of determination.

Funk, after having criticised this method, has substituted for it one not so well based. He admits that the threads at the surface of the flowage are concave arcs of a parabola, whose position and size he indicates; and according to which the heights of the flowage y, at different distances x from the dam, will be given by the equation

$$y=2\text{H}-px-\sqrt{\text{H}\,(\text{H}-\tfrac{1}{2}px)}.$$

I have shown elsewhere how much these results differ from those of observation; and I cite this, as well as the preceding hypothesis, only as matter of history.

165. In our day, Bélanger, Vauthier, Coriolis, &c., have applied to flowage the laws of permanent motion. It would seem as if the formulæ (125 and 126) which give the slope of the surface of a water course, when one of the sections of its current is known, as well as the declivity and shape of its bed, would solve effectually the problem, and determine the curve which the flowage should take when its elements are known.

Induced by the example of authors whom I have quoted, as well as by some peculiar observations of my own, I at first thought it might be so; but I have since entertained doubts respecting it.

The theory of permanent motion, as we have already observed (157), requires, in the bed of the water course to which it is applied, that there should be no abrupt or marked change either in slope or width; and this is rarely the case with rivers. Moreover, the water of flowage seems only to be superimposed above the current, and not to participate wholly with its motion; the engineers who took the levels of the Weser, (a part of which, touching the back-flowage of 21320 ft. in length, I have already reported, in a notice printed in the *Annales des ponts et chaussées*, tom. XIII., 1837), have observed that at a distance of 3884 ft. from the dam, the velocity at the surface was nearly insensible, while that of the bottom was quite strong. The water of the flowage, especially near the dam, presents a sheet slightly inclined, it is true, but its surface remains nearly plane, and is not sensibly affected by great inequalities in the bottom and width of the bed. All this would lead us to believe, that the water of *remous* is not similarly circumstanced with that of ordinary streams; and that the theory, which can scarcely be applied to these, can with still less safety be applied to *remous*. I must say also of this formula, which has but very few data, and where the first deviation affects all the rest of the calculation, it is positive that the trials which I have made with it have indicated slopes very different from those actually taking place.

I here cite from my observations on the back-flowage of the Weser (to which I have before referred), the form of which has been determined by levels made with great care. The slope of the bed, for a length of 55775 ft., as well as on the 22960 ft. occupied by the swell, was sensibly uniform, and equal to $.000454 = p$; in the same space, the mean width was 354.33 ft. $= l$; the depth of the water immediately above the dam was 9.816 ft.; and as that of the natural stream, on the supposition of uniform motion, would have been 2.467 ft., there remained for the surelevation of the water 7.349 ft. $= H$; at the time of levelling, we had $Q = 2651.92$ cub. ft. Thus the formula of Sec. 125, where, in this case, $c = 354.348 + 2h$, $s = 354.348h$, and where the depths diminish as we go up stream, becomes

$$p' = .00001762 z' \cdot \frac{l+2h}{h^3} + .00000051251 z' \frac{l+2h}{h^2} - .87024 \left(\frac{1}{h_1^2} - \frac{1}{h_0^2}\right)$$

As far as 18074 ft. from the dam, I have taken for z' distances of about 1500 ft., so that the extremities may coincide with the levelled stations; beyond this 18074 ft., the values of z' were less. The results of calculation, as well as of observation, are noted in the columns of the following table. The first indicate a very regular curve, and asymptotic to the natural current taken above the flowage. But the slopes resulting from that curve are much less than those found by levelling; most often, they were not the half. Only towards the extremity of the flowage, the differences were less, the two slopes then approaching nearly those of the natural current. In this extreme part, those of observation present great irregularities, the water of the swell having no great depth, being exposed to the action of great inequalities of the bottom. The formula of 126 has given exactly the same slopes as those of 125.

ABSCISSAS OR DISTANCES.	ORDINATES OR SLOPES.		
	By observation.	FORMULA above given.	FORMULA of St. Guilhem.
feet.	feet.	feet.	feet.
1637.1	.052494	.013123	.000000
3166	.12139	.03208	.016404
4967.2	.19029	.052494	.062337
6588	.23294	.082022	.14435
8274.4	.36746	.12139	.26903
9957.5	.50827	.17388	.41339
11520	.64633	.24278	.55447
13136	.84647	.34777	.69555
14750	1.1575	.50525	.85959
16289	1.3681	.70539	1.0826
17109		.8760	
18074	1.5321	1.1253	1.4796
19288	1.6765	1.5157	1.834
20336	2.4836	1.8865	2.1655
20837	2.9954	2.1391	2.3720
21621	3.2579	2.4803	2.7165
22326	3.4974	2.7920	2.9560

166. Such differences existing between the results of observation and those of the formulæ, forbid my recom-

mending their use; and were I called upon to indicate approximately the elevations of water produced by a proposed dam, I should use in preference an equation which the engineer St. Guilhem has arranged, so as to obtain a curve, like to the flowage of the Weser, of the Werra, and others cited in the above named notice. The elevations indicated by it are those which would really occur, if the flowage in question was similar in all respects to that of the Weser, etc.; and they appear to be analogous to all those formed in ordinary rivers, great or small, when dammed up in their course. This equation,

$$(y+px)^3=\frac{H^8}{H^5+\frac{4}{9\times 3.2809}(px)^6}+(px)^3,$$

is that of a curve asymptotic to the natural current: y representing the elevation above the natural surface for a distance x. Its results for the Weser are placed in the last column of the preceding table; they follow very closely those of observation in the middle portion of the flowage, where there is the greatest need for recourse to calculation.

The size and form of the bed do not, it is true, enter as constituents of its expression, but we have seen that, to a certain extent, the flowage is independent of these elements: as for the discharge, it is found in the value of H. After all, this empirical and approximate formula should be used no longer than until it can be replaced by another, based upon a generally admitted theory, and upon the results of observation.

Amplitude. 167. If the flowage were simply water superimposed on the primitive current, uninfluenced by its velocity, its surface would extend horizontally from C to K, the point where the horizontal line drawn through the summit of the flowage meets the surface AB of the natural current. CK would be the *hydrostatic* amplitude, and would have $\frac{H}{p}$ for its expression.

But the real or hydraulic amplitude is not the same; it is generally much greater. Dubuat (164) admits $\frac{1.9H}{p-p_1}$ for its value: and as p_1, the slope of the fluid surface near the dam, is always very small, the hydraulic amplitude will be nearly double the hydrostatic amplitude. Funk has seen that value to be too great, and he fixed it at $\frac{3H}{2p}$; that is to say, that the real amplitude will be one and a half times the hydrostatic amplitude. As a mean term, it is nearly so; for in other respects, this value is usually modified by local circumstances, and sometimes to a great extent.

The theory of permanent motion, according to the formula of St. Guilhem, conducting to an asymptotic curve, would give an infinite extent to the flowage; its surface would be continually approaching that of the natural current without attaining it. But at a distance from the dam nearly equal to the value of the amplitude, the space which separates the two surfaces, according to these theories, is so small as to be inappreciable, and may be regarded as nothing. Moreover, the mutual adhesion of the particles of water, and the greater velocity of those of the primitive current, will tend to diminish, and always will diminish, the extent of the flowage which would have taken place were the fluid particles entirely independent of each other; so that, I repeat it, the extent will be very often less than that assigned it by Funk from his observations.

168. Let us apply our provisional formula to cases of the most frequent occurrence. **Examples.**

I. On a large river, discharging 2825.3 cub. ft. per second, at the time of low water, and whose slope is quite uniformly .000264, we are about to establish a dam 9.8427 ft. in height, and 705.39 ft. in length, and this in a place where the mean depth is 3.1168 ft.; what will be the rise 29528 ft. up stream?

Taking the value of y from the above equation (166), we have

$$y=\sqrt[3]{\frac{H^8}{H^5+\frac{4}{9\times 3.2809}(px)^6}+(px)^3}-px.$$

The value of H will be (163)

$$9.8427-3.1168+.42913\sqrt[3]{\left(\frac{Q^2}{L}\right)};$$

the last term, here expresses the height to which the flowage is raised above the crest of the dam, and since $Q=2825.3$ cub. ft., and $L=705.39$, this term will be 1.0859 ft.; thus $H=7.8118$ ft.; also, $p=0.000264$, and $x=29528$ ft. Consequently,

$$y=\sqrt[3]{\frac{7.8118^8}{29091.2+30398}+473.71}-7.7954=1.1124;$$

that is to say, that at the distance of 29528 ft. from the dam, the raising of the water produced by it would be 1.1122 ft. The depth of the current in this place, according to the level previous to the construction, was 2.788 ft.; it will therefore become 3.897 ft. We will admit it to be at most 3.6089 ft.

II. On the same river, and with the same data, we wish to find at what distance from the dam the rise of the water above its crest shall be only .16404 ft.

The equation of the curve, where $y=.16404$ ft. and $H=7.8118$ ft., will be

$$(.16404+px)^3-(px)^3-\frac{H^8}{H^5+\frac{4}{9\times 3.2809}(px)^6}=0.$$

Now substitute successively for x different values, until the equation is satisfied; thus

It will be for $x=44292$ ft., $+31.43$ cub. ft. $=0$.
" " $x=39371$ ft., -15.186 cub. ft. $=0$.
" " $x=41011.2$ ft., $+2.52$ cub. ft. $=0$.

Thus, at a distance of about 40683 ft., the surface of the flowage will again be at .164 ft. above that of the old current. We conclude from this, that beyond the 41011 ft., this difference will be insensible, and consequently, that the amplitude is 41011 ft.; this would not be 1.4 times the hydrostatic amplitude, which is $\frac{7.8118}{.000264} = 29593$ ft.

III. In a river which conveys about 706.3 cub. ft., and the slope of which is .00032; in a place where the mean depth is 1.3779 ft. and the breadth of the channel 393.7 ft., it is required to establish a dam, which would procure a depth of 3.2809 ft., necessary for the navigation of boats, against the lower face of another dam 47573 ft. above, and where there is only 1.476 ft. in the deepest part. It is necessary, then, that the projected dam should raise the water 1.8045 ft., at least. What should its height be to produce this effect?

Designate this height, the quantity sought, by ξ. Since $Q =$ 706.33 cub. ft. and $L = 393.708$ ft., the water will be raised above the dam .63854 ft.

$$.63854 = .42917 \sqrt[3]{\left(\frac{Q}{L}\right)^2}:$$

thus (163) $H = \xi - 1.377978 + .63854 = \xi - .739438$. We then have $y = 1.8045$ ft., $x = 47573$ ft., and $p = .00032$; thus the equation becomes

$$4936.8 - 3528.03 - \frac{(\xi - .739438)^3}{1 + \frac{1686127.9}{(\xi - .739438)^5}} = 0.$$

Substituting 16.453 ft. for ξ, we have $+ 3 = 0$,
" 16.46 ft. " " " $-.63 = 0$,
which gives for ξ the height of the dam, say 16.455 ft.

But is it advisable to build a dam of such a height? Those engineers who have adopted the principle that, without unusual motives, dams should not exceed ten feet in height, would answer in the negative, and would conclude that, below the existing dam, and in a given length of 47573 feet, there should be two dams in place of one.

169. The flowage (*remous*) which we have just

"*Remous*" peculiar to certain streams.

considered has a concave surface; it loses itself insensibly at its extremity in the natural current, and its extent far exceeds the hydrostatic amplitude. But there are others, rarely met with, it is true, which are characterised by wholly different and nearly opposite qualities; their surface is slightly convex, and is very much so at the ends; they are detached from the current by an abrupt departure, and have a length less than that of the hydrostatic amplitude. These different circumstances are strikingly manifested in the experiments made by M. Bidone at the hydraulic establishment of Turin.

Fig. 34.

The canal on which Bidone operated was of masonry. It was 1.0663 feet broad and the same in depth: the bottom was inclined, and for a length of 32.809 feet, that of the field of observations, the inclination increased nearly gradually from .0623 ft. to .1246 ft. Three currents of water, the quantities of which were exactly known, were introduced successively in it. When the regime of each was well established, and all the circumstances of the natural current, the depths, velocities, &c., were noted, it was barred up, by means of small wooden dams, whose heights were progressively increased. Then the height, the amplitude of the flow, the hydrostatic amplitude, &c., were carefully measured. The form of the flowage, with the rebound, of one of these experiments, is represented in Fig. 34. The result of all these observations is placed in the following table, for the details of which see the work of the author.*

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* Mémoires de l'Académie des Sciences of Turin. Tome XXV., 1820.

Water discharged per second.	CURRENT above the *remou.*			Height of Dam.	*REMOU.*		AMPLITUDE hydrostatic.	DIFFERENCE between the amplitude by		REBOUND by	
	Velocity.	Depth	Slope.		Height above Dam.	Amplitude.		Observation.	Calculation.	Observation.	Calculation.
cu. ft.	ft.	ft.		ft.							
0.7346	4.465	0.154	0.035	0.439	0.335	14.206	21.949	8.399	7.973	0.276	0.272
				0.525	0.328	15.978	24.410				
				0.617	0.335	18.701	27.166				
				0.712	0.335	21.424	29.659				
1.2396	5.522	0.203	0.032	0.446	0.449	12.008	24.016	12.238	12.172	0.420	0.394
				0.528	0.459	14.567	26.805				
				0.620	0.469	17.126	29.561				
				0.705	0.472	19 259	31.858				
				0.794	0.469	21.949	33.760				
1.6493	6.352	0.243	0.032	0.443	0.548	11.024	25.919	14.961	16.109	0.509	0.518
				0.528	0.551	13.419	28.248				
				0.614	0.548	15.420	30.545				
1	2	3	4	5	6	7	8	9	10	11	12

170. It follows from these experiments :

1st. That the height of the flowage above the crown of the dam is independent of the elevation of the crest above the bottom; and that it varies only with the quantity of water discharged. For the three discharges it was .334, .466, and .548 ; the formula .42917 $\sqrt[3]{\left(\frac{Q}{L}\right)^2}$ would have given respectively, .332, .4745 and .574 ft. Here, as in the experiments made at the water-works of Toulouse, beyond a certain limit, the coefficient 0.64 fails by excess, and as much more as the height of water on the crest is greater compared to the height of the dam.

2d. Naturally, the extent of the amplitude increases with the height of the dam, but not in the same ratio.

3d. In comparing the real amplitudes with the corresponding hydrostatic amplitudes, I have observed, not without some surprise and satisfaction, let their magnitude be what it would, that their differences remained the same for a like discharge, or rather, for the same velocity; but that it increased with the velocity. The case is similar to that of currents, already mentioned (133), which on issuing from a gate, enter a canal, where water previously passed is running, but with less velocity, and consequently with greater depth ; the current drives this water before it a certain distance. So here, the natural current meeting the water of the *remou*, which seems inclined to return up stream by virtue of its tendency to a level, drives it, and in some way compels it to retrace its path. The force which it there exerts, like to that which bends a spring, will be an active force, and its effect, the length of the driving back, will be proportional to the square of the velocity. This length, starting from a point where a horizontal line, drawn through the summit of the *remou*, meets the surface of the current, is the difference of the two amplitudes: it will, therefore, be proportional to v^2; and for the above experiments, it will be quite accurately represented by $.39928v^2$, as may be readily seen by a comparison of columns 9 and 10 of the table, the numbers of the 10th column being calculated by means of this formula.

The velocity being less at the sides than in the middle of the current, the running back will be less near the sides, and the flowage will extend farther; in fact, in all the experiments of M. Bidone, its length was greater by from .065 to .131 ft. A manifest proof that the running back is occasioned by the velocity of the current, and that it should increase with it.

4th. Upon the length depends the height of the rebound which takes place at its extremity. The surface of the *remou* at the rebound being sensibly horizontal, that height will be the fourth term of a proportion, of which the three first are the hydrostatic amplitude $\left(\frac{H}{p}\right)$, the height of the *remou* near the dam (H), and the length of the running back ($.39928v^2$), it will therefore be $.39928pv^2$. The numbers of the last column, calculated by this expression, and which differ but little from those of experiment, show that it is very nearly so.

In canals of great velocity, $p = .0001127 \frac{v^2}{h}$; thus, for the height of the rebound, we should have $.00004459 \frac{v^4}{h}$, h being the depth of the current just above the rebound.

In most rivers, where generally v is less than 3.28 ft., and p less than .001, the rebound would seldom exceed .00328 ft.; it would be insensible.

171. Notwithstanding the apparent difference between the ordinary *remou* and those just discussed, M. Bélanger has tried upon them the formulæ of permanent motion; from them we may effectually deduce some of the most remarkable

features of those *remous*, the height of the rebound, for example. For this purpose, we recur to the equation (123)

$$p' = \left(\frac{v^2}{2g} - \frac{v_0^2}{2g}\right) + \int \frac{c}{s}(.00011142v^2 + .0000242647v)\, dz:$$

neglecting the last term, which expresses the resistance of the bed, since, in the very short space dz occupied by the rebound, this resistance is extremely small compared to the other quantities, we have simply

$$p' = \frac{v^2}{2g} - \frac{v_0^2}{2g} = a - a_0;$$

if, in this expression, v and v_0 are the velocities taken at two points, the one just above and the other immediately below the rebound, p' being the slope or difference of level between the two points, will also be the height of the rebound required; a and a_0 are the heights respectively due to v and v_0.

Let h be the depth of water immediately before the rebound, and h_0 that just after the same, we shall then have $p' = h_0 - h$. The velocities being in the inverse ratio of the sections, or the depth of water in rectangular canals, the proportion

$$\sqrt{2ga_0} : \sqrt{2ga} :: h : h_0, \text{ will give } a_0 = a\frac{h^2}{h_0^2} = a\frac{h^2}{(p'+h)^2};$$

so that for such canals the equation will become

$$p' = a\left\{1 - \frac{h^2}{(p'+h)^2}\right\};$$

whence is deduced the expression given by M. Bélanger,

$$p' = \frac{a}{2} - h + \sqrt{a\left(\frac{a}{4} + h\right)}.$$

The value p' of the rebound will be positive only when $h < \frac{a}{2}$; that is to say, there will be no rebound in a water course, save when the depth of the natural current is less than half the height due to its velocity: and as this is most generally very small, it necessarily follows that the depth will be smaller still.

From what has been said, we see that *remous*, like those described by M. Bidone will only occur in water courses of great velocity, and of very small depth; and such water courses, for any notable length, are rarely found in nature.

2. *Remou or Backwater produced by contracting the Water-way.*

Height of *Remou.* 172. If a construction in a river does not extend the whole width of the bed, and obstructs but a part of it, all the water obliged to pass through the other part, that is, through a narrower space, must pass there with greater velocity; the excess of velocity can only be produced by an elevation of the fluid surface above the construction and contracted space, so that the fluid, at the moment of its entrance into this space, experiences a fall, the cause of its increase of velocity.

The height of this fall will also be given by the equation $p'=\frac{v^2}{2g}-\frac{v_0^2}{2g}=\frac{Q^2}{2g}\left(\frac{1}{s^2}-\frac{1}{s_0^2}\right)$ which has just indicated the height of the rebound in a certain flowage. Let x be the height of fall, L the mean breadth of the stream above the contracted space, l the width of the contracted part, and h the depth of the water in that part; its section s will be lh, or rather mlh, m being the coefficient of contraction at its entrance; for the section s_0 of the current immediately above the fall, we have L $(h+x)$, $h+x$ being the depth of the water there, and L the breadth. Thus, observing that x is the slope designated above by p', we shall have

$$x=\frac{Q^2}{2g}\left(\frac{1}{m^2l^2h^2}-\frac{1}{L^2(h+x)^2}\right).$$

Eliminating x, we shall have an equation of the third degree, which would give directly its value; but it may be obtained more simply by substituting in the above equation different values for this unknown quantity, until its two members are reduced to equality.

Backwater occasioned by bridges. 173. Bridges built on rivers, by contracting the water-way, cause, immediately above them, a raising of the level of the same nature as that just described, and which is determined in the same manner.

The sum of the intervals between the piers will be the width of the contracted space through which all the water passes; it is the width designated by l in the above formula, and L will be the breadth of the river above the bridge. Eytelwein takes for the coefficient of contraction m 0.85, when the piers present their up-stream face square against the current, and 0.95 when they are terminated by an acute angle. These limits may, however, be exceeded; thus, the effect of contraction may be diminished by giving to the cut-waters of the piers a form such that their horizontal section may be an equilateral triangle, with sides curved in the arc of a circle, as seen in Fig. 48; or, still better, in an elongated semi-ellipse AMCM′B; this last form being that which, according to experience, affords the least contraction. This should be employed when we would give to a river the best possible discharge; still, the semi-circular form is generally adopted, perhaps because it gives less projections and more elegance to constructions.

Q	l	h	m	x by	
				observation.	calculation.
cub. ft.	feet.	feet.		feet.	feet.
2048	241.80	4.675	.90	.1640	.0525
15256	310.37	8.248	.90	.6857	.7386
27511	290.36	12.733	.90	.8563	.8760
28853	299.55	12.139	.90	.9711	.9908
25958	299.55	10.998	.90	1.0302	1.0597
35175	320.22	14.570	.81	1.1319	1.1221
39660	311.03	16.106	.81	1.2369	1.2566
46546	314.97	17.622	.81	1.2312	1.3977
83700	434.39	18.429	.81	1.7717	1.8340

Let us apply the above formula to observations made at the bridge of Minden, upon the Weser. Funk, who reports them, says, " immediately above the bridge, in 1804, very exact measurements were made at eight different heights of the water." I add, in the above table, as a ninth observation, the relative measurements of the extraordinary freshet of 1799. The values of m are those which Funk himself has adopted; but nevertheless, he remarks that much uncertainty exists upon this matter, "because," says he, "of the works which surrounded the piers, of the different forms of the cutwaters of the bodies placed on the up-stream side to arrest and break the ice, and of the different manner in which the water entered beneath the vaults of the arches, in times of freshets."

In comparing the heights of the backwater given by calculation with those of observation, it is seen that our formula gives the effects of contractions produced by bridges as well as could be hoped, in a matter where all determination rigorously exact is almost impossible.

In the example just given, we have a river carrying a very considerable volume of water, and a bridge which contracts its bed nearly one half, and yet the height of the back flow which it caused was only from .6562 to .9843 ft. In high water, it once exceeded 1.3124 ft.; and in an unusual freshet, it was not 1.8045 ft.

Fall of water under a bridge.

174. Not only is the surface of a fluid mass which passes between two piers, and within any narrowing of the bed in general, raised on the up-stream side, as we have just seen, but it is also lowered in the narrow space, and even a little beyond, as indicated in Fig. 35. In consequence of the total fall, the water a little below the narrow space possesses a velocity sensibly greater than before. With this greater velocity, a greater inclination and a less depth, it will more easily reach the bottom, and will there exert a more powerful action. It will, therefore, be below the contracted way that the current will tend more particularly to hollow out the bed, and to undermine the masonry which confines it.

Fig. 35.

The contraction which occurs at the entrance of each of the arches of a bridge, occasions there not only one, or, more often, two superficial converging currents, but also, it causes inferior currents, thought to be more rapid and injurious. Local circumstances vary their direction, as well as their action upon the bottom; for example, we have remarked, after great freshets, that, in small arches, those less than 25 ft. span, the two oblique currents uniting before their exit, the bed had been deepened most towards the middle, and that in large arches, on the contrary, the deepening was found to be along the piers, and especially near the shoulder angles, at the down-stream ends.

Immediately behind the piers, the water is usually nearly stagnant, and the river deposits there part of the materials which it conveys. It sometimes happens, however, that the currents coming from two neighboring arches converge and unite, wholly or in part, below the intermediate pier; between the pier and the point of junction, a whirling may be produced, which, acting upon the bottom, may undermine the pier; it is proper, for this reason, to lengthen it, and it is partly with this view that a down-stream starling is added. The shoulder angles on the up-stream sides are likewise dangerously exposed; the fall above the bridge, which causes the inferior currents above mentioned, forms, in great freshets, when the starlings are very obtuse or have plane faces, as it were, a cataract, the action of which is exerted near the angles; the evil is prevented, or at least considerably diminished, by giving to the starlings the forms indicated in the preceding number.

ARTICLE FOURTH.

Considerations relative to the action of water on Constructions.

In continuation of my remarks on the subject of bridges, I should be glad, in this fourth article, to discuss the reciprocal action of running waters, and of constructions made in their bed upon each other, and more especially, to point out the means of preventing the ruin of these works; but there is nothing general and precise upon this subject; and a series of local facts would be out of place in this elementary treatise on Hydraulics. I shall consequently confine myself to the few following observations.

The action of water in great freshets.

175. In great freshets, the water produces extraordinary effects upon the bodies exposed to their action, which are by no means, at least apparently, proportional to those we commonly see produced; so that from the ordinary effects, we cannot conclude what has or might have been done by those freshets which hardly happen once in a century. I cite two examples, which seem worthy of remark; they are taken from the same locality, from the Falls of the Sabo on the Tarn, a league above d'Albi. The river there is, as it were, dammed up by a mass of rocks, in the middle of which, at a distant period, and possibly in circumstances having no analogy with the actual state of things, it opened a passage, like an enormous slit, where it falls in cascades, having in all nearly a height of 65 ft.

The rocks are of micaceous or talcose schist, soft, and containing quartz stones. Their surface, which is nearly always above that of the water, yielding to the erosive action of the atmosphere, is decomposed; the schist is reduced to earth, and the quartz stones remain isolated.

In freshets, some are driven into the depressions or cavities of the surface. If the freshet increases, and the velocity of the current becomes very great, it often produces whirlpools above these cavities; there the water seizes the quartz pebbles, and, impressing on them a violent rotary motion round a vertical axis, like a drill, it hollows out of the rocks, already softened by the moisture, perfectly cylindrical holes, with smooth faces, and sometimes 6½ ft. deep; at the bottom of some are still to be seen the stones which have served as borers. This fact shows how great is the action of whirlpools in great freshets upon the bottom of rivers, especially when the current carries pebbles along with it; these are then true whirlpools of stones.

At a period when, in the same place, the Tarn was raised 40 ft. above its usual height, the water rushed through the rift in the dam of rocks with frightful velocity; on the right and the left of the principal current, there was a counter current, which ran back along the adjacent banks with such force as to overthrow, and towards the up-stream side, the great poplars with which one of the banks was covered; I was much surprised in witnessing such an overthrow, some days after it occurred.

What engineer has not seen, after a great freshet, his dams of masonry as it were furrowed by the stones which have passed over them? Who has not seen his pavements, &c., even when constructed of large cut stone, worn down, and in some points turned upside down? Few of our constructions resist the strong freshets that take place in a century; perhaps there is not to be found in France twenty great bridges which have lasted four hundred years. Not that those which have fallen had not a mass strong enough and well enough constructed to resist the shock of the water, but because the fluid undermined their foundations, and excavated the earth on which they were established.

Observations concerning undermining.

176. It will be, then, the chief care of the engineer to guard against this undermining. What he should do for this purpose has been explained in works on hydraulic architecture, as well as in those concerning the art of bridges and roads, chiefly in the works of Perronnet, and Gauthey's treatise upon the construction of bridges; I shall say no more on this, but confine myself to an observation which is more peculiarly in my province.

The study of the soil on which the engineer proposes to establish a hydraulic construction, should be his chief duty. In the tertiary earths of the mineralogists, we find frequently beds of stone alternating with strata almost earthy, such as soft marls, and even with sand banks. When, by sounding, we have reached a layer of the first kind, or what is termed solid, it is necessary to determine its thickness, and to be well assured that there are not, at a small distance below, less solid beds. As the layers of the same soil are not usually entirely horizontal, examination should be made in places where the earth may have been bared, a little above or just below that where the construction is to be made. We should endeavor to examine the bed which has been reached by the sounding-rod, as well as those lying immediately beneath it; so that we may be well acquainted with its character and thickness. But if the locality does not admit of such an examination, it will be necessary to continue the sounding still further; for, I repeat it, the main object is to be well assured of the solidity of the soil on which we have determined to build.

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Difference in the effects of water.

177. The action of water is entirely different on bottoms of a different nature; and works which may produce a marked effect upon one river, or a certain portion of it, may produce none upon another. For example, in the moors of Gascony, where the rivers flow with but a slight inclination, on a very fine and moveable sand, M. Laval, by means of wicker dikes, between which were thrust pines and other trees covered with their branches, narrowed and deepened at his pleasure the bed of these rivers; * whilst upon the Loire, works otherwise quite solid, dams of masonry, transverse and but slightly elevated above the mean level of the water, fixed upon one bank, and jutting quite far into the current, could not produce upon the opposite bank a deepening sufficient for a channel of navigation; the excavation which they occasion in one point is often followed by a filling or deposit in the succeeding point.† Since I have been led to speak on the subject of deepening the channels of rivers for any great extent, I will remark, that we can only secure our purpose by enclosing the current between two longitudinal dikes, beneath the surface or not, either continuous or formed of a series of small dikes, with intervals between them through which the water in time of freshets may pass, to wash out the space left between the dikes and the old banks.

The difference in the manner of operating, according to the localities, is also found in the protection of a bank exposed to a current, which bank might be injured, but for opposing some obstacle against it; this defence is sometimes made by a stone jetty, sometimes

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* Annales des ponts et chaussées. Juillet-Août, 1831.

† Idem, tome V., 1833.

by a revetment of fascines, such as was adopted with great success upon the banks of the Rhine.*

178. Constructions, in all respects similar, not only produce different effects, but sometimes such as are of a directly opposite character. Thus, it is generally admitted, that dikes properly established upon a bank preserve and fortify it, by causing deposits in the vicinity of the points where they are established. In fact, during ordinary freshets, the water remains nearly stagnant, or it turns feebly in the angle formed by the bank and the dike, particularly on the up-stream side, and makes deposits there. But in unusual freshets, when the velocity is very great, this turning may become a rapid whirlpool, to attack and wear away the adjacent bank; it acts upon it not only by its mass, but also by the centrifugal force of its particles, a force due to the velocity of rotation; and here the construction would occasion the ruin of the bank it was designed to protect.

When a dike, or a series of dikes, is designed to attack the opposite bank, or to destroy a deposit of sand formed there, it is often directed down stream, so as to make an angle of about 135° with the bank upon which it is fixed.† It is thought that by this disposition, the current losing but a little of its velocity against these dikes and being directed by them upon the opposite bank, will act there with greater force. But it has happened that in the up-stream angle of which we have spoken, a sand-bar has been formed, with its point presented to the current with an acute angle; thus, the proposed effect did not take place, and it would have been as well to have located the dike perpendicular to the bank.

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* Bélidor, Architecture hydraulique, tome IV. M. Defontaine, work already quoted (152).

† Bossut et Violet: Recherches sur la construction des digues. 1764.

Position and form of Dams.

179. After this diversity in the effects of water, according to the difference of soils and of local circumstances, we should not be surprised at the difference of opinion entertained by skillful men, upon the most ordinary constructions; for example, upon dams by means of which we bar up entirely the course of rivers, whether for an increase of depth for the purposes of navigation or to procure a greater fall, and consequently a greater motive power in the establishment of mills. I shall dwell a few moments on this important question of dams.

In many countries, they are usually placed oblique to the river. It is said in this case, that the water has a less destructive action upon them in times of freshets, especially in the up-stream parts; as to the down-stream part, where sluices, navigable ways and mills are usually built, they are, it is said, sufficiently protected by the constructions which such establishments require. Some prefer to give their dams a broken form, that of a rafter presenting a salient angle to the current, especially when it is intended to build mills at each end. Others build them as much as possible perpendicular to the course of the river; observing that, being shorter, they are less expensive; that also, contrary to the common opinion, they have not to support a greater hydrostatic pressure, and that the difference in the action of the impulse is small. I will observe, that whatever be the direction given to the dam, more particularly when it is placed perpendicular to the current, care must be taken to secure its extremities well into the quays or other adjacent constructions, or to found them safely in the banks.

M. Borrel, engineer, on the subject of the position of dams, has made a remark worthy of consideration, especially whenever the points on which they are to be built are not controlled by peculiar circumstances. In every river with a gravel bottom, he observes that natural bars are formed in certain parts, which will be re-formed soon after their removal; they are a necessary consequence of the form of the bed, and they denote the place where the action of the water upon the bottom is least destructive, and consequently, where the most suitable location for the dam is to be found; the direction of the ridge of the bar, disregarding trifling irregularities, would be that which it would be well to adopt.

180. The opinions of constructors are at least as various in respect to the form and profile to be given to dams. Most frequently, their thickness equals about three times their height, and their upper surface is inclined towards the down-stream side at an angle of 20°. The objection to this form is that it presents too great a surface to the action of stones, drift and ice, brought down in freshets, and on the breaking up of the ice; moreover, it preserves the whole force of the water, and directs it against the bottom. To remedy these defects, experienced engineers have given to their dams a section nearly rectangular, with a breadth but little greater than their height, the upper face inclining slightly up stream, and the two side faces having a slope at most of one in six; at their foot, on the down-stream side, they construct a bank or berm. The water which passes such dams, say their partizans, the inspector M. Bertrand among others, falling in cascade upon this bank, is deadened; it loses its velocity, and retains no longer the power to do mischief. But for

this purpose, the berm should be broad, and of very good masonry, otherwise the water will soon destroy it, and so quickly undermine it. M. Girard, who has made the effects of water upon these dams his peculiar study, remarks that between the foot of the dam and the bottom of the cascade a whirl is produced, with its axis horizontal and parallel to the dam; and that this whirl, whose destructive action is still more increased by the bodies falling with it, wears with such force both upon the foot of the dam and the ground beneath it, that few berms, unless built upon the solid rock, can effectually resist it.*

Finally, skillful men, giving to dams all their former width, have made the upper surface of a curved form, convex at top, and concave at the base: the nature of the curve is of little importance, whether it be a sinusoïde, an arc of a circle, etc., provided there are no sharp angles, and that its last element is horizontal and nearly level with the bottom of the river. The objection to this form is, that it exacts more careful fitting, consequently, greater expense; and, more especially, that it impels the water in a horizontal direction, with all the velocity due to its fall, consequently disturbing the river at a great distance, to the injury of navigation. But, on the other hand, it is the form which gives the least force to the water for undermining the foot of the construction. I should observe, however, that if the bottom affords slight resistance, and a part of its surface should be washed away, there might be formed beneath the lower surface of the current, launched horizontally, a counter-current, which, joining the first at the foot of the dam, would produce there one of those whirls, with a horizontal axis, whose destructive effects we have already pointed out. It is probable that, to prevent these, Perronnet, the most celebrated of our engineers, after having adopted the form just investigated for a dam in the canal of Burgogne, fixed many beds of fascines before its foot.*

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* Annales des ponts et chaussées, tome X. 1835.

Finally, this last kind of dam is little used, it is so costly. The second spoken of, that with a nearly square section and a berm, has prevailed lately, and for some years, among skillful men. But it seems they are now returning to the first, that with a plane inclined to the down-stream side, particularly where the bottom is easily washed away; some, however, substitute a series of steps for the plane.

CHAPTER III.

ON THE MOTION OF WATER IN CONDUIT PIPES.

Similarity of motion in pipes and canals.

181. In a long inclined pipe, as in a canal, the water moves in virtue of its weight, or rather, by that part of its weight rendered active by the inclination of the pipe; the accelerating force in both cases is *gp* (104). So that if, at the upper part of a reservoir, M were fitted at AB, either a canal or a long pipe, admitting that no obstacle opposed the action of this force, the fluid would pass from the point B with a velocity due to the height EB.

Fig. 36.

In a canal open on the upper part, no pressure is exerted on the fluid which enters it, whilst there is commonly a pressure on the head of pipes. For example, if we place the pipe AB at CD, we shall have at C a force of pressure, in consequence of which the water will enter into the pipe with a velocity due to the height AC. According to the first principles of accelerated motion, this velocity must be added to that which the fluid acquires by the effect of the slope from C to D; so that, abstraction being made of every obstacle, it will pass out with a velocity due to AC+FD, or to ED, a height which represents the force in virtue of which the flow tends to take place. This last case can also be referred to that of canals; if we prolong CD to G, at the level of the reservoir, and construct a canal from G to D, the water will tend still to go out with a velocity due to ED. Thus in every case, in pipes as well as in canals, the accelerating force and the effects which it tends to produce are the same.

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* Lecreux, Recherches sur les rivières, p. 266.

Under the influence of such a force, the motion in pipes should be continually accelerated; and yet, at a very small distance from their origin, it is sensibly uniform. It follows, therefore, that beyond that distance, at every instant an opposite force destroys the effect of the first. This opposite force can only be the resistance of the sides of the pipes, which, as in canals, proceeds from the adherence of the fluid particles to those sides and among themselves (106).

Thus in the pipes we have the same accelerating force and the same retarding force as in canals; the motion is of the same nature, and we might say that the case of pipes is only a particular case of canals, the case where the upper part of the canal is closed.

This difference, however, in the form of the bed, occasions, during motion, peculiar circumstances, which demand special considerations: these will be the object of this chapter.

ARTICLE FIRST.

Of Simple Conduits.

In hydraulics, and particularly in the art of fountain-makers, the name of conduit is given to a long line of pipes, exactly joined together. The conduit is *simple*, in opposition to a system of conduits, when it consists only of a single line of pipes, conveying even to its extremity all the water which it receives at its origin.

1. *Straight Conduit, of Uniform Diameter.*

Mode of expressing resistance.

182. For greater simplicity, unite in one the two forces which tend to produce the velocity of exit, the pressure AC at the head of the conduit, and that of FD, which proceeds from the slope: for this purpose, imagine that the given conduit CD is placed horizontally, at HI, at the bottom of a reservoir whose depth AH is equal to $AC+FD=ED$. Nothing will be changed in the data of the problem; we shall always have the same force and the same resistance, this last being independent of the position of the conduit.

The force of pressure in virtue of which the water tends to flow out, or, more immediately, the vertical height ED, the difference of level between the orifice of exit and the surface of the fluid in the reservoir, is called the head upon the conduit. We shall habitually designate it by H.

If the conduit opposed no resistance to the motion, making abstraction of all contraction at the entrance, the water will flow out with a velocity due to all that height, as we have just seen. But such is not the case; the resistance of the sides, opposing an obstacle, diminishes that velocity; it absorbs, consequently, a portion of the motive head H. The flow takes place only in virtue of the remaining part; this part only is the height due to the velocity of exit, and also to the velocity on all the points of the conduit, since the motion in it is uniform, and since its section is throughout uniform. Let v be that velocity, $\frac{v^2}{2g}$ will be the height due, or the effective portion of the head; $H - \frac{v^2}{2g}$ will therefore be the portion absorbed by the resistance; it will serve to measure it, it will represent it.

183. We have just represented by the height H the effort or the force of pressure which urges the water in the pipe, by the height $\frac{v^2}{2g}$ the force which produces the flow, also by a linear quantity $H - \frac{v^2}{2g}$ the resistance or negative force; and yet it is a principle in mechanics, that the forces of pressure or the efforts are equivalent to weights, and ought to be expressed by weights. We will explain. **Observation.**

We have already seen (14) that the absolute pressure on a horizontal fluid surface or portion of that surface designated by s was $psH^{\text{lbs.}}$, p being the specific weight of a cubic foot of the pressing liquid. Since, according to the laws of hydrostatics, the pressure is equal on all parts of that surface, it will be sufficient and proper to consider only one; this will be an infinitely small one, which may be supposed always of equal magnitude; then s being constant, the pressure will depend only on the specific weight, or on the nature of the liquid and the height of the column: it is in this sense that the height of the column of mercury in the barometer expresses the pressure of the atmosphere. If the pressing liquid remain the same, as will always be the case with water, in this chapter, we may neglect its weight p, which is constant, and the pressure will be represented only by H; it will be exclusively proportional to it.

If we adhere rigorously to the principle, we should regard H as the weight of the fluid line which presses and urges along in the conduit the particle which is immediately below it, and we should represent it by a line, as, in elementary statics, we represent by lines the forces which are also weights.

Value of Resistance.

Fundamental Equation.

184. Since the resistance proceeds from the action of the sides, it will be proportional to their extent, that is, to the length of the conduit and perimeter of its section, which is here the wetted perimeter; for we suppose that the flowing takes place with a full pipe, otherwise we should have the case of a simple canal. On the other hand, the greater the section, the more the resistance of the sides will be distributed among a greater number of particles; consequently, it will affect each of them and the total mass less: it will therefore be in the inverse ratio of that number, and consequently of the magnitude of the section. Here also, as in canals (107), it will be proportional to the square of the velocity plus a fraction of the simple velocity.

According to this, if L is the length of the conduit, S its section, C the contour or wetted perimeter, a and b two constant coefficients, the expression of the resistance will be

$$a\,\frac{CL}{S}\,(v^2+bv),$$

and we shall have (as in Sec. 111),

$$H-\frac{v^2}{2g}=a\,\frac{CL}{S}\,(v^2+bv).$$

185. It remains to determine the coefficients a and b. Prony, who first undertook their determination in a proper manner, made use, for that purpose, of fifty-one experiments made by our most skilful hydraulicians, and which Dubuat had already employed for establishing his formulæ. From them he deduced

$$a=.0001061473;\quad b=.16327.$$

Of the fifty-one experiments, eighteen were performed by Dubuat himself, on a tin pipe of .0886 ft. diameter and 65.62 ft. long; twenty-six by Bossut, likewise on tin pipes 0.0886, 0.1181, 0.1772 ft. diameter, and of lengths varying from 31.96 to 191.84 ft.; and seven were made on the great conduits of the park of Versailles, one of .443 ft. diameter and 7480.68 ft. long, and another 1.608 ft. diameter and 3835.489 ft. long.

Twelve years after, Eytelwein treated anew the question of the motion of running waters: he thought proper to take into consideration the contraction of the vein at the entrance of the pipes, and m being the coefficient of that contraction, he established

$$H - \frac{v^2}{2g.m^2} = .000085434\frac{CL}{S}(v^2 + .2756v).$$

But m, the effect of which is, however, insensible in large conduits, is found implicitly in the value of a, given by experiment. Consequently, having regard to the most accurate observations, and particularly to those of Couplet, I adopt the equation

$$H - \frac{v^2}{2g} = .000104392\,\frac{CL}{S}(v^2 + .180449v).$$

For canals, we had (111 and 112)

$$H - \frac{v^2}{2g} = .0001114155\,\frac{CL}{S}(v^2 + 0.217786v).$$

These two equations are similar and very nearly identical, as they should be (181). The small differences in the numerical coefficients probably proceed only from errors in the observations. If it is so, as the observations can be made with much more accuracy on conduits than on canals or rivers, it is to be presumed that the coefficients of the equations for conduits are also the more accurate.

186. The section of pipes being a circle, if D represent the diameter, we shall have $S = \pi' D^2$, and $C = \pi D$; and, putting for π, π' and g their numerical

value, the fundamental equation of the motion of water in conduits will become

$$H - .015536v^2 = .000417568 \frac{L}{D} (v^2 + .180449v).$$

The velocity is rarely in the number of quantities given or sought in problems to be solved; it is almost always supplied by the discharge. Let Q be that discharge, or the volume of water flowing per second. We have $Q = \pi' D^2 v$ or $v = 1.27324 \frac{Q}{D^2}$; this value of v, put into the above equation, transforms it into

$$H - .025187 \frac{Q^2}{D^4} = .0006769 \frac{L}{D^5} (Q^2 + .141724QD^2)$$

Such is the formula usually employed for the solution of questions relating to the motion of water in conduit pipes; having regard, however, in its applications to practice, to the observations to be made in Sec. 205. Of the four quantities, Q, D, H and L, three being known, the formula will give the fourth.

Equation for great velocities.

187. When the velocity is great, that is, exceeding two feet per second, the resistance is sensibly proportional to the square of the velocity; the term containing only its first power would disappear, and, according to the experiments of Couplet, we should have

$$H - .0155366v^2 = .0001333 \frac{Lv^2}{D};$$

or, in terms of Q,

$$H - .0251817 \frac{Q^2}{D^4} = .0007089 \frac{LQ^2}{D^5}.$$

It is to be remembered, that the second member of the above equations is the value of the resistance proceeding from the action of the sides of the conduit.

Expression of Discharge.

188. Taking the value of Q from the general equation, it becomes

$$Q = -\frac{.070862LD^2}{L+37.20D} + \sqrt{\frac{1477.3HD^5}{L+37.20D} + \left\{\frac{.070862LD^2}{L+37.20D}\right\}^2}.$$

In long pipes, where 37.20D is very small compared to L, it may be neglected; the second term under the radical might also be neglected, and for ordinary cases of practice we shall have

$$Q = \sqrt{\frac{1477.30HD^5}{L}} - .070862D^2;$$

or,

$$Q = 38.436\sqrt{\frac{HD^5}{L}} - .070862D^2.$$

189. In great velocities,

$$Q = 37.548\sqrt{\frac{HD^5}{L+35.5D}} \text{ or } Q = 36.769\sqrt{\frac{HD^5}{L}}.$$

If the velocity be required, we have its value by dividing the discharge Q by the area of the section $.7854D^2$.

Expression for the Diameter.

190. The diameter of conduit pipes is very often the quantity to be determined. To undertake its determination most easily, put the fundamental equation (186) under the following form:

$$D^5 - \left(.000095938\frac{D^2QL}{H} + .0251817\frac{DQ^2}{H} + .0006769\frac{LQ^2}{H}\right) = 0.$$

Omit for a first approximation the first two terms in the parenthesis, and we have

$$D = \sqrt[5]{.0006769\frac{LQ^2}{H}} = .2323\sqrt[5]{\frac{LQ^2}{H}}.$$

This value will be a little too small; we should then make small additions, until the first member is reduced to zero. The quantity which leads to this result will be the diameter sought.

For velocities above two feet, we have simply and directly

$$D = .2349 \sqrt[5]{\frac{LQ^2}{H}}.$$

Nothing need be said concerning H and L. The equation of Sec. 186 gives them by a simple transformation.

191. We give examples for the determination of the discharges and the diameters.

I. We have a conduit 0.82022 ft. in diameter, and 4757.3 ft. long; required the volume of water it will deliver under a head of 17.454 ft.

We have then, $D = .82022$ ft., $H = 17.454$ ft., $L = 4757.3$ ft., and $L + 37.2D = 4787.812$ ft.; and consequently,

$$Q = -\frac{.070862(.82022)^2 \times 4757.3}{4787.812} + \sqrt{\frac{1477.3\ (.82022)^5}{4787.812} + \left\{\frac{.070862(.82022)^2 \times 4757.3}{4787.812}\right\}^2}$$

$= -.04737 + \sqrt{1.9994 + .0022439} = -.04737 + 1.4147 = 1.36733$ cubic feet.

The simplified formula would give $Q = 1.4186 - .04767 = 1.37093$ cub. ft. That for great velocities (189), and otherwise applicable to the actual case when the velocity is 2.588 ft., would give $Q = 1.3568$ cub. ft.

II. Required the diameter of a conduit 2483.6 ft. long, which is to conduct 3.1431 cub. ft. per second, under a head of 3.2809 ft.

Substituting these numerical quantities in the equation of Sec. 190, it becomes, every reduction being made,

$$D^5 - (0.22827D^2 + 0.075828D + 5.0624) = 0.$$

Neglecting at first the second and third terms, we have $D = \sqrt[5]{5.0624} = 1.3831$ ft. This value being too small, after several trials, is raised to 1.4127 ft., which is the diameter sought.

The formula for great velocities, and here $v = 2.0046$ ft., would have given $D = .2349 \sqrt[5]{\frac{2483.6\ (3.1431)^2}{3.2809}} = 1.3984$ ft.

Equation when Pipes are terminated by Ajutages.

192. Thus far, we have supposed the pipes entirely open at the extremity; but almost always, they are terminated by mouth-pieces, cocks, or, in general, by additional tubes, which contract the opening. In such cases, the velocity of the fluid at its exit is not the same as in the pipe, and consequently, the equations of motion which are given in §§ 185 to 188, and which are based on the supposition of that identity, cannot be applicable. The first member of those equations, $H—.01555366v^2$, presents the part of the head absorbed by the resistance of the pipe; and this portion is the head H, minus what remains at the extremity of the pipe, to produce there the velocity of exit (182); if this velocity is designated by V, the first member of the equation will in general be $H—.01555366V^2$. The second member is the expression of the resistance of the sides (187), which is a function of the velocity in the pipe or of *v; v* must then remain as it was in that member, which will not be changed in value.

193. In pipes, still more, if possible, than in other cases of a fluid moving without breaking its continuity, the velocities are in the inverse ratio of the sections; so that if *d* is the diameter of the additional tube at the orifice of efflux, *m* the coefficient of contraction applicable to it, D always being the diameter of the pipe, we have $V:v::\pi'D^2:\pi'md^2$, whence $V=v\frac{D^2}{md^2}=1.27324\frac{Q}{D^2}\times\frac{D^2}{md^2}=1.27324\frac{Q}{md^2}$. The equation of motion will then become

$$H—.0251817\frac{Q^2}{m^2d^4}=.0006769\frac{L}{D^5}(Q^2+.141724QD^2).$$

Of the five quantities which it includes, four being given, the value of the fifth will be shown.

Let it be required, for example, to determine the diameter to be given to a circular orifice in a thin plate fitted to the end of a pipe .26248 ft. diameter and 1745.493 ft. long; the quantity of water discharged to be .706332 cub. ft. per second, and the head being 14.764 ft.:

The above equation will give

$$d=\sqrt[4]{\frac{.0251817Q^2D^5}{m^2\{HD^5-.0006769L(Q^2+.141724QD^2)\}}}$$

Substituting the numerical values ($m=.62$), reducing and extracting the fourth root, we find $d=.076773$.

194. For velocities above two feet, we have

$$H-.0251817\frac{Q^2}{m^2d^4}=.0007089\frac{LQ^2}{D^5};$$

$$Q=37.548\sqrt{\frac{HD^5}{L+35.47\frac{D^5}{m^2d^4}}};\text{ and}$$

$$D=.2349\sqrt[5]{\frac{LQ^2}{H-.0251817\frac{Q^2}{m^2d^4}}}$$

I give two examples.

I. To the pipe already examined in Sec. 191 is fitted a conical tube of .098 ft. diameter; every thing else remaining the same, it is required to assign the discharge which will take place.

Here $D=.82022$ ft.; $L=4757.3$ ft.; $H=17.45$ ft.; and for m, considering the convergence of the tube (50), take .90. Consequently, $m^2d^4=.000076022$, and $35.47\frac{D^5}{m^2d^4}=173214$.

Thus $Q=37.548\sqrt{\frac{17.45\times .82022^5}{4757.3+173214}}=.22654$ cub. ft.

The complete equation of Sec. 192 would also have given .22654.

It may be remarked that if, instead of an additional tube .098 ft. diameter, we had taken one .4101 ft., (half the diameter of the pipe,) the discharge would have been . . . 1.295 cub. ft.
With a diameter of .6151 ft., (¾ that of pipe,) . 1.360 "
Without additional tube, ... 1.367 "

Which shows that when the diameter of an ajutage is great, compared to that of the pipe, (so as to be more than one half thereof,) the discharge differs but little from that obtained from the pipe being quite open.

In many of my experiments on the conduits of Toulouse, I was struck by this fact; the difference was even less than that indicated by theory; it was insensible. For example, at the extremity of a pipe .164 ft. diameter and 1391 ft. long, were fitted, in succession, plates with gradually decreasing circular orifices; and under the constant head of 53.48 ft., we had the following results. The diameter of the conduit being .164 ft., the first result was obtained without any plate, the pipe being entirely open. It is to be remarked, that the results of calculation approach nearer to those of experiment as the velocity of the water in the pipe was smaller.

DIAM. of orifice.	DIAM. of orifice.	DISCHARGE.	
		By calculation.	By experiment.
inches.	feet.	cub. ft.	cub. ft.
1.97	.164	.0756	.0607
1.38	.115	.0742	.0607
1.18	.098	.0731	.0607
.79	.066	.0646	.0558
.59	.049	.0519	.0470
.39	.033	.0297	.0290

II. To determine the diameter of a pipe 2736.35 ft. long, which, under a head of 21.326 ft., must discharge .3885 cubic ft. per second, by many orifices situated near each other, which, taken together, are equivalent in area to a circular orifice of .13124 ft. (about 1.57 inches) diameter, the coefficient of contraction is estimated at 0.85.

We have $m^2d^4 = (.85)^2 \times .13124^4 = .0002143$, and $.0251817 \times \frac{Q^2}{m^2d^4} = 17.732$; and consequently,

$$D = .2349 \sqrt[5]{\frac{2736.35\,(.3885)^2}{21.326 - 17.732}} = .60795 \text{ ft.}$$

2. *Pipes bent and contracted at some points.*

195. We have just considered pipes as being rectilinear, and of equal section throughout their length;

Three kinds of Resistance.

but usually they present bends; and sometimes there are parts of less section, either for a very small extent and forming a sudden contraction, or for a considerable length.

The water moving in such pipes, on arriving at the bends, is obliged to change its direction. In this change, it loses a part of its velocity; the resistance causing this loss is like an effort opposed to the motive effort, or to the first head; it destroys a part of it.

At sudden contractions, the water experiences still another loss; having to pass through a narrower section, it must have a greater velocity; a new effort is necessary to compel it to receive this velocity; this is a new absorption of the total head. Thus, water moving in pipes, experiences or may experience three kinds of resistance; that due to the action of the sides, by far the most considerable; that proceeding from bends; and that from sudden contractions. The forces or partial heads employed to overcome these resistances, are subtracted from the total head; it is in virtue only of the remaining part that the flow takes place; this part alone is the head due to the velocity of exit.

We have treated of the resistance of the sides in detail (184—188), and will now examine the two others.

Resistance of Bends.

196. Every moving body, which, after having followed one direction, suddenly changes it, loses a part of its velocity, represented by the verse-sine of the angle formed by the two directions. If, during its motion, it follow a curved line, it changes its direction, it is true, every instant; but the loss of velocity at each change is only an infinitely small quantity of the second order; and consequently, although the number of losses is infinite, the total loss will be an infinitely

small quantity of the first order, which may be considered as nothing; in other words, every body in motion which arrives tangentially at a curve, and which follows it any length, retains on quitting it the same velocity it had on its arrival. Whence it follows, that if the curve of a pipe be well rounded, whatever be the nature of the curve, and if the fluid exactly follow the curvature, it will experience no loss of velocity, no resistance.

But such is not the case; the particles of which the fluid is composed being independent of each other, while those in contact with the sides follow the curvature, the rest being directed against the sides, will be reflected by them or by the particles interposed, at an angle which may be quite large. For example, the central fillet aC tends to strike at C the side ACB, and then to incline along Cb, making an angle of reflection equal to the angle of incidence, which would be half the supplement of the angle of the curve aCb. The reciprocal action of the particles on each other will cause, in the total fluid mass, a loss of velocity, which loss will generally be less than that of the central fillet taken separately, but always greater than that of the fillets near the sides. Fig. 37.

This diminution of velocity and consequently of discharge, although real, will usually be very small. Thus, Bossut having taken a pipe .088587 ft. diameter and 58.2834 ft. long, extended it horizontally and in a straight line; under a head of 1.0662 ft., he obtained .736 cubic ft. in one minute; then, having bent it into a serpentine form with six curves, well rounded, it is true, he obtained, all else being equal, .720 cubic ft. per minute. (*Hydrodynamique*, § 659). Still, by increasing the number and abruptness of the curves, the diminution of the discharge can be rendered quite considerable, as seen in the following example: Rennie made a lead pipe 15 ft. long and one half an inch diameter; he fitted it horizontally to a reservoir, and under a head of one ft., he obtained 1.921 cubic ft. in one minute; then he bent the same pipe so as to form a series of fifteen semi-circular cavities or convexities, with a radius of about 3¾ inches; he fixed it in this new state to the reservoir, and the product of the flow was only 1.709 cubic ft; so that the fifteen curves reduced the discharge in the ratio of 100 to 89; under four times the head, the reduction was from 100 to 88.*

197. As to the laws followed by the resistance of curves and the measure of that resistance, it is to Dubuat that we are indebted for the first researches made on that subject. He took different pipes, at first straight, and he measured the head necessary for them to discharge a certain volume of water in a certain time; then he bent them in various ways, and in such a manner that the central fillet tended to make angles of reflexion of a determined number and magnitude; and he again ascertained the head under which they discharged the same volume of water in the same time. The difference between the two heads for the same pipe, when straight and when curved, was evidently the head due to the curves, and consequently the measure of their resistance. He thus made twenty-five experiments, the principal of which are introduced into the following table:

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* Philosophical transactions of the Royal Society of London. 1831.

PIPE.			VELOCITY of water.	RESISTANCE due to the curves.	Coefficient deduced, for ft.
Diameter.	Length.	Angles, No. and value.			
inches.	feet.		ft., per sec.	feet.	
1.07	10.391	1 of 36°	7.546	.0666	.00338
1.07	10.391	2 36	7.546	.1332	.00338
1.07	10.391	3 36	7.546	.2211	.00375
1.07	10.391	4 24.57	7.546	.1332	.00338
1.07	10.391	10 36	6.362	.5243	.00375
1.07	12.300	4 36	5.158	.1457	.00396
1.07	12.300	4 36	2.605	.0364	.00387
1.07	65.456	4 36	2.546	.0348	.00387
2.13	22.671	4 36	7.664	.2576	.00302
2.13	22.671	4 36	5.217	.1181	.00314
2.13	22.671	6 24.57 5 36.00 1 56.23	7.664	.7674	.00378

Dubuat concluded from his experiments, that the resistance of curves is proportional to the square of the velocity of the fluid, to the number of angles of reflexion, and to the square of their sine.

In this hypothesis, the coefficient varies within small limits, the mean term being .00375. So that, if v be the velocity, n, n', &c., the number of the angles of the same magnitude; i i' the respective number of degrees, the value of the resistance will be

$$.00375v^2\,(n \sin^2 i + n' \sin^2 i' + \ldots);$$

or, in function of Q, s^2 being the sum of the squares of all the sines, $.006079\,\frac{Q^2}{D^4}\,.s^2$.

Application and Remarks. 198. In applying this formula to a given pipe, it would be necessary to determine the number and value of the angles of reflexion for each curve. Now, a simple drawing shows, 1st, that in a pipe bent into an arc of a circle, and there will be arcs of no other kinds, the semi-diameter of the pipe, divided by the radius of the arc, gives the verse-sine of the angle of reflexion, and consequently its cosine and its value in degrees; 2d, that the num- Fig. 38.

ber of degrees of the arc, (that is, the supplement of the angle of the curve,) divided by double the angle of reflexion, indicates the number of the angles.

Take, for example, a pipe .82 ft. diameter, conveying 1.766 cub. ft. of water, which has a curve of 95°, the radius of curvature being 6.89 ft.; demanded, the resistance occasioned by the curve. From what has been said, the verse-sine of the angle of reflexion will be $\frac{.41}{6.89} = .0595$, and its cosine equal $1 - .0595 = .9405$, which belongs to the angle of 19° 52′: this is the angle of reflexion. The arc of curvature $= 180° - 95° = 85°$, divided by double the angle of reflexion, 39°.73, will give the number: this will be taken as 3, the quotient being 2.14. The sine of 19° 52′ is .3398, and its square $= .1155$; the resistance required will consequently be

$$.00608\,\frac{(1.766)^2}{(.82)^4} \times 3 \times .1155 = .0145 \text{ ft.}$$

a quantity extremely small; and yet the curve is quite sharp and the velocity considerable. For Rennie's pipe of fifteen curves (196), the above mode of calculation would have indicated a resistance of .633 ft.; and experiment even gave 1.161 ft., as we shall soon see (which would raise Dubuat's coefficient of .00608 up to .01115). But such a case is never presented in practice, and even with double the coefficient, the resistance rarely exceeds one half or three quarters inch loss of head. It is diminished still more, or rendered inappreciable, by taking a large radius of curvature; the greater it is, the more angles of reflexion we have, to be sure, but they are smaller, and the sum of the squares of the sines, and consequently the resistance, is less.

Effects of Angles.

199. If the effect of well rounded curves is insensible, it would not be so with angles properly so called. An experiment of Venturi shows their influence. This savan made three tubes, fifteen inches long and 1.3 inch in diameter; one was straight, the second had a curve of 90°, gently rounded, and the third had an abrupt angle of the same number of degrees: under a head of 2.887 ft., they filled a vessel containing 4.838 cub. ft. in 45″, 50″ and 70″ respectively. The bad effect of angles is still more manifest in the experiments of Rennie (196); with his pipe, fifteen feet long and one half inch diameter, under a head of four feet, he had per minute a discharge from the straight pipe . . . . .4196 cub. ft.

From the pipe with fifteen semi-circular bends, . .3694 cub. ft.
" " " " one right angle, ...3334 "
" " " " twenty-four right angles, . .1519 "

So that a single angle of 90° reduced the discharge more than the fifteen curves. This single fact shows with what care all angles should be avoided in the establishment of conduits.

In seeking the head required to cause the three pipes with curves or angles to give a discharge of .4196 cub. ft., equal to that obtained when there was neither curve nor angle, we have 5.151, 6.332 and 30.546 respectively. Substracting four feet, there remains for the resistance of the curves and angles (197), 1.151, 2.332 and 26.546 ft. Whence we conclude, that the resistance of a single angle of 90° was more than double that of fifteen curves, and that of twenty-four angles was only 11.4 times greater than that of a single one. This last result shows that the resistance of the angles or curves is not proportional to their number, as Dubuat admitted. I had already remarked this want of proportionality, in my experiments on the motion of air in pipes. (*Annales des Mines*, 1828, p. 453.)

Resistance from Contractions.

200. The sudden contractions referred to are occasioned by a diminution of the section of the conduit, for a very short extent.

To give an exact idea of the resistance which they oppose to motion, suppose that in a pipe, we place, perpendicularly to its axis, a diaphragm or thin plate pierced with an orifice. When the fluid in motion arrives at this, the vein will be contracted, and will also be reduced to the size of the opening; it is through such an opening, thus reduced, that it is necessary to force its passage, by taking a velocity as much greater as the opening is smaller; and this velocity will always be superior to that which would take place in this part of the pipe, without the diaphragm. The excess of force necessary to produce the excess of velocity, the direction of the motion remaining the same, will evidently be the effect of the sudden contraction; it will be the resistance opposed.

Let B be the diameter of the orifice, m its coefficient of contraction. The velocity through this passage being necessarily greater than in the pipe, according to the inverse ratio of their sections, will be $v\,\frac{D^2}{mB^2}$; the force to produce it, or the head due, will then have for its expresssion $.0155366v^2\,\frac{D^4}{m^2B^4}$. The head belonging to the velocity in the pipe was simply $.0155366v^2$. The excess of head, or the force proceeding from the sudden contraction, will therefore be

$$.015536v^2\left(\frac{D^4}{m^2B^4}-1\right)=.015536v^2D^4\left(\frac{1}{m^2B^4}-\frac{1}{D^4}\right).$$

In function of the discharge, this resistance will be expressed by

$$.0251817Q^2\left(\frac{1}{m^2B^4}-\frac{1}{D^4}\right).$$

M. Navier, admitting that the fluid vein, on passing from the sudden contraction, quickly resumed the section and the velocity belonging to the pipe, which is never the case, as we shall see in the following number, instead of the difference between the squares of the two terms $\frac{1}{mB^2}$ and $\frac{1}{D^2}$, took the square of their difference $\left(\frac{1}{mB^2}-\frac{1}{D^2}\right)^2$. But can a result deduced from a positively false supposition be adopted?

We shall, however, very rarely have occasion to make applications of the above formula, for in a conduit we ought to have no sudden contractions; but if accidentally one should be presented, it will enable us to appreciate its effect. The effect will generally be very small; in my experiments made with the stop-gates established on the conduits of Toulouse, having once diminished the section of one of them .94, I had but one hundredth diminution in the discharge.

201. If, on the same pipe, below the first sudden contraction, there be a second, a third, &c., the resistance of each will be determined by the above expression, and their sum must be taken.

But that these resistances be added thus together, it is necessary that they be independent of each other; that is to say, that the fluid, after its passing the first contraction, must resume the general velocity in the pipe before its arrival at the second. If it were not so, which would be the case where the orifices were very near each other, the fluid vein, after passing the first, would preserve wholly or in part the excess of velocity which had been impressed upon it in order to cause it to pass, it would require a less effort to make it traverse the second, and as much less as the distance is smaller.

Eytelwein made many experiments which put this fact to the full proof. He took tubes about $1\frac{1}{32}$ inch diameter, and of the lengths noted in the first column of the opposite table; at each of their extremities was a thin plate of copper, pierced with an orifice of about $\frac{33}{64}$ inch diameter. They were fitted horizontally to a reservoir, and the discharge of each ascertained: this discharge, compared to the theoretic discharge, which is represented by 1, is placed in the second column; it gradually diminishes, and consequently the resistance increases in proportion as the distance between the two orifices is greater. Eytelwein also placed in a tube of about $1\frac{1}{32}$ inch diameter, and about one quarter inch apart, four small plates, pierced equally with an orifice of about one quarter inch diameter; the discharge obtained was .622. Afterwards, the plates were put about $12\frac{3}{8}$ inches apart, and the discharge was only .331.

DISTANCES.	DISCHARGE.
inches.	
.276	.626
.512	.622
1.024	.614
2.047	.568
3.110	.509
5.157	.487
12.362	.481
24.724	.478

202. What has just been said upon sudden contractions caused by thin plates pierced with small orifices,

is equally applicable to those produced by very short tubes, of smaller diameter than the diameter of the conduit.

I cite an experiment of Venturi, (his 24th). This ingenious philosopher composed his apparatus of two Fig. 39. kinds of tubes, placed alternately; one set, B, B, were about 1$\frac{25}{32}$ inch long, and about $\frac{8}{10}$ inch diameter; the diameter of the others, C, C, was about 2.13 inches; and their length sometimes 3.46 inches, and sometimes 6.77 inches. Venturi at first made use only of a single tube C; then of two, of three, of four, and finally of five; he fitted successively these various sets to a reservoir, and caused an efflux under a head of 2.887 ft. The following are some of the discharges obtained:

With the single tube B,	.04443	cubic ft.
" " one " C added,	.03295	"
" " three " C "	.02522	"
" " five " C "	.02020	"

I have attempted to compare these results with those derived from the methods of calculation above indicated; the differences were sometimes great and sometimes inconsiderable; thus, for the last case, I found $Q = .01854$ cubic ft.

Effect of Enlargements.

203. Notwithstanding the great irregularities presented by these experiments, they are very remarkable, principally because they exhibit, in a very prominent manner, the effect of enlargements existing in a conduit; an effect quite as prejudicial as that of contractions, taken above a certain limit.

The entire apparatus, which was 3.199 ft. long, may be considered as a pipe of $\frac{8}{10}$ inch diameter, having the five enlargements C. It gave, as we have just seen, a discharge of .02020 cubic ft.

Venturi then took a tube of the same length, but having throughout a uniform diameter of $\frac{8}{10}$ inch, and obtained a discharge of .03270 cubic ft.

The enlargements thus diminished the discharge in the ratio of 100 to 62.

Contractions at the entry of Tubes.

204. There is also one other contraction which ought to be mentioned, resulting from that which the whole fluid mass experiences at its entrance into a pipe of smaller diameter than that of the pipe immediately preceding it. The resistance proceeding from this contraction will evidently be the same as if at the entrance of the pipe were placed a plate pierced with an orifice whose section should be to that of the pipe as m to 1, m being the proper coefficient of contraction; its expression will therefore be

$$.0251817 \frac{Q^2}{D^4} \left(\frac{1}{m^2} - 1\right);$$

a particular case of the general formula (200), in which $B = D$.

The value of m can only be taken by approximation. For a very short pipe, like cylindrical ajutages, it will be .82 (41). But in pipes, properly so called, it will nearly approximate to 1, and as much more as the pipe is longer, and even, according to Prony, as the diameter is greater; so that in large pipes, the effect of contraction is very small. It is rendered still less, by flaring the pipe at the entrance, and by fitting the parts of less diameter to those of greater diameter above them by short conical pipes, gradually leading from one to the other.

Finally, and as we have remarked (185), the effect of contraction at the entrance of a simple conduit is implicitly comprised in the values of the coefficients of the fundamental equation; and its effect at the entrance of a pipe which branches off from a larger conduit will be comprised in the determination of the head of such branch; so that in every case we may disregard it.

Modifications for the application of Formulæ.

205. The coefficients of the formulæ just given, particularly that which concerns the principal resistance, that due to the action of the sides, were determined by experiments made chiefly on pipes of small diameter and quite short in length (185). They were tubes well bored, well jointed and clean. Can such formulæ be applied without modification to conduits which are not in the same condition, to such as serve for great distributions of water? This question ought to be examined.

The pipes of which conduits are formed are almost always more or less deformed in the process of moulding or of casting; their section is not always circular, and consequently, all else being equal, it is smaller than it ought to be. The interior surface is covered with superfluous ridges and asperities, which retard the motion. Where there are joints, the axis of the whole conduit is not always an unbroken line; the interior surface is not perfectly cylindrical; the edges of some of the pipes advance inward and form projections; the fluid lines which arrive at the projecting parts are arrested, divided, and sometimes thrown back; hence disturbances in the motion, losses of motive force, and consequently a diminution in the discharge. Even when the pipes are of good calibre, and of an interior quite uniform and regular, at every joint, at least, there will be a small annular void or break in the continuity, which would produce, to a certain extent, the effect of projections, and which, repeated at every step (so to speak) on a conduit which has more than a thousand, cannot but be attended with a sensible reduction in the discharge. M. Gueymard, mining engineer, has with reason insisted on this cause of reduction; he sought to diminish its effect in the establishment of the fountains of Grenoble, and with success.

Moreover, when pipes are crooked in the vertical plane, and almost all are so, if at the summit of the projecting parts there are not vents, the air which the water always carries along with it, and which is disengaged in greater or less quantity, will rise to the more elevated parts; it collects and remains there, and chokes up the passage: the bad effects have been observed. The most limpid waters in appearance always carry foreign substances, and particularly extremely fine earthy particles, which are deposited in certain parts of the pipes; in process of time, they narrow up the section and diminish the discharge. Not to speak of calcareous and selenitic particles, which, though dissolved in the water, are precipitated on the sides of the pipes, and cover them with a stony crust, which, continually augmenting in thickness, will finally entirely obstruct them, if not removed in time; that evil belongs only to some localities. There are even ferruginous deposits made from point to point, in the form of tubercles, in the pipes of Grenoble, and which, continually increasing in number and size, diminish the discharge, until they have reduced it more than one half in less than eight years; the aërated water which runs in cast iron pipes, attacks the substance of them, and forms an hydrate of iron, which is deposited in long nipples, on lines parallel to the direction of the current, and in a greater quantity on the lower part; under these nipples, the cast iron is, as it were, corroded to the depth of .08 to .12 inch. (See concerning these very remarkable phenomena, which have attracted the attention of philosophers, the *Annales des Mines*, 1834, page 203, &c.)

Abstraction made of these local circumstances, and consequently of general causes mentioned above, it has very often happened in experiments made on conduits, supposed to be in good condition, that the discharge has been found a quarter or a third less than that indicated by the formulæ; scarcely ever has it been equal to it. I have cited many of these experiments in my *Histoire de l'éstablissement des fontaines à Toulouse*.

From these generally known facts, the hydraulic engineers of Paris, when they make use of the formulæ of discharge, diminish their numerical coefficient one third. I have adopted an analogous method, by increasing one half the quantity of water which a conduit about to be established must convey; if, for example, it be designed for ten cubic feet per second, I call it fifteen cubic feet per second, and make my calculations accordingly in respect to diameters, &c. I advise engineers charged with projecting or executing a plan for distributing water, to employ the formulæ here given with such a latitude; they will in this manner prevent the mistakes which they would often experience, did they confine themselves to the results given by pipes made with a precision which their own will never have.

206. In the great systems of distribution, and in all that re- Pouce d'eau.

lates to public fountains, instead of expressing the discharges and quantities of water to be conducted in decimal fractions of a cubic metre per second, they are expressed in *pouces d'eau* (water inches), a peculiar unit, independent of time, and more suitable for such a purpose.

The old fountaineers, who introduced this unit and this denomination, gave it, as we have seen (67), to the quantity of water which runs from an orifice of one inch diameter, pierced in the side of a basin against which the fluid is kept one line above the summit of the orifice.

The exact determination of this quantity is necessary in a great number of cases, and it is necessary to translate it into a precise expression, which would lead to a verification in a case of dispute. Mariotti first made an attempt for this purpose; his estimation would correspond to 697.86 cub. ft. in twenty-four hours. Afterwards, the water inch was estimated at 676.73 cub. ft. Finally, of late years, in applying the metrical system of weights and measures, it was raised to 20 cub. metres = 706.33 cub. ft. Prony proposed to call it the "*double module*" of water.

Thus the *pouce d'eau* (water inch), as now established, and as I have exclusively employed it in the establishment of the fountains of Toulouse, gives 706.33 cub. ft. in twenty-four hours, or .008175 cub. ft. per second.

3. *Pressure on the sides of Pipes.*

After having treated of the circumstances of the motion of water in pipes, let us now examine the effects resulting from the pressure which the fluid exerts against the sides of the pipes containing it; then we will explain the most important consequences of this examination.

Its nature and expression.

207. Let a horizontal conduit AB be fitted to a reservoir kept constantly full.

Fig. 40.

First shut the extremity B. Each of the points of the conduit will experience a pressure measured by the height or head AC; and if, on any of them, H, I, K, &c., take at pleasure, be inserted vertical tubes, the water will rise until the weight of the columns HL, IM, KN, is in equilibrium with the pressure exerted on those points; consequently, in each, it will rise to the level of CD.

Open now the orifice B; suppose the sides of the tubes oppose no resistance to the motion, as in the case of a very short tube, and that there is no contraction at the entrance A. The water will run in the pipe and will pass out with a velocity due to the total head AC. All the force of this head will therefore act parallel to the axis of the pipe; there will result no action perpendicular to that direction, and consequently no pressure on the sides; we shall then have the case of water moving in canals where there is no pressure tending to elevate the surface. The fluid of the tubes HL, IM, will therefore descend to the upper part of the fluid in the pipe.

208. If the orifice B be opened only in part, so that it shall be less than the section of the pipe, the phenomena will no longer be the same. The water will always pass out with the velocity due to AC; but the velocity in the pipe will be less, according to the inverse ratio of the sections. Let v be that smaller velocity, $.0155366v^2$ will be the force or part of the head AC employed to produce it; still acting parallel to the axis, it will exert no pressure on the sides. But the remaining portion of the total force, or $H-.0155366v^2$, by making $AC=H$, acting on all the particles, extending in all directions, will press the fluid upward in I, K, &c.; and it will ascend in the vertical tubes to a height equal to $H-.0155366v^2$; which will be limited by the horizontal line EF, CE being equal to $.015536v^2$. Hence the great principle which Bernoulli established by calculation, which is confirmed by

experiment, and which he made the base of his *Hydraulico-statique* (*Hydrodynamica, sectio* XII.); to wit: *The pressure which water in motion, in a pipe, exerts against any point of its sides, is equal to the* (*effective*) *head on that point, minus the height due to the velocity in the pipe.*

209. The resistance which the sides of pipes oppose to motion, does not in any manner destroy this principle; it only diminishes H, or the head which would have existed without the resistance on the point under consideration. Let us enter into some details.

The resistance is proportional to the length of the pipes (184); that is, to the length of the passage made by the water; thus on the same pipe, it will increase progressively from its origin A, where it is zero, to its extremity B, where it is $.0007089 \frac{LQ^2}{D^5}$ (§ 187). So that if on BD we take FG equal to that expression as representing the resistance at B, and draw the straight line EG, the resistance at H, I, K, &c., will be represented by the lines ae, $a'e'$, $a''e''$, &c. (since $ae : a'e' : a''e'' : \ldots . FG :: Ee : Ee' : Ee'' \ldots : EF$). Designate by $r, r', r'' \ldots .$ R these resistances. At each of the points just indicated, I for example, the column MI (measure of the pressure in the state of rest) will fall 1st. Me' $(=.015536v^2)$; for in this case, as in the preceding, this portion of the motive force being directed along the axis of the pipe, will exert no pressure on its sides; 2d. $a'e'$ $(=r')$; this other part of the total force having been absorbed, as it were annihilated, by the resistance which the pipe opposed to the motion of the fluid from A to I, can have no other action on this last point; the pressure here will therefore be simply measured by $a'\,I = H - r' - .015536v^2$. In general, the pressure on any point of a horizontal pipe, in which r represents the resistance experienced from the origin of the pipe, is expressed by $H—r—.015536v^2$.

At the extremity of the pipe, where the resistance is R, the pressure $GB = H - R - .015536v^2$. If this extremity were entirely open, we should have (182) $R = H - .015536v^2$, and consequently, $GB = 0$; that is, the pressure at the extremity of the pipe would be nothing, and the columns, measuring the pressure at its different points, would have the line EB for their upper limit.

210. We come at last to the case of an ordinary inclined pipe. In the state of rest, the columns indicating the pressure would rise to the horizontal line CD, the level of the fluid in the reservoir; this is the law of communicating tubes; these, and consequently the pressures, will be unequal; each will have for its measure the difference of level between the point where it is exerted and the surface of the reservoir. In the state of motion, the columns will undergo the same diminutions as in the preceding number, and through the effect of the same causes; their summit will only attain the line EG, which will be the limit (they would be limited by EB, if the pipe were entirely open); consequently, the pressure on any point, of which H_0 is the distance below the reservoir, will be expressed by $H_0—r—.015536v^2$. Fig. 41.

The expression would be the same for a bent pipe like AH′I′K′B. Only the summits of the columns would no longer be on a straight line; the resistances, being proportional to the lengths of the pipes, will follow the ratios of AH′, AI′; but not those of Ee, Ee', a condition necessary in order that the points E, a, a', be in a straight line.

Total head and Effective head.

211. We have called *head on the pipe*, and have designated by H, the difference of level between the surface of the fluid in the reservoir and the orifice of efflux; it would be the height due to the velocity of efflux, if the pipes opposed no resistance to the motion (182). But the resistance diminishes this *total head;* so that the *effective head* of the pipe, or the height, in virtue of which the fluid really passes out, will be less by the whole resistance which it will have experienced, from the origin to the extremity of the pipes; R being that resistance, the effective head will be $H-R$.

By analogy, for every other point of the pipe, its *total head* will be the height of the reservoir, H_0, above it; and its *effective head* will be that same height, diminished by the resistance experienced by the fluid from the origin of the pipe to that point, or H_0-r.

Difference between the head and pressure.

212. Since the pressure on this same point is $H_0-r-.015536v^2$, $.015536v^2$ will be the difference between the pressure and the head. In general, the height due to the velocity of the water, at any point of a pipe, is the difference between the effective head, or head properly so called, and the pressure on that point. There will, therefore, be an error when one is taken for the other; but in great pipes, where the height due to the velocity is very small, the error is almost always of no consequence.

The Piezometer and its indications.

213. The tubes which we have supposed to be placed on the pipes, and which, by the height to which the fluid rises in them, measure the pressure which takes place on the parts to which they are applied, take the name of piezometers (*πιεσις*, *πιεσος*, pressure, and *μετρων*, measure).

They serve to give us a physical representation of what is meant by resistance and loss of head.

Suppose one to be established at any point of a pipe, at H_0 below the level of the reservoir. From what has just been said, if the water were at rest in the pipe, it would rise in the tube to the height H_0; when the flow takes place, it will fall, and will remain at the height $H_0 - r - h$, h being the elevation due the velocity v. The fall, or difference between the two heights, will therefore be $H_0 - (H_0 - r - h)$; and designating it by α, we have $\alpha = r + h$, or $r = \alpha - h$; that is to say, *the resistance experienced by the water from the origin of the pipe to one of its points, will be represented by the difference of level between the surface of the reservoir and the summit of the fluid column in a piezometer applied to that point,* (*minus the height due to the velocity in the pipe;* a quantity always very small). If we increase or diminish the volume of water which flows in a pipe, and consequently its velocity, by enlarging or lessening the orifice of efflux, the fluid of the piezometer will fall or rise very nearly proportionally to the square of that volume, $\Big\{$the fall must be $Q^2\left(.0006769\frac{L}{D^5} + \frac{.025187}{D^4}\right) + QL\frac{.00009594}{D^3}\Big\}$; and we shall be able to compare the results of theory with those of experiment.

For a second point of the pipe, taken lower than the first, for example, we should in like manner have $r' = \alpha' - h$, since the velocity and its height due h remain the same throughout the pipe. Subtracting from this equation the first, $r = \alpha - h$, we have $r' - r = \alpha' - \alpha$. Now, $r' - r$, the difference between the two resistances, is evidently the resistance experienced between the first point and the second, and $\alpha' - \alpha$, the difference of the lowering in the piezometric columns below the reservoir, will be the difference of level between the summits of the two columns: thus *the resistance which the water experiences from one point of a pipe to another, or the loss of head from the first to the second, is equal to the difference of level between the summits of the fluid columns of two piezometers established on the two points.* If the diameter of the pipe on which the second piezometer is placed be different from the first, then the height h' due to its velocity will not be equal to h, and we shall have $r' - r = \alpha' - h' - (\alpha - h) = (\alpha' - \alpha) - (h' - h)$; that is, the resistance from one point to another would be the difference of level between the two piezometric summits, minus or plus the difference between the two heights due to the respective velocities, according as the velocity at the point lower down is greater or less than the other.

We see by these examples how the piezometers place before the eye the resistances of pipes and the variations which occur in them; and consequently, how these indications can be rendered useful. Such an instrument, of glass, which I fitted up on one of the pipes of the water-works of Toulouse, and which rises in the Hotel de Ville, in front of the water engineer's office, indicates to him, at every instant, the state of his service, and the perturbations which it suffers.

Thickness of Pipes.

214. Upon the pressure which water running in pipes exerts against their sides depends also the thickness to be given to those sides.

Let H be the greatest pressure or head which a pipe will have to sustain, D its diameter, t the thickness sought, and f the force capable of breaking the material of which it is composed; this will be the weight which, being suspended to a vertical bar of the material of a square inch transverse section, would occasion the rupture. The resistance which the pipe opposes, in the unit of length, will be tf, or $2tf$, if it be required to separate entirely one part from the other, since $2t$ $(= aa' + bb')$ is the breadth of the part to be broken. Observe now the effort exerted by the fluid to cause such a separation. Take a pipe uniformly equal in length to the unit of length selected; place it horizontally; imagine it divided into two halves by a horizontal plane, that the lower half is firmly fixed, and that the upper half can only be separated from it by being driven vertically upward, as would be done by the pressure of the water. Imagine the interior surface of that half divided into a very great number of longitudinal elements, of which the extremely small breadth will be σ; each will experience from the fluid, in a direction perpendicular to itself, a pressure or force equal to the weight of a prism of water which would have $1 \times \sigma$ for its base and H for its height, and which would consequently weigh $w\sigma H$ (w being the weight of the unit of water). This force, estimated in the vertical direction, would be reduced to $w\sigma' H$, σ' being the horizontal projection of σ; the sum of all these forces will then be wH,

Fig. 42.

multiplied by the sum of the σ', and this is evidently equal to the diameter *ab* or D: thus the total effort tending to separate the two halves of the pipe will be wHD. In the case where it would simply be in equilibrium with the resistance, we should have $w\mathrm{DH} = 2tf$, from which to deduce the value of t. For cast iron, the material of which pipes are usually made, it is generally admitted, that a bar drawn in the direction of its length is ruptured by a weight of 19919 lbs. avoirdupois per square inch; thus we have, per square foot, $t = \frac{62.449\mathrm{HD}}{2 \times 144 \times 19919} = .0000109\mathrm{HD}$; with such a thickness, the pipe would be on the point of bursting. But it is necessary to keep far from such a minimum: the custom is to give to materials employed in constructions such dimensions at least that the effort which they will have to sustain will cause no permanent alteration in them; this effort is estimated, for cast iron, at from two and a half to three and a half times less than what would cause rupture: thus it would be necessary to multiply the thickness already found by two and a half or three and a half; for greater security still, I multiply it by 4.2, and have

$$t = .0000456\mathrm{HD}.$$

It is not merely necessary that pipes be able to resist their ordinary greatest pressure, but also extraordinary pressures, or shocks of the water-ram ("*coups de bélier*"), to which they are exposed, when the mass of water which they conduct is suddenly arrested, as by the too sudden closing of a cock. Thus, although the head on pipes rarely exceeds fifty or sixty feet, it is commonly supposed to be 328 ft.; and it is under such a pressure, equivalent to that of nearly ten atmospheres, that pipes ought to be tested: H then being 328 ft., we have $t = .01496\mathrm{D}$.

Such is the thickness to be given to cast iron pipes, if the material be entirely compact and without defect, and if there are means of casting it so thin. But this is not the case; cast iron is a porous substance, which permits the water to percolate under great pressures; it commonly contains flaws which considerably diminish the thickness of the parts where they are found; when run too thin, it sets before entirely filling the mould; besides, rust attacks and corrodes the pipes, and at length may reduce their thickness very much. So that there is a thickness below which we should not descend, and a constant quantity ought to be added to 0.015D. Some add .01968 ft., others .02624 ft.; I would avoid all objections, by taking into consideration all the difficulties of the foundry, by raising the quantity to .0328 ft., and definitely establish the formula

$$t = 0.03289 + 0.015D.$$

However, there is no need of adding the second term to pipes below .3937 ft. diameter; they will all have .0328 ft. thickness. That of the rest, expressed in feet, will be respectively for pipes of

.3937,	.4921,	.6562,	.8202,	.9842,	1.3123,	1.640	ft. diameter,
.0351,	.0393,	.0426,	.0459,	.0492,	.0525,	.0590	ft. thickness.

Lead pipes offer less resistance than those of cast iron. M. Jardine subjected to trial one of 0.166 ft. diameter and 0.0166 ft. thick: it began to dilate under a head of 803.8 ft., and burst under one of 1000.6 ft. This experiment, giving $t = .0001244HD$, indicates a resistance which is not a ninth part of that presented by cast iron. This less tenacity, and the more than double price, has caused lead pipes, before generally employed, to be given up, and the use of cast iron pipes has become almost exclusive for all great establishments.

Wooden pipes resist a great pressure and they are not dear; but they rot, they must often be changed, and they require peculiar care.

As to pipes of potter's clay, they can only be employed under a small head; it is difficult to lute the joints well, and they are liable to break; consequently, their use is not to be recommended.

ARTICLE SECOND.

Systems of Pipes.

It is very seldom that we have a simple pipe, conveying to its extremity all the water which it received at its origin; almost always, on different points of its length, there are quantities of water taken which are conveyed by secondary pipes; from them, pipes of a third order branch off, &c.; so that, in great distributions of water, such an assemblage or system of pipes presents, as it were, a trunk which is ramified and sub-ramified in various manners.

Problem to be solved.

215. To determine the circumstances of the motion of water in the different parts of such a system, and that by the simple knowledge of the dimensions and the respective positions of these parts, is a complicated problem, of which no solution has yet been given; and still, the determinations which the engineer has to make refer almost always to an assemblage of pipes, and not to a single isolated one.

To have a good idea of the bases on which I have established the solution which I am about to submit, and which is applicable at least to some questions, let us imagine a system already existing, fitted to a reservoir kept constantly full, and delivering water through mouth-pieces fixed at the extremity of each of its branches; let it be required to determine, for example, the quantity of water which passes through each mouth-piece (though this is not my immediate object); it is evident that we could immediately calculate that quantity, if we knew the effective head at the extremity of the branches, that head being the height due to the velocity of efflux (211). Now, from what has been said (211—213), the effective head is the entire head, minus losses of head or resistances which the fluid particles arriving at the mouth-piece in question have experienced in their passage through the system from the reservoir to that mouth-piece. So that the problem is reduced to the determination of these various losses of head.

The different losses of head.

216. These proceed, first, almost entirely from the resistance which the sides of the pipes oppose to the motion; secondly, from the resistance due to bends; thirdly, from change in the direction of the motion

when the water passes from the principal pipe into a branch, and from a branch into a sub-branch; fourthly, from disturbances in the motion, occasioned by diversions of water or *érogations* at the head of each branch or sub-branch. As to the resistances proceeding from obstructions, it is superfluous to mention them; we ought not to admit a permanent obstruction in a pipe; if there be one accidentally, we have indicated the mode of calculating its effect (200). We have seen that all resistance to the motion of the water in a conduit is like an effort opposed to the motive effort or entire head, destroying a part of it; it produces a loss of head; it is equivalent to a loss of head.

We have discussed in detail the first two of the four losses which we have noticed, and we shall limit ourselves to bearing in mind that they are given, for the action of the sides, by the expression (186)

$$.0006769L\left(\frac{Q^2}{D^5}+\frac{.141724Q}{D^3}\right);$$

for bends, by (197)

$$.00608\,\frac{Q^2}{D^4}\,s^2.$$

It remains to examine the two other losses.

Loss due to change of direction.

217. When a body which moves with a velocity v in one direction, is constrained to take another making with the first an angle i, its velocity in the new direction is only v cos. i. So when a fluid, having the velocity v in a pipe, passes into a branch, (abstraction made of other forces which may act upon it,) it will have only the velocity v cos. i. The force or head due, which was $.015536v^2$ in the conduit, will be only $.015536v^2$ cos.^{2} i; it will then have lost in height or head $.015536v^2$ (1 — cos.^{2} i), or $.015536v^2$ sin.^{2} i.

Almost always the branches are fixed perpendicularly upon the conduits, so as afterwards to be deviated by bends more or less abrupt. In this case, $i = 90°$, sin. $i = 1$, and the loss of head, recollecting that $v = 1.27324 \frac{Q}{D^2}$, is

$$.025187 \frac{Q^2}{D^4};$$

that is to say, the head or force proceeding from the velocity which the water in the pipe has is entirely lost; it has no component in the direction of the branch; the fluid enters the branch only in virtue of the pressure or piezometric height which is found in the conduit, in front of the point where water is taken out.

Loss due to érogation.

218. At this point, at the entrance of the branch, there is still another loss of head. To measure its magnitude, MM. Mallet and Génieys, hydraulic engineers of Paris, placed a piezometer on a pipe of 9.8425 inches diameter, a little above the point where a pipe of 3.189 inches branched off, and they established a second on the 3.189 inch pipe, a short distance from its origin. The column in the second was 4.7244 inches lower than in the first, when the discharge through the pipe was .15362 cub. ft.; the velocity in the pipe was 2.7789 ft., and the height due this velocity .11998 ft.; this last quantity being required over and above the elevation of the first piezometer, to impress the above-mentioned velocity, there remains for the effect of the "érogation" only .2737 ft.; a quantity 2.28 times greater than the height due. The velocity in the branch being made 3.2907 ft., the difference between the two instruments was .502 ft.; the height then due being .1683 ft., there remained for

érogation a quantity 1.94 times greater than that height. It is concluded from these experiments, that the loss of head occasioned by the érogation is equal to about twice the height due to the velocity in the branch.

Although the above results may have been influenced by particular circumstances, the conclusion drawn from them is admitted, until other observations shall have introduced more light upon the matter.

All uncertainty in respect to the value of the loss of head due to érogation, as well as those proceeding from bends and changes of direction, are of but little consequence in practice, these values being altogether minute compared to the other quantities which enter into the equations, particularly when compared to the loss resulting from the action of the sides, and this was determined by the aid of more than fifty experiments (185).

219. For some time, I feared that the érogations for the branches might extend their effects to the conduit itself, below the points where the érogations were made, and that the head might suffer a considerable diminution. If it had been so, the solution of the problem here given, and which was implicitly given in my *Traité sur le mouvement de l'eau dans les conduites*, published in 1827, would have been entirely defective. To decide this important question, I made, in 1830, the following experiments:—

On a pipe of about 3.15 in. diameter and 2090 feet long, at 1414 feet from the origin, I placed a tube with a cock, through which I made a greater or less quantity of water to pass; this was the érogation. At 1.64 ft. above, as well as 2.30 ft. below, was fitted up a large piezometer. The head at the origin of the conduit remaining nearly the same (24.279 feet), and its extremity being entirely open, the volumes of water indicated in the following table were allowed to pass; in the annexed columns are noted the volumes passed at the extremity, as well as the heights above the points of érogation at which the water stood in each piezometer. As these heights correspond so nearly, it may be concluded that they are the same above and below the érogation. This equality of pressure is maintained in many other experiments which I have executed with the same apparatus.

WATER PASSED IN 1″.		PIEZOMETER.	
At the érogation.	At the extremity.	upper.	lower.
cub. ft.	cub. ft.	ft.	ft.
.0000000	.0598263	6.234	6.267
.0095002	.0526571	5.085	5.085
.0295247	.0360582	2.986	3.051
.0462647	.0204483	.591	.558
.0487369	.0181527	.394	.328

Thus the taking of water from a pipe does not sensibly diminish the pressure, and consequently the head on the points which are below the point where the opening is made; and in a system of pipes, there are no other losses of head except the four which have been investigated.

Equation of motion in branches.

220. Take now a branch or sub-branch of any order n, discharging all its water into the air through an ajutage fitted to its extremity.

Take d_n = diameter of the ajutage at the orifice,

m_n = its coefficient of contraction,

H_n = the total head of the branch or the difference of level between the orifice of its ajutage and the surface of the reservoir,

D_n = diameter of the branch,

L_n = its length,

Q_n = quantity of water which it passes,

S^2_n = the sum of the squares of the sines of the angles of reflexion of the different curves,

$[R]$ = the sum of the resistances or losses of head experienced by the water which flows in the branch, from the origin of the system to the point of junction of the branch. If, in following the course of the water which arrives at the branch, we represent by r and r' the losses of head due to the resistance of the sides and curves on the principal pipe, but only up to the point of the insertion of the first branch;

by r_1, r_1', r_1'', r_1''', the four losses of head on this first branch and up to the second only; by r_2, r_2', r_2'', r_2''', the losses on this second up to the third, and so on successively up to the n—1 branch, to which the branch n is fitted, we shall have

$$[R]=r+r'+r_1+r_1'+r_1''+r_1'''+\ldots r_{n-1}+r'_{n-1}+r''_{n-1}+r'''_{n-1}.$$

This granted, since the sum of the losses of head, subtracted from the total head, gives the head or height due to the velocity of efflux (215), or since the entire head is equal to the sum of the losses plus that height, which is (193) $.0251817\,\frac{Q_n^2}{m_n^2 d_n^4}$, the equation of motion will be

$$H_n=[R]+.0251817\,\frac{Q_{n-1}^2}{D_{n-1}^4}+.0006769L\left(\frac{Q_n^2}{D_n^5}+\frac{.141724Q}{D_n^3}\right)$$

$$+.00608\,\frac{Q_n^2}{D_n^4}\cdot s_n^2+.0503634\,\frac{Q_n^2}{D_n^4}+.0251817\,\frac{Q_n^2}{m_n^2 d_n^4}.^{*}$$

When the branch is entirely open at its extremity, there being no ajutage, $m_n^2 d_n^4=D_n^4$.

The above equation will enable us to determine, directly or indirectly, either of the variables which it includes, by a knowledge of the rest.

Expression of the loss of head from one point to another.

221. In its application, we very often have need of establishing explicitly the ratio between the loss of head and the quantities which are connected with it; we shall recollect for this purpose, that the loss of head between two points of a system of conduits is equal to the sum of the resistances experienced by the fluid, in its passage from the first of these points to the other.

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* In the equation as I present it, I have followed strictly the laws and reasoning of the author, and feel justified in adhering to the results given by them.

TRANSLATOR.

222. Let us apply the principles and the formulæ just given to the determination of the diameter of the pipes of a great system of conduits. This application will enable us to examine some cases not yet investigated, and to make some observations of direct utility for practice.

Example of a great distribution of water. Diameter of Pipes.

About 260 water inches ("*pouces d' eau*"), or 183651 cub. ft., are to be distributed at different points of the city. These points, as well as the quantity of water to be discharged at each, are assigned by the government.

The engineer charged with the distribution will first determine the level of the places indicated below the ordinary level of the water in the feeding basin; these differences of level will be the total heads of the pipes and branches which must terminate at these points. He will trace on a plan the pipes and branches; he will measure their length and also the angles and their bends; and according to the observations made in Sec. 205, he will augment by the addition of one half the quantity of water appropriated to each of them. These data of the problem to be solved, for the portion of the system comprised in figure 43, are noted in the following table: B, C and D are the knots or points of division of the waters.

Fig. 43.

We have further, $ai = 623.371$ ft.; $ij = 278.876$ ft.; $il = 1315.641$ ft.; $op = 364.180$ ft.; and $pq = 557.753$ ft. . . . The angles of the bends are at l 130°, at k 140°, at m 110°, at n 75°, and at r 90°; the radius of curvature for all is 9.843 ft.

The descent cannot be made by a uniform loss of head from the reservoir to each of the extreme points; and even could this be done, it would scarcely ever be proper to do it; it will be for the engineer to fix the proper loss from the reservoir to each of the principal points of division of the waters to be distributed; he must do so in a manner to draw from these waters all the advantages which they can afford, and with the least possible cost. If in the first and consequently largest branches of the system, he admits a very great loss, there might not remain in the branches following head enough to convey the waters to their destination, so that in case of necessity a greater supply than ordinary may be carried down: on the other hand, if the loss admitted be too small, the diameters of the first branches would be too large, and would cost too much. For example, in the projected distribution, or at one of the extreme points d, it is required to have a jet of water rising about 24.607 feet above the pavement, which in that part of the city is 38.715 feet below the level of the reservoir, and where, consequently, the loss of head from A to *d* could only be 14.108 feet, it will be distributed as follows: 3.281 ft. from A to B, the point of the principal division; 6.234 from B to C, another point of division; there will consequently remain 4.593 ft. from C to *d*. From B to D, admit a loss of 7.218 ft.

These secondary data, added to those already had, either by the disposition of the ground, or in consequence of a controlling power, will suffice to determine the diameters.

POINTS OF DISCHARGE.				PIPES.		
		WATER DISCHARGED.				
Designation.	Level below reservoir.	Water incs. ("*pouces d' eau.*")	Cubic ft. with increase.	Designation.	Length.	Calculated diameter.
	feet.				feet.	feet.
(B)				AB	2483.641	1.45
a	26.575	4	.04909	*ia*	2103.056	.202
b	33.793	2	.02472	*lb*	836.629	.119
c	55.119	5	.06145	*jc*	853.034	.13379
(C)		75	.91964	BC	1328.764	.8530
d	(38.714) 14.108	63	.77167	C*d*	2231.011	.7382
e	(27.231) 8.202	35	.42910	B*o*	354.337	.51156
f	28.872	5	.06145	*oq* *pf*	921.933 659.461	.2467 .1784
g	32.153	3	.03673	*qg*	354.337	.1286
h	31.168	2	.02472	*gh*	160.764	.1004
(D)		67	.82146	BD	3979.731	.7710
		261	3.20003			

1) Begin with that of AB, the first of the principal pipes; it is the trunk of the tree.

It receives at A all the water to be distributed, the volume of which is supposed to be 3.1976 cub. ft. per second: at *i* and *j* it leaves .1342 cub. ft. per second, and carries the remainder to B. For rigorous accuracy, we should give each of the three parts, A*i*, *ij* and *j*B, an appropriate diameter; but the difference would be so small, that it is proper to make all the pipe of the same diameter, such as it should have if all the water taken at A were conveyed to B. We shall then have

Q = ... 3.1996 cub. ft.;
L = the length AB = ... 2483.641 feet;
D = the diameter sought;

bearing in mind that the loss of head from A to B is 3.281 ft.

The pipe starts from a reservoir or gate-house (*chateau d'eau*); it descends at first vertically, but soon takes a horizontal direction; it thus makes a curve of 90°, in which the water suffers two reflexions of 22° 30′ each; and $2 \times \sin.^2 22° 30' = .2929 = s^2$.

The resistance of the sides will be (216) $\frac{17.2128}{D^5} + \frac{.76236}{D^3}$.

That of the curves will be $\frac{.018232}{D^4}$.

Equating these two resistances with the loss of head, 3.281 ft. (221), we have

$$\left(\frac{17.2128}{D^5} + \frac{.76236}{D^3}\right) + \frac{.01823}{D^4} = 3.281,$$

from which we deduce D, by means of successive substitutions. After several trials, it will be found that the value of D satisfying this equation is 1.45 nearly.

I dwell for a moment on the principal pipes, and here point out three arrangements which I made on the pipes of the fountains of Toulouse, the good effect of which an experience of ten years had warranted; they were taught to me by the first principle of every great and good distribution of water: *the insurance of the continuity of the discharge at all the principal points*, so as, if possible, never to be interrupted.

Instead of a single pipe, or line of pipes, conveying a certain volume of water, two have been established, side by side, each conveying half the volume. Thus, in our example, instead of the seventeen inch pipe, conveying 3.1996 cubic ft. per second, we would have two, each with 1.5998 cub. ft., but always under the condition of not losing more than 3.281 ft. of head; their diameter would then be about thirteen inches. Whilst one might be undergoing repairs, the other could furnish the supply; it would, indeed, deliver less water, but not so much less as might at first be believed; thus, in one of my experiments, when the volume of water delivered by two pipes side by side, starting from the same reservoir and ending at the same box (*boite*), was

.4203 cub. ft. per second, it was found to be .4026 cub. ft. per second when one of them was closed. The doubling of the pipes increases indeed the cost of the first establishment; it increased the cost about thirty per cent. at Toulouse, where the three principal conduits were doubled.

In order that double pipes may have all their advantage, it is necessary, as in the case just mentioned, that their extremities be fitted to a common reservoir. For this purpose, both of them may end in a cast iron drum, or small distributing chamber, from which the pipes for conveying the water may proceed. At the principal point of distribution, at Toulouse, there was one of 3.281 ft. diameter and 1.969 ft. high; its convex surface had seven tubes, to which as many pipes were fitted.

Finally, the principal pipes ought to be placed in subterranean galleries. To inspect them then is very easy; every loss of water is soon perceived and quickly remedied. As to secondary pipes, which are buried about 3½ feet below the paving of the streets, a loss in them is of less consequence; when the water fails at a fountain fed by one of them, temporary recourse is had to a fountain supplied by a neighboring pipe, which is generally at only a short distance. But at the principal points of the distribution, I repeat it, the water must never fail.

2) Let us return to the determination of the diameters, and that of the branch *ia*. It must convey to *l* .07381 cub. ft. per second, and then from *l* to *a* only .04909 cub. ft. Here also, for greater simplicity, as well as to gain the advantages of a large diameter, we will determine this on the supposition of $Q_1 = .07381$;

The length *ia* is . . . . 2103.056 ft. $= L$;
The diameter will be . . . . D_1;
The head or descent of the mouth of efflux below the reservoir is . . . . $26.575 = H_1$;
That mouth is equivalent to a cylindrical ajutage whose diameter should be . . . . . . .131235 ft.$= d_1$;
its coefficient of contraction should be . . . . $.82 = m_1$.

Take now the equation of the branch (220).

Its first member $H_1 = 26.575$ ft.

The first term of the second member will be the resistance experienced by the water of the branch on the principal pipe from A to *i* only, and consequently on a length of 623.371 ft. For the resistance of the sides, we shall have

$$\left.\begin{array}{l} .0006769\times623.37\left(\frac{(3.1996)^2}{1.45^5}+\frac{.141724\times3.1996}{(1.45)^3}\right)=.73673 \\ \text{that of the curve } \frac{.00608\times(3.1996)^2}{(1.45)^4}\times.2929=\ .00412 \end{array}\right\}.74085$$

The second term, $.0251817\frac{Q^2}{D^4}$, which indicates the height due to the velocity in the pipe A*i*, a height or head entirely lost for the branch, will be . . . . .058321 ft.

The third term, or rather, the binormal expressing the resistance of the sides of the branch, will be

$$.0006769\times2103.056\left(\frac{.73811^2}{D^5}+\frac{.141724\times.73811}{D^3}\right) \dots\dots = \frac{.0077558}{D_1^5}+\frac{.014891}{D_1^3}$$

There will be no term relative to curves, since the branch is straight.

For the next, giving the loss of head produced by érogation, we shall have $\frac{.0503634\times.73811^2}{D_1^4}=\frac{.00027438}{D_1^4}$.

Finally, the last term, $.0251817\frac{Q_1^2}{m_1^2d_1^4}$, representing the height due to the velocity of efflux, will be . . . . .68787 ft.

So that the equation of the branch will become

$$26.575=.74085+.058321+\left(\frac{.0077558}{D_1^5}+\frac{.00027438}{D_1^4}+\frac{.014891}{D_1^3}\right)$$

or

$$25.08796-\left(\frac{.0077558}{D_1^5}+\frac{.00027438}{D_1^4}+\frac{.014891}{D_1^3}\right)=0.$$

D is obtained by the above method; it will be .202 ft.

3) We come now to the sub-branch, fitted to the above described branch, at *l*, a distance of 1315.64 ft. from the origin.

Its length is . . . . 836.63 ft. $=L_2$.

Its diameter we designate by . . . . D_2.

The quantity of water it is to convey . .024721 cub. ft. $=Q_2$.

It is discharged through a conical ajutage having a diameter at its point of delivery of . . . . .068899 ft. $=d_2$.

Its coefficient of contraction will be . . . . . 0.90 $=m_2$.

The extremity *b* is below the reservoir . . .33.793 ft. $=H_2$.

There is a bend just after the point *l* of its junction, which we will neglect.

The first member of the equation of the sub-branch is 33.793 ft.

The first term of the second member ought to express all the resistances experienced, both upon the principal pipe as far as *i*, and those upon the branch as far as *l*. The first are equal (as we

$$\left.\begin{array}{l} \text{have already seen) to} \; . \; . \; . \; . \; . \; . \; . \; . \; . \; .74085 \\ \begin{array}{c}\text{On the branch}\\ \text{we have for}\end{array} \left\{\begin{array}{ll}\text{resistance of sides} & 15.6408\\ \text{érogation} \; . \; . \; . & .16553 \\ \text{change of direction} & .063977\end{array}\right\} 15.8703 \end{array}\right\} 16.6111$$

The second term, the height due to the velocity in the branch, is .082768 ft.

The third term, or the resistance of the sides of the sub-branch, is equal to . . . . $\frac{.00034451}{D_2^5} + \frac{.0019796}{D_2^3}$

The following, relating to the érogation, is . . . $\frac{.000030638}{D_2^4}$

Finally, for the last term, the height due to the velocity of issue, we have . . . . .83927 ft.

So that the equation of the sub-branch, proper reductions being made, is

$$16.260 - \left(\frac{.00034451}{D_2^5} + \frac{.0017996}{D_2^3} + \frac{.000030638}{D_2^4}\right) = 0.$$

Whence, by substitution, we derive $D_2 =$ . . . . 0.119 ft.

These two examples which we have given will suffice to show the manner of calculating in detail and with exactness the diameter of every branch and sub-branch. In what follows, we shall only dwell upon some circumstances peculiar to the case in hand, and shall adopt abridged modes of calculation.

4) In the branch *jkmnc*, we devote our attention to the numerous bends which it presents.

Here we have L = 853.034 ft., Q = .06145 cub. ft., and H = 55.119 ft. The branch is open at its extremity, so that the effect of érogation is twice the height due the velocity of efflux, and the loss of head arising from these two causes will be $3 \times .0251817 \frac{Q^2}{D^4}$. The resistance experienced in the pipe from A to *j* is 1.2598 ft., and the equation becomes

$$53.8592 - \left(\frac{.0021804}{D^5} + \frac{.0050287}{D^3} + \frac{.00028549}{D^4} + \frac{.000023037 s^2}{D^4}\right) = 0.$$

The last term expresses the resistance of the bends: but to have one of its factors, s^2 (197), we must know D, which is precisely the quantity sought. Neglecting the last term of the equation, we obtain at first an approximate value, D = .13379 ft. With this diameter, and following the method given in Sec. 198, we find that the bends present sixteen angles of reflexion, of 6° 41′ each, and consequently, that $s^2 = 0.2168$. This number gives for the last term of the equation $\frac{.000004995}{D^4}$, a quantity too small to influence the value of D; and this value may remain 0.13379

With such a diameter, the resistance of the bends will be .01557 ft. That of "érogation," comprising the height due to the velocity of efflux, will be equal to .889 ft., and that of the sides will be 52.953 ft. We see by this example how small the action of bends is in itself, and compared to that of the sides; and yet, we had here unusual bends, and a great velocity, as high as 4.26 ft. Even when the radius of curvature, in place of being 9.84 ft., had been only 1.64 ft., the resistance amounted only to .574 ft.; even in this case, we might omit it without sensible error.

I have already observed (198), that the resistance may be entirely neglected, by giving to the bends a great radius of curvature. At Toulouse, I have made them as great as 12.53 ft. Moreover, for pipes of a diameter in frequent use, that of .164 ft. for example, I have had the pipes cast in pieces 3.28 ft. long, and presenting an arc of 15°; at each bend is placed the number proportioned to its magnitude; six for a right angle.

5) The diameter of the pipe which extends from the chamber B to the chamber C will be determined in a manner analogous to that employed for AB: the sum of the resistances will be equal to 6.23 ft., and we have D = ... 0.853 ft.

6) We shall have, in adopting the same method for the pipe *Crd*, where we admit a loss of head 4.59 ft., a diameter = 0.738 ft.

7) That of the pipe BD, with 7.22 ft. loss of head, will be = ... 0.771 ft.

8) At *e*, quite near to *o*, is a great fountain, allotted to discharge through seven mouths 0.4291 cub. ft. of water, at 19.029 ft. above the pavement, and consequently, 8.202 ft. below the reservoir; this will be the entire head at the points of delivery. We must then deduct, 1st, the loss of head from A to B, which is

3.2809 ft.; 2d, the height due the velocity in AB, or .05832 ft.; 3d, 2.69 ft., for the resistance of the seven lead pipes, which, departing from the point *o*, conduct the water to the seven mouths, and which are 39.37 ft. long, with a diameter of .1312 ft. Thus we can only reckon on a head of 2.172 ft. to convey the .5509 cub. ft. of water, which should pass from B to *o*, a length of 354.33 ft. Employing the second formula of Sec. 190, we have for the diameter of B*o*

$$D = .2344\sqrt[5]{\frac{354.33 \times (.5509)^2}{2.172}} = .51156 \text{ ft.}$$

9) The pipe is continued beyond *o*: it conducts .1229 cub. ft. to *p*, a distance of 364.1799 ft.; and then .06145 cub. ft. to *q*, a distance of 557.7530 ft. further. It is considered advisable to give the same diameter to the pipe between the points *o* and *q*, throughout the distance of 921.933 ft.; for this purpose, we suppose that it conveys such a quantity of water, that with this quantity and the same diameter, the loss of head from *o* to *q* will remain the same as that which would have occurred with the above quantities, and which has been already fixed at 9.8427 ft. Let x be the mean quantity sought; since the resistances, with an equal diameter, are proportional to the lengths of the tube, and to the squares of the volumes of water, we shall have

$$921.933x^2 = 364.1799\,(.1229)^2 + 557.750\,(.06145)^2;$$

whence $x = .090835$ cub. ft.; and for the diameter we have

$$D = .2344\sqrt[5]{\frac{921.933 \times (.090835)^2}{9.8427}} = .24679 \text{ ft.}$$

10) At the point *p* is a branch *pf*, 659.46 ft. in length, conveying .06145 cub. ft., and terminated by a conical ajutage, .0656 ft. in diameter; we take .90 for its coefficient. From the entire head, 28.8719 ft., we must deduct, 1st, the loss of head from A to B, 3.2809 ft.; 2d, that from B to *o*, 2.16539 ft.; 3d, that from *o* to *p*, got by the usual calculation, 7.0211 ft.; 4th, that due to the change of direction at *p*, .15748 ft.; 5th, finally, the height due the velocity of issue, 6.3321 ft. The equation of motion, in substituting then the resistance of the sides, expressed by a single term (187), will be

$$9.9149 - \left(\frac{.0007089LQ^2}{D^5} - \frac{.0503634Q^2}{D^4}\right) = 0,$$

which gives, in substituting for L and Q their numerical values, D = .1784 very nearly.

11) The branch *qg* has a conical ajutage .049213 ft. in diameter. From the entire head of 32.1528 ft., subtracting the losses of head from A to *q* (3.2809 + 2.16539 + 9.8427), there remains 16.8638 ft. for the head at commencement of branch, and the formula of Sec. 194 will give

$$D = .2349 \sqrt[5]{\frac{354.337\ (.036729)^2}{16.8638 - \frac{(.036729)^2 \times .02518}{(.90)^2\ (.049213)^4}}} = .12861 \text{ ft.}$$

12) So for the branch *qh*, terminated by a thin plate, with an orifice of .04593 ft., and whose head at the commencement is 31.16855 — (3.2809 + 2.16539 + 9.8427) = 15.87956 ft., we have

$$D = .2349 \sqrt{\frac{160.76\ (.02472)^2}{15.8795 - \frac{.0251817\ (.02472)^2}{(0.62)^2\ (0.0459)^2}}} = .1004 \text{ ft.}$$

In great distributions of water, the pipes are not usually cast to the dimensions indicated by calculation. Thus, in the above described project, I should only admit six calibres, to wit: 1.47, .98, .82, .49, .26 and .16 ft.; and I should refer to each of them the nearest diameters given by calculation. I remark on this subject, that we should never allow a calibre below that given by calculation; on the contrary, it would be better to adopt larger dimensions. We must provide against earthy deposits, and contractions; moreover, we should be able to convey a larger quantity than is required by the ordinary service, in case of fires. Similar reasons forbid our going below a certain calibre; at Toulouse, I never went below .164 ft., and I do not believe it well ever to go below this limit.

As to the lengths to be given to pipes, they should be as great as the founders can furnish them; 8.202 ft. if they can be had, and 6.56 ft. at least, exclusive of jointage.

Practical remarks.

223. We have seen (205) that pipes seldom give the quantity of water which they ought to furnish according to the formulæ, and which they would furnish, if they were established and maintained in a perfect manner. We have remarked that the principal causes of this were, 1st, asperities or parts which project into the interior, and breaks in the continuity at the

joints; 2d, air which collects and remains in the summits of vertical flexures; 3d, muddy deposits, which principally settle in the lowest parts.

On the subject of the first of these causes, we cannot but recommend much severity in the reception of the pipes; those should be rejected whose diameters are smaller even by a small quantity than what was prescribed; those which are deformed, and those whose interior surfaces present superfluities, or are not clear. All these conditions ought to be specified in the schedule of clauses and conditions imposed upon the contractor. It is also necessary to take much pains in laying the pipes after they are received and approved; to make the axis of the whole to form exactly a straight line or a series of straight lines (curves excepted), and that the interior sides be as closely united as possible, so that the water will pass along on all its points without disturbance. With more reason, all contractions, which proceed either from the filling of the joints penetrating into the interior of the pipes, or from the openings of the cocks having a less section, must be avoided: as a first principle, there should be no contractions in a pipe.

There should be furnished a vent for the air which is conveyed to the higher portions, by placing at the culminating point a pipe with a coupling, to which a leaden pipe rising higher than the level which the water can attain, may be affixed, or a float-valve, or a cock. The tube is the most sure vent, and should be especially employed where it can be fitted up without inconvenience or without being exposed to damage, which is very rarely the case. Float-valves are principally suitable for large galleries, where they can be often visited. As to cocks, notwithstanding the simplicity of the means, they require a great deal of attendance; they must be frequently and regularly opened. The hydrants ("*bornes-fontaines*") established on the culminating points of streets of double slope, for the purpose of washing the two declivities, very conveniently perform the office of vents.

At the lowest parts of pipes, in valleys or depressions, large discharge cocks should be fitted, to be opened from time to time, to clean out the pipes, by making as much water pass through them as possible: the earth and mud deposited in the ordinary flow of the current will be taken up and carried along by the water, when animated with a greater velocity. The chambers for distribution, mentioned in Sec. 222, are very suitable for this cleansing; the fluid having there scarcely any velocity, naturally deposits in them the substances which it brings along; at the bottom of the lateral surface is a large tubular opening, closed up by a plate retained in its place by bolts, which may be unscrewed when it is desirable to wash out with much water. This method is employed with much success for the pipes of Toulouse; the mud and even sand, which the water deposits in great quantities, notwithstanding its previous clarification, are entirely removed from the tanks, and following along little scouring drains, are delivered into the common sewers of the city.

The entrance of all the pipes starting from the reservoirs or from the tanks, and that of the branches near the point of junction, ought to be provided with stop-cocks, designed to shut off or let on the water at will.

For pipes of more than four inches diameter, stop-cocks ("*robinets-vannes*") are used, the opening of which is shut by the aid of a conveniently arranged plate, raised or lowered by means of a screw. For less than four inches, turning-cocks are used.

We will not dwell on the form and construction of the different cocks, vent holes, pipes, etc., upon their connection, on the laying of pipes, nor, in general, on what pertains to the art of the fountain-maker. For what concerns these subjects, which would be out of place in a manual of hydraulics, the reader is referred to works treating specially upon them; among others, to those of MM. Girard,[*] Mallet,[†] Génieys,[‡] and Gueymard,[§] as well as the authors of the *Histoire de l' établissement des fontaines de Toulouse.*[††]

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[*] Description des ouvrages à exécuter pour la distribution, dans Paris, des eaux de l' Ourcq. 1808.

[†] Bulletin universel des sciences, 5th section, 1826; et Notice sur le projet d'une distribution générale d'eau dans Paris, avec des détails y relatifs, recueillis en Angleterre. 1829.

[‡] Essai sur les moyens de conduire, d' élever et de distribuer les eaux.

[§] Sur la conduite des eaux dans tuyaux cylindriques. Annales des Mines, Tome V. 1829.

[††] Nouv. Mémoires de l'Académie des Sciences de Toulouse, Tome II. 1830.

CHAPTER IV.

JETS D'EAU.

Natural height of jets.

224. If on the upper part of a small chamber, placed at the extremity of a pipe proceeding from a reservoir kept constantly full of water, a hole be pierced, a jet will pass out, which will rise, or rather tend to rise, to the height which the water of a piezometer placed on the chamber would attain during the flow. This height, on account of the upward direction of the motion in the tank, will be the effective head on the orifice of efflux; and its value will be found by subtracting from the entire head of the reservoir above the orifice, the sum of the resistances experienced on the whole length of the pipe.

Real height.

225. The real height of the jet will be somewhat less. Many causes contribute to diminish it. The principal is the resistance of the air; its effect, it is true, is insensible for heads below three feet; but above that, it has an appreciable value, and greater in proportion as the head augments, the resistance being proportional to it. Besides, this resistance of the air produces in jets a separation of the fluid lines, which considerably accelerates the destruction of the ascending force. Among other causes of diminution in the height, must be placed the obstacle which the upper part of the spouting column opposes to the free ascension of the lower part; this obstacle would be nothing, if the fluid particles were entirely independent of each other, since the velocity of all would decrease according to the same law; but the adhesion which connects them, causes them to exert an action on each other; the enlargement

of the column in its upper part, which can only happen in consequence of that action, proves the existence of it. The falling down of the upper layers, after the extinction of their velocity, upon the lower layers, would considerably diminish the elevation, if the enlargement just spoken of, increasing very rapidly at the top of the column, in consequence of the law of its formation,* did not impress upon the fluid particles an almost horizontal impulse, which removes them, and causes them to fall on the side; nevertheless, some fall back upon the column, and hinder its attaining its natural height. This can be shown by slightly inclining the orifice of efflux; then the jet, receiving no shock from the particles which fall back, rises higher; thus Bossut, by slightly inclining the apparatus which gave him a vertical jet of 11.221 ft., had 11.385 ft. (nearly two in. higher).

The effect of these combined causes can be determined only by experiment. Mariotte has investigated it. (*Traité du mouvement des eaux.*) At the bottom of a reservoir or drum of about 12.8 in. diameter, set up in a high place, he fitted a vertical tube of 3.19 in. diameter, the length of which was gradually augmented to 65.62 ft. The extremity was curved upwards, and was successively covered with different plates, pierced with circular orifices of different sizes, the edges of which were very smooth. The resistance of the pipe could scarcely diminish the height of the reservoir (two or three centimetres) from .79 in. to 1.18 in.; thus it might be disregarded, and the height of the reservoir taken for the effective head. In the following table is given the results of six experiments, made with an orifice of .53 in. diameter; one experiment given by Bossut (*Hydrodyn.* § 607) is added. The series of ratios noted in the table show that the dimensions in the elevation of the jet follow nearly the ratio of the squares of the heights of the reservoir; so that if h is that height, or, in general, the effective head, and h' the real height of the jet, we shall have $h'=h-\mu h^2$. The values of the coefficient μ, derived from these experiments, are placed in the last column.

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* This law is expressed by the equation $y^4 = \frac{hm^2d^4}{h-x}$, which belongs to an hyperbole of the fourth degree: h is here the effective head on the orifice, d its diameter, m the corresponding coefficient of contraction, y the diameter of the column taken at the height x above the orifice.

At the summit of the column, where $x=h$, the diameter or enlargement would be infinite.

The force of projection due to this abrupt enlargement is combined with the action of gravity, and the water falls back in the form of a paraboloid or trumpet-shaped mouth-piece, under which the jet is seen. The fountain of the Trinity at Toulouse presents this form in a perfect and very agreeable manner, when the fluid column, with a base of two inches diameter, rises to a height of from sixteen to twenty inches.

HEIGHT.		DIMINUTION or difference.	SERIES OF RATIOS		μ
Head.	Jet.		of the diminution.	of the sq. arcs of the head.	
feet.	feet.	feet.			
37.7303	34.0886	3.6417	1.000	1.000	.0025602
37.2382	33.7933	3.4449	.951	.974	.0024993
27.8220	25.8107	2.0113	.549	.543	.0025907
26.0175	24.3443	1.6732	.464	.476	.0024993
13.1564	12.7955	.3609	.098	.121	.0020726
5.8728	5.7416	.1342	.031	.024	.0032308
11.7128	11.2207	.5921	.134	.097	.0035660

The mean value of μ, in the experiments of Mariotte, is .0025602. That of Bossut gives .003566. Although every thing tends to the belief that the first result is the more exact, yet, as only a simple approximation is required, and as no great error is to be feared in making the diminution a little too great, I shall admit a mean term, and consequently a very simple expression,

$$h'=h-.003047h^2.$$

226. Great jets rise higher than small ones; having more mass, the resistance less promptly destroys their velocity, and they are less divided.

Bossut, under a head of 11.713 ft., after having obtained, with an orifice of .709 in. diameter, a jet of . . . . . 11.221 ft.
with an orifice of .177 in. diameter, had only . . . 10.696 ft.

Mariotte, under a head of 26.018 ft., with an orifice of .532 in. diameter, had . . . . 24.344 ft.
and with an orifice of .248 in. diameter, but . . . 23.622 ft.

The difference, which here is small, becomes insensible when the jets are not raised above 6¼ feet, and the diameter of the orifice is not diminished below .276 in.

But if it is smaller, and the head be great, the diminution of the height becomes considerable, and as much more so as the head is greater. Thus Mariotte, with an orifice of .089 in. diameter, found a diminution of .625 ft., under a head of 4.790 ft.
a diminution of 3.182 ft., under a head of . . . . 14.928 ft.
" " 7.546 " " " " . . . . 28.872 ft.

Effect of Ajutages.

227. The jets just investigated passed through circular orifices pierced in thin plates. These orifices are those which carry the jets to the greatest height, and give them the smoothest form; on examining them as they pass out, one would believe he saw a bar of the purest crystal. Thus, when the elevation and the beauty of the jet are had in view, these orifices are preferred.

Cylindrical ajutages are less satisfactory in these two respects. They diminish the velocity of efflux in the ratio of 1 to .82 (42), and consequently the heights of the jets in the ratio of the square of these two numbers, or of 1 to .67; that is, the height of a jet coming from such ajutages will be only two thirds of the effective head, or rather, only two thirds of the height which would have been attained through an orifice in a thin plate. Besides, after the exit, the fluid threads composing the jet scatter, and the water has a troubled appearance.

Conical ajutages, having coefficients of diminution of velocity which vary from .85 to .95 (50), will give heights from .72 to .90 of those due to orifices in a thin plate. Their jets, moreover, are smooth and transparent at the exit.

Fig. 44.

228. Very often the ajutage is inclined. The jet then describes a curve, and we must determine its greatest elevation CD, and its *amplitude* AB, that is, the greatest horizontal distance it can attain.

Amplitude and elevation of inclined jets.

Without the resistance of the air, the curve described would be a parabola (37). It is so, in fact, under a head of a few feet; but higher, it is altered; this alteration somewhat diminishes the elevation and amplitude, but not enough for the error resulting from the supposition of an exact parabola to become of importance.

If n is the coefficient of the velocity for the ajutage employed (47), nv will be the real velocity of exit, and n^2h, or the height due to this velocity, will be the force of projection, h always being the effective head. Calling i the angle of the inclination of the ajutage, which is the angle of projection, taking the horizontal line AB as axis of abscissas, we have for the equation of the parabola described by the jet (*Mécanique* de M. Poisson, No. 208),

$$y = x \text{ tang. } i - \frac{x^2}{4n^2 h \cos.^2 i}.$$

229. If we designate by A the amplitude, observing that it is only x, for the case of $y=0$; and recollecting that tang. $i=\frac{\sin. i}{\cos. i}$, we have

$$A=4\,n^2\,h\sin. i\cos. i=2\,n^2\,h\sin. 2\,i.^*$$

230. AC, half of this amplitude, put for x, in the equation of the curve, gives for the ordinate CD (=E), representing the greatest elevation of the jet,

$$E=n^2\,h\sin.^2 i.$$

General Problem.

231. The problem proposed for jets d'eau, taken in all its generality, will be thus enunciated: at a given point, to produce a jet which carries a certain quantity of water to an elevation and a distance also given. The question is reduced to determining the kind, inclination and diameter of the ajutage to be established at that point. Since the point where the jet must issue is given in position, we know the entire head, or its depression below the reservoir which is to furnish the water; we must calculate the resistance it will experience in the pipe established or to be established between the reservoir and that point; this resistance is to be subtracted from the entire head, and we shall have h. Q, A and E are given; and it is required to determine i, n and d, this last letter representing the diameter of the exit of the ajutage.

Dividing the equation of Sec. 230 by that of Sec. 229, we have

$$\frac{4E}{A}=\frac{\sin. i}{\cos. i}=\text{tang. } i.$$

Thus the angle i, under which the ajutage must be inclined, will be ascertained.

The sine of this angle, put into the two equations just investigated, will give n; and this coefficient, by the aid of the table at Sec. 50, will indicate the kind of ajutage to be used; that is, what its degree of convergence should be for the case demanded.

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* Vide Farrar's Mechanics, page 192.

The same table will also give the coefficient m of the discharge, for the formula $Q = m\pi' d^2\sqrt{2gh}$; from which to deduce d.

The problem will thus be solved.

Example of a Wheat-Sheaf Jet.

232. A collection or sheaf of jets is to be established, for which .57213 cub. ft. of water per second is designed. In the middle, a vertical jet is wanted, and around it, on two concentric circles, sixteen other jets, so inclined that their water, in falling, may present nearly the form of a hemisphere. The place from which the jet starts is 29.528 feet below the reservoir, and the loss of head in the supply pipe is 4.921 feet; so that there still remains an effective head of 24.607 feet. The jet at the centre is to pass through an orifice in a thin plate; it should be larger than the others, and the quantity of water appointed for it is six "*pouces d'eau*" = .049443 cub. ft. per second. The elevation which it will attain will be $24.607 - .0030489\,(24.607)^2 = 22.7655$ ft. Since the water of the inclined jets, in falling, should form nearly a hemisphere, they should be allowed an amplitude of 22.7615 ft., and we will decide upon 19 ft. 8 incs. as the elevation to be attained by the first row of orifices, and 16 ft. 5 incs. as that of the second row; each of the first eight will discharge .03673 cub. ft. = 4½ "*pouces*," and each of the last eight will discharge 3½ "*pouces d'eau*" = .02861 cub. ft. Such, with somewhat less dimensions, is the sheaf-formed fountain ("*la gerbe d'eau*") which I have established at the Place des Carmes, in Toulouse.

For the orifice in the centre, we have only the diameter to determine; it will be

$$D = \sqrt{\frac{Q}{m\pi'\sqrt{2g}\sqrt{h}}} = \sqrt{\frac{.049443}{0.62 \times .78539 \times \sqrt{64.364} \times \sqrt{24.607}}}$$

$= .0505$ ft.

For each of the ajutages of the first row, we have tang. $i = \frac{4E}{A} = \frac{4 \times 19.6854}{22.9663} = 73^\circ\ 45'$; consequently, $n = \sqrt{\frac{E}{h \times \sin.^2 i}}$

$$= \sqrt{\frac{19.6854}{24.607 \times \sin.^2 73^\circ\ 45'}} = .86795.$$

The table (Sec. 50) shows that to such a coefficient of velocity belongs an angle of convergence of nearly 7°; its coefficient of discharge, m, according to the same table, will be 0.93. Thus, for the diameter to be given, we have

$$D = \sqrt{\frac{.036729}{.93 \times .78539 \times \sqrt{64.364} \times \sqrt{24.607}}} = .035546 \text{ ft.}$$

Recapitulating, there will be required for each of these ajutages
a diameter of efflux of . . . . .035546 ft.
a convergence of . . . . 7°,
and an inclination of . . . . 73° 45′.

By analogous calculations, we shall find for the ajutages of the
second row a diameter of . . . . .0318 ft.
an angle of convergence of . . . . 2°,
and an inclination of . . . . 70° 43′.

Finally, to arrange both of these properly, we take a brass plate .0426 ft. thick, to which we give the form of a spherical cap, with a radius, for example, = 1.64 ft.; it will be above the box or trunk ("*souche*") whence issue the jets,—a box to which we may give the form of a cylinder .984 ft. in diameter and as much in height. In the middle, or culminating point of the cap, as a centre, and with a radius of .462 ft., (that is to say, at a distance of 16° 15′, the complement of inclination to be given to the ajutages of the first row,) we describe a circumference, on which are placed the eight ajutages, at equal distances apart, and exactly in the direction of the radius of the spherical cap. For the eight of the second row, we describe another circumference, with a radius of .549 ft., or 19° 17′; and in establishing them, we observe that each of them must correspond to the middle of the interval between those of the first row.

Both will consist of small bronze cylinders, 1.18 in. in diameter and the same in length; they will be bored longitudinally, so as to have the above diameters and flaring openings; the flaring is determined by the dimensions to be given to the diameter of the orifice of entry, knowing the diameter of efflux and the length of the ajutage; a length which should exceed double this last diameter (50). I remark, that if this length is such that the entering end of the ajutage exceeds the thickness of the cover of the box, and so penetrates the interior, in order that no extraordinary contraction may result from it, to diminish the projectile force and the discharge, it is necessary that the thickness of the ajutage, around the orifice of entrance, should be at least 0.27 in. (3 *lignes*). The lateral surfaces of the ajutages are cut in the form of a screw, so that they may be screwed upon the cap, in the holes which have been bored for that purpose; their upper extremity, like the head of a nail, has a greater diameter than their body; it will have, upon its sides, two notches, so that by a screw-jack they can be taken out or replaced at pleasure. The same arrangement must be given to the middle orifice.

SECTION THIRD.

WATER AS A MOTOR.

Water in motion acts as a motive power, in communicating by its *impulse* a part of its velocity to bodies opposed to the direction of its path. It acts also in a negative manner, in destroying or reducing, by its *resistance*, the velocity of bodies either moving in it or upon it, as that of ships, for example.

There is still another motive action of water, and one of great interest to us, and it is that which this fluid exerts upon machines, impressing motion upon them, by means of which the manufacturing arts execute a great part of their varied operations.

The difference in the nature of these two actions divides this section in two parts, or sub-sections, entirely distinct.

SUB-SECTION I.

IMPULSE AND RESISTANCE OF WATER.

CHAPTER I.

IMPULSE OF WATER, OR HYDRAULIC PRESSURE.

Nature of the impulse of fluids.

233. When a solid body, endowed with a certain velocity, encounters and gives a shock to another body, moving with less velocity, or in a state of rest, it communicates all the motion which it can impart in an instant, whose duration we cannot appreciate; at the end of this instant, all action on the part of the body imparting the shock ceases, and the entire effect of *percussion* is produced. The impulse of a current of water is of another character; for in this case, there is a multitude of particles, succeeding each other without interruption, which act upon and continually press the body impinged upon. Their effect is like that of a spring which acts against an obstacle, preserving always the same tension; or like that which a weight exerts upon bodies which cannot yield to its action; and this impulse may therefore be likened to a weight. Moreover, experiments leave us no doubt respecting this view of the subject; if we fix a plate at the extremity of a balance, and direct upon it a jet of water, issuing from a vase kept constantly full, there will always be found a weight which, placed at the other extremity of the beam, will maintain its equilibrium, through the whole term of the running of the water. This weight, being thus in equilibrium with the effort or force of the impulse, will be equal to it; it will represent it. We see

from this fact, that this force is but a simple *pressure;* and that the impulse of a current is the *hydraulic pressure* due to the motion of the fluid, while the *hydrostatic pressure* is that which proceeds solely from its weight.

We have to consider, in this chapter, the impulse which is produced by an isolated vein acting against a surface; that experienced by a body plunged wholly or in part in a fluid of indefinite extent; and that which takes place when the fluid moves in a water-course, where the body receiving the impulse occupies nearly its whole section.

ARTICLE FIRST.

Impulse of an isolated vein.

Theoretic expression of the direct impulse of a vein.

234. Let us take first the most simple case, that where a plane surface at rest is exposed perpendicularly to the impulse of a fluid vein.

Fig. 45.

Let there be a vein issuing from the horizontal tube AB, directed so as to give an impulse against the vertical plate MN, which is fixed at the extremity of an angular lever COD, moveable around the point O. We are to determine the value of the weight P, which, placed at D, (OD being equal to OC,) will maintain the plate in its position. By reason of the equality in the arms of the lever, and the direction in which this weight tends to push the plate, it is evidently the same as if it were applied immediately to its posterior face, and then exerted its action directly opposed to that of the current; and in order to destroy, at each instant, the quantity of motion of the current, it must be equal to it.

Designate by s' the section of the fluid vein at B, by v its velocity at the same point, and by τ an extremely small time; $s'v\tau$ will be the volume of water passed in this time; and $s'v\tau\varphi$ will be its weight, φ being the weight of a cubic foot of the impinging fluid. The quantity of motion being the mass multiplied by the velocity (and the mass being the weight divided by g) will be $\frac{s'v\tau\varphi}{g}\cdot v$, or $2hs'\tau\varphi$, representing by h the height due to the velocity v, since $\frac{v^2}{g}=2h$. Against this quantity of motion, it is necessary that the weight P should oppose an equal quantity. Now, it is admitted in mechanics (Poisson, § 128), that the quantity of motion produced in the extremely small time τ, by a weight P placed in the basin of a balance, or suspended from the arm of a steelyard, is $P\tau$. We have then $2hs'\tau\varphi=P\tau$; or simply,

$$P=2s'h\varphi.$$

But $2hs'$ is the volume of a prism, which has s' for its base and $2h$ for its height, and $2hs'\varphi$ is the weight of a like prism of impinging water. Thus, the force or *effect of an impulse exerted by a fluid vein upon a plane surface at rest, and exposed perpendicularly to its action, is equal to the weight of a prism of this fluid, having for its base the section of the vein, and for its height twice the height due to its velocity.*

Such is the expression first given by Newton, and afterwards generally admitted by all authors.

Let us see how far it has been modified by experiment.

Laws and expression of impulse, according to experiment.

235. Bossut having fitted horizontally, upon the end of a balance beam, a circular plate .223 ft. in diameter, caused to fall upon it a jet, through a vertical and cylindrical ajutage, .0754 ft. in diameter; and he found

that the weight necessary to maintain an equilibrium was,
under a head of 4.265 ft., 1.4772 lbs. avoirdupois;
and under a head of 2.132 ft., .7386 " "

For the ajutage of .0754 ft. diameter, he substituted one of .0443 ft. diameter, and he had,
under a head of 4.265 ft., .8422 lbs. avoirdupois;
and under a head of 2.132 ft., .4211 " "
(*Hydrod.*, §§ 855 et suiv.)

In these experiments, when the heads were diminished one half, the effects of the impulse have decreased in the same proportion; and as the heights due the velocity of issue, through the same ajutage, follow the ratio of the heads, we conclude, from observations which we shall report, as well as from a great number of others, that the *effort of impulse of a fluid vein, is proportional to the height due to the velocity of the vein;* or, which amounts to the same, *to the square of this velocity.*

236. It is moreover quite natural to admit that the effort is also proportional to the number of impinging particles, that is to say, to the section of the fluid vein, at its issue from the orifice. In reality, in experiments which we shall cite, the two sections having varied in the ratio of 100 to 36, the weights measuring the efforts of the impulse have followed this same ratio.

237. We may thus establish, observing that φ, in the case of water, is 62.43 lbs. per cubic ft.,

$$P = 62.43 \text{ lbs. } n s' h;$$

n being a coefficient to be determined by experiment.

Value of coefficient.

238. The value of n will depend principally upon the extent of surface which receives the impulse, its distance from the orifice of issue of the vein, and, to a certain extent, upon the nature of this surface.

In order that the shock may produce its whole effect, the surface impinged upon should be sufficiently extended to arrest all the fluid threads, and destroy the velocity which they had in their primitive direction; and that it may be so, in consideration of the dispersal of the threads after their exit, it is proper that this surface should be from six to eight times greater than the orifice; these experiments, as well as theory, give $n=2$, very nearly. In the observations of Bossut above mentioned, we have $n=1.95$; in those of Bidone, upon six well polished brass plates, whose diameters varied from 0.1772 ft. to .8005 ft., impinged upon by three veins, issuing with an extreme velocity of 30.381 ft., from three pipes, 0.0656 ft., .0885 ft. and .1181 ft. in diameter, n has ranged from 2.04 to 2.23.* But if the plate is not large enough to intercept all the fillets, there will pass all around it a great quantity, which exerts no action upon it; so that when it was only equal to the section of the vein before the dispersal, Langsdorff and Dubuat found the value of $n = 1$ only.

Whatever may be the dimensions of the surface impinged upon, to obtain the greatest percussion which it can receive, it should be at a certain distance from the orifice. If it were applied immediately at the orifice, it would only have to support the hydrostatic pressure of the column above it, 62.43 $s'h'$; h' being the height of the reservoir above the centre of the orifice. But when it is removed, the effect of the impulse increases more and more to a certain distance, beyond which a less velocity and the separation of the threads cause a diminution. Bidone has found, that for his vein, .0886

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* Expériences sur la percussion des veines d'eau, par George Bidone. Turin, 1836.

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ft. diameter, the maximum distance was at .5239 ft., and he then found $n=2.22$.

The nature and degree of polish of the plates exposed to the action of the veins have also an influence upon the result produced; thus Zuliani has found it much greater upon an iron disc than on one of wood, all things being equal in other respects.

Effect due to flanged rims.

239. This result is considerably increased, by surrounding the plate which receives the impulse with a flanged curb or rim.

In 1812, Morosi, after having observed the action of a jet upon a simple square plate, under three different heads, and having found it five, seven and nine pounds, fixed upon each of the four edges a border or rim .0459 ft. in height; and, under the same heads, the action of the jet was as high as eleven, fifteen and twenty pounds; that is to say, more than double. Even if this result be exaggerated, we are none the less positive as to the considerable effect of these borders; experiments made by M. Bidone afford us precise measures of these effects. This savant took the three circular plates of which we have already spoken (238), and surrounded them with a cylindrical rim, whose height was gradually increased from .0074 ft. to .1607 ft.; he then exposed them perpendicularly to the action of three veins, and thus made 180 experiments. I place opposite, the principal results obtained with the vein of .0886 ft. diameter upon the plate of .2657 ft. We here see that the actual effect of the impulse increased with the height of the border, until this height attained 0.262 ft.; after which it diminished, though the height of the rim

Height of Border.	Coefficient n
ft.	
0.0000	2.220
.0072	3.349
.0148	3.713
.0223	3.900
.0259	3.929
.0295	3.909
.0443	3.831
.0889	3.536
.1332	3.447

increased. The greatest value of n was 3.93; and then the result was 1.77 times greater than without the rim.

The fluid threads, after striking the plate, spread out upon it, radiating through all its parts; arriving at the rim, they follow it, and then quit it as if sent back by it; they return among themselves under the form of a hollow vein, whose base rests upon the perimeter of the rim. If, in this return, they have a direction parallel and a velocity equal to that of their departure, the force exerted by them against the plate will be double, and we shall have $n = 4$: for it is a principle of mechanics, that when a body, after having struck an immovable obstacle, is sent back by it in its first direction, and with an equal velocity, the force exerted against the obstacle is double; we have an example of this in the collision of elastic bodies.

Daniel Bernoulli, and then Euler, have given a theory and an expression of the percussion of fluids, which they afterwards abandoned, as not susceptible of application. To the term of Newton, $2\varphi s'h$ (234), they joined $-2\varphi s'h \frac{\cos.\,\omega \sqrt{h'}}{\sqrt{h}}$, in which h' is the height due the mean velocity of the fluid thread on quitting the plate, and ω the angle which, in quitting, it makes with the primitive direction of the vein. When the threads leave the plate in its direction, always supposed to be perpendicular to that of the vein, and preserving their original velocity, $h' = h$, $\omega = 90°$, $\cos.\,\omega = 0$, and the second term disappears; we then have only the generally admitted expression, and $n = 2$. But if all the fillets return parallel to their first direction, and with the same velocity, $\omega = 180°$, $\cos.\,\omega = -1$, the second term becomes equal to the first and addition; so that we have $n = 4$. Euler has explicitly signalised this extreme case; and experiment has approached very nearly the result indicated by this illustrious analyst; it has given 3.93. As for the intermediate cases, how shall we have ω and h'?

240. Independently of the permanent action of a vein which flows upon a body, we have to consider the force of the *first blow* of the shock, properly so called. M. Bidone, who sought to determine its extent, has sometimes found it double. For example, in one of his experiments, after having equilibrated the force of permanent action of a vein by a weight of 7.89 lbs., suspended at the extremity of the horizontal arm of a balance, he sustained this arm, and stopped the flow: shortly after, he renewed the flow, and it required a weight of 15.45 lbs. to prevent the arm being raised by the first shock. Having also examined the action of such a shock upon plates with a rim, he has seen n raised as high as 5.36.

First blow of the shock.

This action of the first blow of the percussion of fluids should be most carefully considered by engineers: the dike to which double strength has been given to that required for resisting the effort of a continuous current, might be carried away by the sudden shock of a great wave; the construction capable of enduring the action of a steady but strong wind, may be overthrown in a tempest, by the sudden blow of a squall. Let us remember that the action of the first shock has been as high as 5.36, a quantity nearly triple that indicated by the ordinary theory.

241. If a plane, instead of being exposed perpendicularly to the action of a fluid vein, were under an angle which we will designate by i, the force of pressure of the fillets will be decomposed; a portion directed parallel to the plane will have no effect upon it; and the other portion, which acts perpendicularly, will have for its expression, 62.43 lbs. $ns'h$ sin. i, per cubic ft. Thus, if BC represent the force of the direct action, which is always 62.43 $ns'h$, BD, which is equal to BC sin. i,

Oblique action.

Fig. 46.

will represent that which acts perpendicularly upon the plane. A series of experiments made by Dr. Vince, under angles of inclination of from 10° to 90°, shows that the normal force is actually proportional to the sine of the angle of incidence.

If the force were to be estimated in the direction of the vein, it would be represented by $BE = BD \sin. i = BC \sin.^2 i$. So that the *force of the impulse would be proportional to the square of the sine of the angle of incidence of the vein upon the surface impinged upon.* This theory, which has been generally adopted since Newton's time, is now abandoned. Experience declares against it; it has shown that when the angle of incidence is great, the ratio approaches more nearly the first power of the sine, and after that, the 1½ power; but when the angles are small, it is more complicated, and not so well understood.

Direct action against a plate in motion.

242. We have thus far admitted that the plate which received the impulse was immovable; but most generally it is in motion. Suppose first that it receives the impulse perpendicularly, and moves in the same direction as the vein. Let u be its velocity when motion is well established; it will of necessity be less than that of the fluid vein, or than v. The impulse will not entirely destroy this last, since, after this has taken place, the fluid, moving with the plate, will possess its velocity u. It will then have lost the velocity $v - u$ and the quantity of motion $\frac{62.43}{g} s'v\,(v - u)$. Now, it is this quantity of motion destroyed which measures the force of the impulse (234); we have, then,

$$P = \frac{62.43}{g} s'v\,(v \mp u).$$

The sign + applies to the case where the plate, instead of moving before the fluid, goes directly to meet it. The relative velocity will then be $v+u$, in place of $v-u$.

Oblique action against a plate in motion.

243. Let us admit now that the fluid vein falls obliquely upon a plate constrained to move in a certain direction.

Let AB be the direction of the fluid vein which strikes the plate CD, and BK the direction of motion which it is constrained to follow. Take upon the first, $\text{BE}=v$, to represent the velocity of the fluid; and upon the second, $\text{BF}=u$, for the velocity of the plate; make the angle $\text{ABC}=i$, and the angle $\text{CBK}=j$. The component of BE perpendicular to the plate will be $\text{BG}=v \sin. i$; and that of BF, in the same direction, will be $u \sin. j$; so that the velocity lost in this direction will be equal to $\text{GH}=v \sin. i - u \sin. j$. This loss of velocity, estimated in the direction of motion, will be IK; or, $\text{IK}=\text{GH} \sin. \text{BGK}=\text{GH} \sin. j$. We shall have, then, for the quantity of motion lost, a quantity which measures the force of the impulse received by the plate,

$$\frac{62.43\, s'v}{g}(v \sin. i - u \sin. j) \sin. j.$$

Example. A vein of water issues, under a head of 15.7843 ft., from a conical trough with an orifice of .1640 ft. diameter: it falls upon a plate, making with it an angle of 75°, and this plate moves with a velocity of 7.2179 ft., maintaining an inclination of 64° to the direction of its motion. What will be the force?

The coefficient of velocity for a conical trough is 0.95 (50); thus $v=0.95\times\sqrt{64.364\times 15.7843}=30.280$ ft.: we have also $u=7.2179$ ft., $s'=\pi'(.1640)^2=.021124$ sq. ft., $i=75°$, and $j=64°$, or the $\sin. i=0.966$ and $\sin. j=0.899$. Thus the force exerted will be $\frac{62.43\times 30.280\times .021124}{32.182}(30.280\times .0966 - 7.2179\times 0.899)\, 0.899=25.39$ lbs.

ARTICLE SECOND.

Action of an indefinite fluid.

Circumstances of motion and of the action of the fluid.

244. We call a fluid indefinite, when the space comprised between the sides of the bed or basin which contains it and those of the body which it strikes upon, is so great as to occasion no greater impediment in the motion of the fluid, than if the space were infinite; such, for instance, as occurs when a ship is urged by a current of the sea.

When a fluid impinges against a body which is entirely submerged, it exerts an action not only upon its front face, as in the impulse of an isolated vein, but also upon its lateral and rear faces; and all these actions must be taken into account.

To have a correct idea of what takes place in such a case, we take at first a right prism, and suppose it to be entirely submerged in a current, with its axis in the direction of motion, and consequently horizontal. If the fluid were at rest, each of the points of the surface of the prism would experience a hydrostatic pressure, represented by the depth of this point below the surface of the water; the two bases being equal and of equal depth below this surface, the pressure upon one would be equal to that felt by the other; and there would be no tendency to motion in the direction of the axis. But the moment the water commences running, a different order of facts is presented. The fluid fillets which would traverse the space occupied by the prism, begin to turn aside a little above it; they go on diverging, and so pass around the anterior part; contracted then in a narrower space (for the aqueous mass between the prism and the sides of the bed to a certain extent acts as a resisting body), their velocity increases and is accelerated; then repelled, and, as it were, reflected by this mass towards the prism, a part running along its lateral faces, a part converging, they re-unite behind the posterior face, preserving a portion of the excess of velocity which they have acquired; a portion which will be so much the greater as their passage through the aqueous mass and along the prism may have been shorter. When these fillets commence diverging above the prism, there remains between them and the anterior base a small mass or *fluid prow;* it is pressed against this base by the fillets in motion; its particles tending to escape are carried from the centre to the circumference; those in contact with the base move parallel to it, with a velocity which is rapidly accelerated in approaching its edges. Moreover, when the fillets re-unite in converging behind the posterior base, they enclose between them a *fluid stern*, whose particles they in some way carry along with them; there results a less pressure against this base, and a tendency for a vacuum to be formed behind it. In consequence of these different motions just indicated, the pressure upon the anterior base of the body is augmented; it has become greater than the hydrostatic pressure; upon the posterior base, on the contrary, there has been a diminution, and the real pressure there is less. From this twofold cause, the pressure upon the anterior base will predominate; and it tends to produce a motion in the direction of the axis of the prism.

As for the pressures which take place on the lateral faces, whatever their absolute value, they will always be equal upon two points directly opposite; they will be destroyed, and no motion can be occasioned by them.

This analysis or separation of the elements of action of a fluid in motion upon a body exposed to it, is due to Dubuat. I refer, for more ample details, to his *Principes d'hydraulique*.

The motion of the particles, in the *fluid prows and sterns* of this author, may be caused by whirls, according to the observations of M. Poncelet: we should have two, turning in opposite directions, against each base of the prism. But what is remarkable is, that immediately behind the two whirls adjacent to the posterior base, we have two others, moving in opposite directions to them: we might say that the fluid fillets which form the two first are as if unrolled, to be rolled up again in an inverse manner. Behind this second couple of whirls, there is a third, a fourth, &c., with motions always alternating. In proportion to their distance from the prism, they enlarge in dimensions, and they finish by being lost or absorbed in the great fluid mass. It is in this wise, says M. Poncelet, that motion is destroyed in fluids.

Measures of pressure upon submerged bodies.

245. We proceed to the determination of the intensity of pressures.

Let H be the mean hydrostatic pressure or depth of the prism below the surface of the water; h the height due to the velocity of the current; the hydraulic pressures being proportional to this height, we may represent by mh that experienced by the anterior base, and by $m'h$ the negative pressure, called non-pressure by Dubuat, which takes place at the posterior base; m and m' are two numbers to be determined by experiment. The total pressure at the up-stream base will then be $H + mh$; and $H - m'h$ at the down-stream base. These two forces, acting in opposite directions, their resultant, or the force which urges the prism in the direction of its axis, will be equal to their difference, and we shall have for its expression, that is to say, for the height of the column which measures its force, $H + mh - (H - m'h)$, or $(m + m')\ h$.

246. Dubuat has also determined the values of m

and m'. I record one of his experiments, and the results he deduced from it.

He took three prisms, or rectangular parallelopipeds; the base of each was a square of 1.066 ft. per side; one had only a height of .0295 ft., and was a simple plate; the second had 1.066 ft., and was a cube; and the height of the third was 3.182 ft., or three times the side of the base. He plunged them and held them in a current whose velocity was 3.182 ft. By means of a very ingenious piezometer, he measured, upon 625 points of the anterior base of each of the three bodies, the hydraulic pressure, which was the height of the piezometric column above the surface of the current; and for each, he had a mean of $1.19h$; thus, $m=1.19$. This value was constant, and independent of the length of the prisms.

It was not so with the *non-pressure* measured by the falling of the piezometric column below the surface of the current; in these prisms, the value of m' was respectively, 0.67, 0.27 and 0.15. The *non-pressures* diminished with the cause which produced them, namely, the velocity of the fluid fillets in the rear of the prisms; a velocity which is smaller, in proportion as the body is longer (244).

In short, the total pressure, or the force of the current upon the three prisms, was expressed by $1.86h$, $1.46h$ and $1.34h$. This force would have diminished if the length of the body had continued to increase; but only up to a certain point; beyond which, it would have increased.

247. Is the absolute force of the impulse proportional to the anterior surface of the body acted upon, as has been often admitted?

In the case of very thin bodies, as of simple plates, experience answers in the negative. Thus Mariotte, having exposed to the action of a current of the Seine a square plate, smaller, in the ratio of 100 to 25, than that used by Dubuat for the above experiment, found the effect less in the ratio of 100 to 16, being reduced to the same velocity. This observation, and many others, show that the force of the impulse increases in a ratio greater than that of the surfaces impinged upon, without, however, indicating the law of its increase.

The ratio approaches equality as the plates increase in thickness; and a great number of observations compel us to admit, that in similar prisms, and, in general, in similar solids, the force of the impulse is sensibly proportional to the surface receiving its action.

We have, then, for the expression of this force, at least for prisms, s being the surface acted upon,

$$62.43\,(1.19 + m')\,sh\,;$$

an expression in which m' will diminish to a certain limit, according to the increase in length of the prism.

Floating bodies. 248. What we have said of bodies entirely submerged, applies to those which are partly submerged, or floating bodies.

In this case, the fluid, on arriving at the anterior face of the body, is elevated above the primitive level; it there forms a rise, whose greatest height is at the middle of the face, and which lowers gradually towards the sides; the fluid follows this slope, and moves continually from the centre towards its edges. It lowers in its passage along the lateral faces of the body; and upon the back face, it is found to be below the general level of the current; it there forms a trough or a depression, around which its particles are highly agitated.

This difference of level, from the front to the rear of the body, is termed "*dénivellation.*"

It occasions a greater hydrostatic pressure, but the total pressure is not augmented. That experienced by the anterior face, notwithstanding the rise, is even less than with bodies entirely submerged; Dubuat found it to be only $1h$; while in the latter, it was $1.19h$. The *non-pressure* upon the posterior surface appeared to him to be a little greater; but the total effect was a little less.

Definite expression of the force of the impulse.

249. From what has been said, if s is the surface receiving the pressure, that is to say, the area of the greatest section made by the submerged parts of the body, perpendicular to the direction of the current; if h is the height due its velocity; making $m + m' = n$, and comprising in the value of the coefficient the correction due to the form of the surface acted upon, the force of pressure experienced by a body plunged wholly or in part in the water is

$$62.43nsh.$$

The coefficient n will be constant for each kind of similar solids; but it will vary with the different kinds of solids, and for each kind, it must be determined by experiment.

Case where the body is itself in motion.

250. If the body exposed to the action of an indefinite fluid animated with the velocity v, is itself in motion, and in the same direction, with a velocity u, the relative velocity of the impulse will be, as for the case of isolated veins, $v - u$. But, according to a theory which we shall hereafter discuss (331), the fluid mass imparting the pressure will not be independent of u; instead of being proportional to the simple velocity v of the current, it will be so to the relative

velocity $v-u$; it will be equal to $\frac{62.43}{g}\,s'\,(v-u)$: so that the quantity of motion lost, or the force of the impulse, will be $\frac{62.43}{g}\,s\,(v\mp u)^2 = 62.43\times 2sh'$; or, more generally,

$$62.43nsh';$$

h' being equal to $\frac{(v\mp u)^2}{2g}$: the lower sign relates to the case in which the body, instead of descending, ascends the current.

Oblique impulse.

251. If the body, while maintaining the direction of the current, presents to it an oblique surface, the effect will be less. We have seen that that of a fluid fillet falling upon an inclined surface, and estimated in its own direction, was proportional to the square of the sine of inclination (241). Moreover, the number of fillets which strike this surface will be also less, in the ratio of the same sine: so that the total effect will be as the cube of the sine; or, what amounts to the same, it will be equal to the direct effort upon a projection of the surface made upon a plane perpendicular to the direction of the current, and multiplied by the square of the sine of inclination. This theory has been admitted a long time; but experiment has shown that it in no wise corresponds with the facts.

ARTICLE THIRD.

Impulse of a fluid contained in a water-course or mill-race.

252. When a stream of water is conducted by a mill-course upon a plate occupying nearly its whole section, its action is nearly similar to that of an isolated vein; for in this case, also, the fluid particles which pass the

section s' of the sluice or water-course, occasion the impulse, and lose their motion, or a part of it, against the plate, deduction being made of those which escape through the small interval between the sides of the water-course and the edges of this plate. Moreover, these sides perform the office, to a certain extent, of the rims of Morosi, and they augment the force of the impulse. Thus, in the expression of this force, $62.43ns'h$, n often exceeds the values which it has when the plate is in the open atmosphere (238).

If the plate recedes before the fluid with the velocity u, we shall have, as in section 242,

$$\frac{62.43}{g} s'v\,(v-u) = 1.94\, s'\,(v-u).$$

253. M. Christian tried the effect of rims upon a plate placed in a water-course of 0.656 ft. in breadth.* He observed that rims fixed upon the horizontal edges, both at the top and bottom, did not augment the impulse; but that it was increased by the lateral edges in the ratio of 100 to 112, when the space between the rims and sides of the water-course was small; and in the ratio of 100 to 123, when it was .164 ft.

The difference in these two cases is due to a cause which I have already noticed. The sides of the water-course themselves produce the effect of rims, and so much the more, as the interval is more contracted: thus, when it was small, the effect was already produced, and the addition of the rims had but little influence. It follows, from this observation, that rims may be employed with more advantage upon the floats of wheels which move in an indefinite fluid, than under any other circumstances.

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* Mécanique industrielle. Tome I., p. 270 et suivantes.

CHAPTER II.

ON THE RESISTANCE OF WATER.

ARTICLE FIRST.

In a large Bed.

Difference between resistance and impulse.

254. All the authors who have, since Newton, given their attention to the motion of solids in fluids, have supposed that the effort required to retain a body struck by a fluid in which it is submerged, was equal to that which must be made to move the same body, with the same velocity, in a fluid at rest: it is this last effort to which we give the name, *resistance of fluids*, or *resistance of the medium* in which these bodies move.

Towards the end of the last century, Dubuat, having conceived doubts as to the equality of these efforts, wished to remove them by direct experiments. He resumed the square plate, which, being struck by a current of 3.182 ft. in velocity, had given him $m = 1.19$ and $m' = 0.67$ (246): he moved it with an equal velocity, in stagnant water, and he had no more than $m = 1$ and $m' = 0.43$; so that the resistance was found to be less than the impulse, in the ratio of 1.86 to 1.43. He concluded from this, that water in a state of rest is more easily divided than when it is in motion.

I shall not raise any doubt as to the exactness of an experiment, otherwise very important; but as many other experiments of different authors have not given the same results, I do not think it worth while to admit generally so considerable a difference in the two cases. The laws which resistance follow are, moreover, mainly the same as those of impulse, as we shall see.

255. The numerous observations of Borda, Bossut,

Ratio of resistance to velocity.

Beaufoy, Macneill, etc., show beyond all doubt, that in ordinary velocities, — those from two to ten or eleven and a half feet, — the resistance is proportional to the square of the velocity.

Below two feet, it diminishes less rapidly than this square, and so much the less, as the velocity is less. Dubuat first perceived and noted the cause of this — the viscosity of the water. This causes a body, moving in this fluid, to carry along in its train a certain quantity with it; but, as in other cases of the lateral communication of motion, it carries so much the less as it moves with more rapidity. It will only be, then, in small velocities that the effect of viscosity will be marked; it will diminish as the velocity increases, and will be insensible when it is great.

For velocities above eleven and a half feet, there is a distinction to be made between bodies entirely immersed and floating bodies, or those only partially immersed.

In the first, the resistance is very nearly proportional to the square of the velocity. Thus Beaufoy, having set in motion, in a great basin, a score of cubes of one foot per side, submerged, having prows and sterns of different forms, found that with velocities from 9.8 to 13.1 ft., the resistance was proportional to $v^{1.98}$; with velocities from 1.64 up to 4.92 ft., it was proportional to $v^{2.02}$; values nearly identical.*

256. For floating bodies, Bossut and Macneill have found that the exponent of v was generally a little over 2 in velocities below 9.843 ft. But the last of these authors has proved, that, for very considerable veloci-

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* Nautical and Hydraulic Experiments, by Colonel Beaufoy. 1834. However numerous and interesting may be these experiments, they can only be regarded as approximate; the great and complicated apparatus used rendered the passive resistances, foreign to the bodies, too great.

ties, it decreases notably, and so much the more, as the motion is more rapid. He took a small boat made of a thin sheet of copper, of the form used in England upon canals for great velocities, having a length of 10.17 ft., a breadth of .6889 ft. at the water line, and weighing 39.24 lbs. It was put upon an artificial canal 68.89 ft. long, 4.003 ft. wide, and 1.017 ft. deep; motion was imparted in a very convenient manner, by a weight which expressed the force of traction and consequently the resistance when the motion had become uniform. The velocities varied from 3.08 ft. to 21.4 ft., and this last is nearly the extreme term which boats and even vessels attain—nearly fifteen miles an hour. Unfortunately, the space run through was much too small, especially in great velocities, for them to be determined with sufficient exactness.* Figure 92, in which the abscissas are the velocities, and in which the ordinates express the resistances, presents the result of the experiments; we have there traced the parabola resulting from the law of the square of the velocities; the curve of resistances follows it very nearly, until towards the velocity of 10.79 ft., and then it continues sensibly in a right line. Whence we conclude, that the resistances at first increased as the squares of the velocities, and that then, beyond that of 10.79 ft., *the increase of the resistance was proportional only to that of the simple velocity.*

This considerable diminution in the increase of resistances, beyond velocities of 9.84 ft., appears to be an effect of the adhesion of the fluid particles among themselves, and of the effort which they oppose to their separation, especially when the attempt is made suddenly.

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[*] On the resistance of water to the passage of boats upon canals. By John Macneill. 1833.

We have an example of such an effort, in the phenomenon which a flat stone and even a cannon ball present, when forcibly projected upon a sheet of still water, and in a direction making but a small angle with it; notwithstanding the great quantity of action with which these projectiles are animated, they cannot surmount the resistance which the fluid opposes to its separation; and, as if repelled by it, they rise again and rebound at its surface. So also, when a boat is drawn with a great velocity, it is visibly elevated above the surface of the water, and so much the more, as the velocity is greater; it would be entirely raised, and would glide along the fluid sheet without opposing any resistance, if the velocity were infinite. Even in this case, we cannot say that the law of the square of the velocity is at fault, since the section immersed, which is one of the factors of resistance, would be nothing. Thus, in the actual state of our information, nothing as yet authorises us to assert, that, other things being equal, the resistance is not proportional to the square of the velocity. We have been dealing with bodies floating upon an indefinite fluid, and with plane surfaces; but we shall see hereafter (274) that there are very considerable anomalies when boats go with great velocities in narrow canals.

Ratio of resistance to the surface.

257. As in the impulse, if the body which moves in a fluid is thin, like a simple plate, the resistance increases in a greater ratio than the striking surface. The more considerable this is, the more the fillets driven before it have to deviate, and the greater the force we must exert against them, independent of their number.

But if the body, moved in or upon the water, has a length at least equal to one of the sides of the face which strikes the fluid, the ratio of resistances approaches that of the surfaces, and we may hold it established that there is a proportionality. The surface to be admitted for floating bodies will be the greatest transverse section of the part of the body immersed in the fluid.

Expression of resistance.

258. Consequently, the general expression of resistance, s being this section, and v the velocity, will be as that of the impulse (249), $\frac{62.43}{2g} nsv^2$, or

$$.9702nsv^2.$$

Even here, the coefficient n will be constant for all solids of the same kind, that is, for all similar solids; but it will vary with the change of form. Its determination is the great object of experiments made and to be made in this department of hydraulics; it will establish the rank which different bodies occupy in the order of least resistance.

Absolute resistance of prismatic bodies.

259. For a simple square plate, 1.06 ft. per side, Dubuat found this coefficient 1.43.

With a cube, the faces being of the same dimensions, he had no more than 1.17; Borda had previously obtained the same value. Another cube of treble size gave 1.21.

As the length or horizontal dimensions increase, n diminishes, the negative pressure on the back decreasing with the length (246). But the diminution has a limit; it is greatest, that is to say, n is at its *minimum*, when the length of the prism equals five or six times the side of the base (or rather $\sqrt{s}$); then $n=1$, and the expression of resistance is simply

$$.9702sv^2.$$

If the length still increases, n, in place of continuing to diminish, will also increase, and the resistance will be greater. The injurious effect of friction, or the adhesion to the wetted sides of the prism, which increases with its length, will more than compensate for the favorable effect of the *non-pressure.* Thus, Beaufoy having moved in water, with a velocity of 6.56 ft., three prisms with square bases, whose lengths, compared with the sides of the bases, were 10, 17.8 and 34.6, found that the resistances gave for the respective values of *n*, 1.14, 1.16 and 1.31.

The lowest limit of the values of *n* for prisms, or, more rigorously, for rectangular parallelopipeds, presenting their bases square against the action of water, will be then 1. But we descend much below this limit, by substituting for the plane base, perpendicular to the direction of the motion, inclined faces, and better still, curved surfaces; as we shall soon see.

Resistance against oblique surfaces.

260. By simply inclining the front base of a prism to the direction of motion, we diminish materially the resistance which it meets. Thus, Bossut having moved upon water a right prism, placed successively in front of it two bodies or bevels, presenting to the action of the fluid, the one a face inclined 43° and the other 25° 26′, and the respective values of *n* have been 1.02, 0.67 and 0.47. Beaufoy has also taken a prism 21.13 ft. long, having a square base 1.22 ft. per side; he moved it under the surface of the water with a velocity 7.02 ft., and he had a resistance of 81.57 lbs.; he then fitted upon it a bevel with the face inclined 9° 36′, and the resistance was no more than 35.27 lbs.; so that it was diminished in the ratio of 100 to 43.

This form of prism cut to an angle being the most simple of those which considerably reduce the resistance, will be found very often among the bodies used in navigation; and many of the boats which we see on rivers are formed of such prisms.

261. The experiments of Bossut, one of the results of which I shall shortly cite, being too little known, and appearing to involve very important considerations, I proceed to explain them with precision, and to indicate some of their consequences.*

Fig. 93. The first of the three bodies employed (A) was a right prism, with a square base, whose side *ab* = 1.745 ft., and the length *ad* 2.287 ft. There was fitted to this, to form the second (B), the prismatic prow *dce*, having the face *ce* inclined 43° 1′ to the fluid surface GH. In the third (C), the inclination *cf* was but 25° 26′. They were suitably placed in the basin of the military school at Paris, and loaded so as to have a constant depth of immersion of 1.106 ft.; then they were drawn by different weights, which caused them to run through the space of 68.89 ft. with different velocities. This done, the two truncated prisms were reversed, so that the inclined face, always placed in front, was turned upwards, in place of being turned towards the fluid, as it was at first. There were, then, five series of experiments, whose results are here given. The velocities noted are those with which the last twenty feet were traversed; the motion then being quite uniform.

BODIES used in Experiment.	VELOCITIES, THE MOVING WEIGHTS BEING			SERIES of	
	$6.967^{\text{lbs.}}$	$10.792^{\text{lbs.}}$	$12.95^{\text{lbs.}}$	velocities.	resistances.
	ft.	ft.	ft.		
A	2.1489	2.3884	2.5984	1.00	1.00
B	2.7001	2.956	3.1988	1.24	0.65
C	3.1693	3.5368	3.8944	1.48	0.455
B. reversed.	1.8635	2.044	2.2474	0.86	1.35
C. reversed.	2.0407	2.2539	2.4934	0.95	1.11
	1	2	3	4	5

On an examination, in each of the three first vertical columns, of the velocity of the five bodies moved by the same weight, we see that it follows very nearly the same law, a law which is indicated by the numbers of the fourth column.

If we consider the three velocities of the same body, drawn successively by the three weights employed, we find that the square of the velocities increases nearly as the numbers 1, 1¼ and 1½, that is to say, as the motive weights, which express the resistances. Very probably, were it not for small errors in observation, the increase would have followed exactly the above-stated ratio; and then the numbers of the last column, which indicate the inverse ratio of the square of the numbers in the preceding, will express the resistances experienced by the five bodies, that of the body A, or of the unbevelled prism, being taken for unity.

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* Nouvelles expériences sur la résistance des fluides. Edition de 1777, p. 81—86.

A comparison of the numbers of the last column shows us that the resistance of the prism bevelled at 43°, when reversed, has been much greater than that of the same prism in its first position, in the ratio of 135 to 65; more than double. The increase has been still greater for the prism bevelled at 25° 26′, since it rose from 45 to 111, or from 10 to 25.

Action of water tending to sink or raise the front of a floating body.

262. What is the cause of so great an increase in the resistance experienced by the same body, especially when the angle of incidence of the fluid, or, at least, its sine, is the same in the two positions? In the first, the fluid has a much greater facility in clearing itself after being struck; and it tends to raise the front of the prism. In the reversed position, on the contrary, it cannot escape but with great difficulty, at the bottom; it remounts the sloping face which it strikes; it tends to sink it, and its resistance is increased.

Fig. 46.

The angles of incidence of the fluid were 90°, 43° and 25°, and the respective resistances of the reversed prisms as 100, 135 and 111; thus, while the angles diminish, the resistances increase, but only to a certain limit, beyond which they diminish also: there is, then, an angle giving a *maximum* of resistance. If it be that which tends to produce the greatest sinking, it will be about 45°; indeed, when a fluid, animated with a velocity or force CB (Fig. 46), which we will make equal to 1, strikes a face MN under an angle i, the component GB, which acts upwards on this face, and tends to sink it, has for its value $\sin. i \cos. i = \frac{1}{2} \sin. 2i$; a quantity which is at its maximum when $i = 45°$. Without attaching any importance to a theory which I am far from believing to be correct, I shall still conclude that the angle tending to produce the greatest sinking will be from 40° to 50°.

For a similar reason, when the front of a floating body is disposed like *ce* (Fig. 93), and inclined about 45°, the fluid will exert upon it the greatest effect to raise it.

Raising of the prow, and immersion of boats in great velocities.

263. This raising of the front by the action of the fluid on which the body navigates, cannot be called in question; it is manifest when the velocity is considerable: thus, whale boats drawn by harpooned whales with the extreme velocity of 39 to 42 ft., have been seen to be so elevated at the prow as to show out of the water 6.56 ft. of their keel, and that when the direction of traction, considerably inclined to the horizon, had a tendency to sink it. (Macneill's *Resistance of water*, etc., p. 27.)

According to observations lately made in England, when a boat is drawn with great velocity, the prow is at first elevated and the stern depressed; but soon the latter recovers its former position, the elevation of the prow is maintained, and the boat travels in a horizontal position, and so much the less sunk in the water as it moves with the greater velocity, as we have already remarked. Thus, Mr. Russell, having moved with velocities varying from 4.43 to 29.36 ft. a small skiff, which, in a state of rest, had a draught of .223 ft., saw the immersion diminish quite gradually from .2165 to .1246 ft. (*Annales des ponts et chaussées*. Tome XIV., p. 156). The raising of the front part facilitates the clearing of the water, which tends to pile up before this part; it favors the immersion of the body, and consequently the diminution of the immersed section: under this twofold relation, so that it is unaccompanied by any sinking in the rear, it cannot but diminish the resistance. From this we see how important to the art of naval constructions is a knowledge of the form and angle best fitted to reduce the resistance.

Effect of prows.

264. Should the prow of a body serving for navigation consist of but one face only inclined and rising above the surface fluid, usually this will be a solid which,

presenting an edge to the water, will cleave and divide it; gliding then on these inclined faces, it will oppose but a very small resistance.

A series of experiments made by Bossut, Dalembert and Condorcet, enables us to appreciate the good effects of such prows, when they consist of but two plane faces, united in the form of a wedge. I cite some of these experiments. To a rectangular parallelopiped 4.26 ft. long, 2.13 broad and 2.75 deep, was fitted a series of prows, whose horizontal sections were isosceles triangles, and whose front angles were more and more acute. In this state, the body was conveniently placed in a great basin, immersed to the depth of 2.13 ft.; it was drawn by different weights, and when the motion was uniform, the time required to run 52.49 ft. was noted. The inverse ratio of the square of the times, which was the direct ratio of the square of the velocities, and very nearly that of the resistances, is indicated in the last column of the annexed table. The resistance of the right prism (without any prow) being taken for the unit, the numbers of this column express the resistances corresponding to the different prows; and they are very nearly the same as the respective values of *n*.

Angle of the prow	Series of resistances.
180°	1.00
156°	0.96
132°	0.85
108°	0.69
84°	0.54
60°	0.44
36°	0.41
12°	0.40

Effect of the stern.

265. Cuneiform sterns, placed in the rear of the prisms of Bossut, also diminished the resistance, as we see in the adjoining table; but they diminish it much less than the prows; thus the stern of 24° reduced it but 16 per cent., while the same angle for the prow caused a reduction of more than 59.

Angle of the stern	Series of resistances.
180°	1.00
96°	0.89
48°	0.86
24°	0.84

The reason is plain: the sterns diminish the *non-pressure* resulting from the vacuum usually made behind bodies which move in a fluid (244); but this *non-pressure* in the rear is much less efficient than the pressure against the front; and this it is which in a great measure is destroyed by the prow.

Effect and advantage of curved surfaces.

266. We diminish still more the resistance experienced by floating bodies, by forming the prow, the stern, and even the sides, with curved surfaces. For the resistance upon the curved parts is much less than it would be upon an assemblage of planes substituted for them (in the same manner as we should substitute for the circumference of the circle, the sides of an inscribed polygon).

An experiment made by Borda proves the advantage of these curved surfaces. He took three right prisms; the base of the first was the equilateral triangle ABC; of the second, the semi-ellipse AMCM'B; and of the third, the mixtilinear triangle, whose two sides were arcs of a circle of 60° each. He caused these prisms to move in air, with the same velocity, placing forward 1st, the plane face corresponding to the edge AB; 2d, the apex of the plane angle corresponding to the point C; 3d, the semi-ellipse; 4th, finally, the summit of the mixtilinear angle; the four resistances found were respectively as the numbers 100, 52, 43 and 39.

These results of moving the bodies in air would probably have been the same, if the movement had been made in water. In fact, Beaufoy, having placed in this fluid and drawn, with a velocity of 9 ft., the prismatic body whose base is represented by the figure 94, and whose height was equal to BC=1 ft., had a resistance of 25.86 lbs. with the prow of plane faces BAF, and 19.137 lbs. with that of the curved face BMANF; Fig. 94.

the diminution was very nearly that of the experiment of Borda, following the ratio of 52 to 38½.

267. There has been an ineffectual attempt, up to the present time, to express analytically the resistance experienced by curved surfaces. Newton, who first attempted such an expression, after having established the resistance for plane surfaces as proportional to the square of the sine of incidence of the fluid, assumed it to be so for the differential elements of curved surfaces, regarding them as an assemblage of infinitely small planes; and with this hypothesis, he determined the resistance for different bodies, terminated by such surfaces. For nearly a century, all mathematicians adopted this basis of calculation. At length, Borda, after having made, in 1763, many experiments upon the resistance which different bodies experienced when moved, either in water or air, has shown that their results were in opposition to this theory, and that it could not be maintained. This theory had indicated, for the four cases of the experiment of Borda, resistances decreasing as the numbers . . . 100, 52, 50 and 49.
Observation has given . . . . 100, 52, 43 and 39.

So that the resistances derived by calculation are much too small for plane surfaces, and notably too great for curved surfaces. Other observations have conducted to the same conclusion: for example, in the experiments of Bossut upon truncated prisms (260), the resistances have been as . . 100, 65 and 45;
the old theory would have given . . . . 100, 46 and 18;
so that in these days, it is entirely abandoned.

Resistance of the sphere.

268. In the application to the sphere, Newton concluded that the resistance, for this solid, was the half of that experienced by its great circle. Dubuat, however, has observed that it was only 0.35 of it (*Principes d'hydraulique*, tome II., p. 263), and the experiments of Beaufoy confirm his remark. This last author took a ball 1.128 ft. in diameter, and moved it, at sufficient depth under water, with velocities from 2.001 ft. to 12.01 ft.; and the resistances which it met indicate for n values comprised between 0.402 and 0.364, or a mean of 0.383; for a thin circular plate of the same diameter, he had 1.12; thus that of the sphere would be 0.342.

Beaufoy also made, by means of this same solid, some other very interesting observations.

For a cylinder having the same diameter of 1.128 ft., and 1 ft. long, he found $n =$ 1.030

The sphere gave 0.383

He divided it in halves; one of them put in front of the cylinder, like a prow, reduced the coefficient to . . 0.328

Put in the rear, it ascended up to 0.888

Finally, one half being placed in front, and the other in the rear, he only had 0.276

And even, in one experiment 0.230

We see, by this example:

1st. That in lengthening a body, we diminish notably its resistance: here the prolonging of the sphere diminished it in the ratio of 383 to 276, or of . . . 100 to 72;

2d. How great is the effect of prows and sterns: they reduced the resistance from 1030 to 276, or from 100 to 27; that is to say, nearly a quarter; and possibly more than a quarter, from . . . . . . 100 to 22;

3d. That the reduction due to the prow alone was 100 to 31;

4th. And that the reduction due to the stern alone was only . . . . 100 to 86.

The prow should be less acute than the stern.

269. Seeing the superiority of prows over sterns, and knowing moreover that the first diminish the resistance in the ratio of their acuteness (264), it would seem, if we had to move in the water a body having an obtuse extremity, and one with a sharper end, that the last would be the best to put in front; experiment teaches us, however, and in a positive manner, that such is not the case. Thus, the prism represented in figure 94 gave $n = 0.327$, when the angle A was in front; and it indicated 0.430, and consequently experienced a much greater resistance, in the proportion of 100 to 132, when the angle D, much the most acute, passed first.

This fact has been known for a long time. Chapmann, a celebrated Swedish engineer, demonstrated it by many experiments, in his treatise on the construction of vessels. In one of them, he took two cones, which he joined base to base, as indicated in figure 95; and when the most acute angle C was in front, the resistance was greater, in the ratio of 100 to 224. Consequently, in a vessel, it would be an advantage to place near its front the greatest transverse section, or *midship-frame*, in technical language. Finally, nature has furnished examples on this subject, in the form which she has given fish; they are larger at the head than at the tail. Fig. 95.

Resistance experienced by vessels.

270. What form, what curvature, is to be given to the different parts of a floating body, so that it shall experience the least resistance as it moves in the water? To answer this question would be to resolve the problem of the *solid of least resistance*, so important for naval architecture. But theory, in its actual state, cannot lead us to its solution (267); and experiment, which we have so much interest in consulting, has not as yet determined it in a direct and precise manner.

I know of but one observation giving immediately the resistance of a body similar to the great vessels used in navigation; it is that made by Bossut with a model of one of our ships of the line, which was 6.397 ft. long and 1.738 ft. wide; and yet, as the author has not given the area of the portion of midships immersed, we cannot determine all the circumstances of resistance. Still, as a right prism, having this midship-frame for its base, and the same length, has been submitted to experiment under the same draught of water, we can judge of it by comparison. Taking, then, for the unit, the resistance of the prism, it was found, in six experiments, that that of the vessel varied from 0.219 to 0.178, or at a mean of .20, that is to say, a fifth part of the other. As the prism was rounded upon its lateral faces, it is probable that the coefficient n of the resistance was below 1, and consequently, that of the vessel would have been less than 0.20. It would have been still less for brigs, store ships, and other vessels, prime sailers, which go with great velocity, and so experience but little resistance.

It is said that for certain boats n would descend as low as from 0.17 to 0.16. This would probably be the case with the *wave boat* of Mr. Russell, which, upon the Union Canal in Scotland, experienced a resistance much less than that of other fast boats. It would have been thus for the boats of the engineer Burden, of which the part immersed in water consisted of two or three bodies, spindle-formed, which would only experience a slight resistance; boats which navigate, it is said, the great rivers of America, with a velocity of eight leagues per hour.

There are, then, ships and boats for which the resistance of the medium in which they sail is only a sixth part of that experienced by a rectangular parallelopiped of the same length, breadth and draught of water; it would be nearly expressed by

$$9^{k}.sv^{2}, \text{ for metres; or } 0.17sv^{2}, \text{ for feet.}$$

Form of vessels.

271. More ample details concerning vessels and boats are foreign to our purpose; they belong exclusively to the department of engineers charged with their construction.[*] I confine myself to giving an idea of the forms which seem to have been adopted.

I shall first remark, that the least resistance is not the sole aim to be attained, especially in great ships. It is also necessary to secure their *stability*, that is to say, the means of resisting the forces which tend to incline them beyond a certain limit, and to upset them: they must also be rendered fit to carry sails, heavy ordnance, etc.; to resist the agitations of the sea; they must have a capacity to hold many passengers or much freight; they must often be made to navigate in shallow water; finally, they should yield readily to the action of the helm. Thus their forms will be different, according to the object for which they are designed; though, for nearly all, it is desirable that, under the same motive power, with the same wind, they may attain a sufficiently great velocity, and consequently, that they should offer but a small resistance.

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[*] See the great Dictionnaire de Marine, par Vial de Clairbois.

For this purpose, and to satisfy the idea that ships, even those of war, should be made suited to cleave the water, towards the middle of the last century, they were narrowed and considerably reduced on approaching the prow and bottom; the same course was taken even towards the stern, to facilitate the clearing of the water, and as if to equilibrate the two halves of the ship; we have the form indicated (Fig. 96) for the sections of a ship of the line constructed at that period. Fig. 96.

Afterwards, these contractions were somewhat reduced, and we have the form given by Fig. 97, and which is now most generally adopted in Europe. Fig. 97. We have only represented the bottom, or that part which lies under water; that which is above, or the top sides, is often of a great height, as we may judge by Fig. 96. At A is the elevation of the ship: we there see the different parts, or frames of carpentry: *ab* is the keel, *bc* the stem, *ad* the stern, and M the midship-frame; $1'$, $2'$, $3'$ and $4'$ are some forward half frames; 1, 2, 3, 4 and 5 are those belonging aft: these last are seen on the left side of the vertical projection B; the first are on the right. In the elevation A, ee' represents the load water line, ff', gg', hh' are the water lines, or the intersections of the bottom with horizontal planes, intersections made at $\frac{2}{5}$, $\frac{3}{5}$ and $\frac{4}{5}$ of the immersed part. We see them marked with the same letters on the horizontal projection, or rather, the half horizontal projection C: they show the narrowings of the ship towards its extremities; and they are so much the greater as they are found nearer the keel.

In recent constructions, we place midships more forward; the water lines are quite convex towards the prow, and they terminate nearly in a right line alongside of the stern: we give more rake, curvature and salience to the stem, and a greater draught of water to the rear. All these dispositions are seen in Fig. 98, which Fig. 98.

represents an elevation and plan of an American schooner, a vessel eminently adapted for speed.

I shall not speak of floating bodies used for interior navigation, more than to say, that to swift boats, those which make four or more leagues an hour, is given an elongated form, for example, 69 ft. long by 5.25 ft. wide, and tapering very much at the prow. (See on this subject a memoir of Mr. Russell, in the *Annales des ponts et chaussées*, tome XIV. 1837.)

ARTICLE SECOND.

Resistance in a narrow Canal.

We have two cases to distinguish: that of boats designed to carry merchandise with small velocity, and that of boats built to carry passengers with great speed.

Resistance in small velocities.

272. When a barge, or any floating body in general, is moved in a canal whose width does not exceed four or five times its own, it has a peculiar obstacle to surmount. The water which is in front, urged by it, will be elevated against this front; falling then from this height, it tends to escape along the sides, as in the case of an indefinite fluid. But it cannot accomplish this with the same facility and readiness; the body urges and carries forward with it a portion, the more considerable, as the interval between its sides and those of the canal is reduced; whence a greater force is required to move with the same velocity.

Bossut also made numerous and interesting experiments to determine this force. In one of them, he provided a prism with a square base, and each side 2.13 ft.; its length was 6.46 ft., and he sunk it one half in the water. It was moved in a large basin, with a velocity of 2.756 ft., without contracting the intervals, which may there be regarded as indefinite; and there was a resistance of 17.263 lbs., which we will represent by 1, in the annexed table. Then, by means of great plank enclosures, the intervals between the bottom and sides were gradually diminished, and reduced to the dimensions noted in the two first columns; there resulted the resistances indicated in the last column.

INTERVAL		RATIO of resistances.
at each side.	at bottom.	
feet.	feet.	
indefinite	indefinite	1.00
indefinite	1.355	1.10
indefinite	0.2952	1.15
2.031	0.3116	1.52
0.7086	0.2788	2.26
0.2001	0.2788	3.15

Dubuat, analysing the different experiments of Bossut, found that even here the resistance increased as the square of the velocities; that it depended neither upon the form of the canal, nor that of the floating body, but solely upon the ratio between their sections. Calling c that of the canal, s that of the portion of the prism plunged in the water, P the resistance which this prism experiences in an indefinite fluid, and P′ that experienced in the canal, he found that we have, with sufficient exactness for his experiments,

$$P' = P \frac{8.46}{\frac{c}{s} + 2}.$$

This relation indicates, that when the section of the canal is 6.46 times greater than that of the prism, it moves as in an indefinite fluid. But in order that this may be the case, the width of the canal should be at least four times that of the body.

In adapting to the bases of the right prisms, which have served to make the experiments, angular prows, we diminish the resistance, but much less than in an indefinite fluid, and so much the less as the canal is more narrow; the body was compelled to push before it very nearly the same quantity of water, whatever might be the form of its front part. Designating by q the ratio between the resistance of the prism with a prow to that of the prism without a prow, moved, both of them, in an indefinite fluid, and calling P″ the effective resistance in the canal, Dubuat was led by analogy to admit

$$P''=P'\left\{1-0.183\,(1-q)\left(\frac{c}{s}-1\right)\right\}.$$

Resistance for the boats of the canal of Languedoc.

Let us compare the result of this formula with that of an experiment made upon the greatest of our canals, the canal of Languedoc, by M. Maguès, chief engineer of the canal, and by myself.

In a place, near Toulouse, where the mean section of the canal was 285.79 sq. ft., a barge, like to the freight boats, loaded with 108 tons, with a transverse section of 73.627 sq. ft., being drawn by two horses, ran 12060.6 ft. in 1^{h} 15^{m}; that is, 2.6798 ft. per second: a dynamometer, fixed, one part to the boat, the other to the trace, marked as a mean of the small oscillations, 264.57 lbs., reduction being made to direct traction.

We have, in this case, $v=2.6798$ ft.; $c=285.79$ sq. ft.; $s=73.627$ sq. ft.; and consequently, $\frac{c}{s}=3.8815$. We will make $q=0.4$; it is, I believe, the smallest value which can be assigned to it in these barges, which are great boxes, fitted with prows and sterns very obtuse, though with curved faces. As for n in the value of P, it is 1. Consequently, we have

$$P=1\times 73.627\times .0155366\,(2.6788)^2\times 62.429^{\text{lbs.}}=512.86^{\text{lbs.}}$$

$$P'=512.86\,\frac{8.46}{3.8815+2}=737.71^{\text{lbs.}}$$

$$P''=737.71\,\{1-0.183\,(1-0.4)\,(3.8815-1)\}=504.31^{\text{lbs.}}$$

Thus the result of calculation is nearly double that of experiment: it is 1.91 times greater. At certain periods, it is true, it was less; but it did not descend below 1.60. So that the formula of Dubuat is not applicable to the barges of the canal of Languedoc.

273. Every day's experience gives a much less resistance.

I state the facts, and first the principal dimensions of the canal and of the barges.

The profile of the mass of water contained in a canal experiences small variations, according to local causes and the droughts of the seasons. As a mean term, we may admit:

Width of the fluid surface,		$57.415^{\text{ft.}}$
Width at bottom,		31.168
Depth of water,		6.5618
Thus the mean section will be		$290.63^{\text{sq. ft.}}$
The great barges are about		
In length,		$85.303^{\text{ft.}}$
The mean width	on deck,	17.388
	at bottom,	14.60
Under a full load of		120 tons,
The draught, at the maximum allowed, is		$5.249^{\text{ft.}}$
The section below the water line is then		$80.732^{\text{sq. ft.}}$

Such a barge, drawn by two common horses, goes from Toulouse to Agde, running a length of 743970 ft., in seven days: we reckon, per diem, 14 hours of travel, of which 3 are employed in the passage of locks, and 11 for the effective travel. So that in 77 hours they go 743970 ft., or 731424 ft., deducting 12545 ft., the length of 84 locks passed: thus, the horse drawing makes 9498.2 ft. per hour, or 2.6378 ft. per second, as his mean velocity; it would only be 2.1096 ft., including the rest at the locks.

In cases of urgency, the journey is accomplished in six days; the time of the day's work is increased; and the two horses drawing the barge travel 123995 ft. per day.

The force they exert upon the traces is very near 286.62 lbs., or 264.57 lbs. in the direction of motion.

With a velocity of 2.5247 ft., and the above dimensions, the formula of Dubuat would indicate 582 lbs., a quantity more than double.

Correcting this formula by the observation which gives us but 264.57 lbs., causing the correction to be made in the multiplier 0.183, and comprising q in its value, the resistance of barges which navigate the canal of Languedoc will be expressed by $P' \left\{1. - 0.26 \left(\frac{c}{s} - 1\right)\right\}$; or, with more simplicity and sufficient exactness,

$$2.6639 \frac{s^2 v^2}{c + 2s} \text{ lbs.}$$

Resistance in great velocities

274. When the velocity of a boat becomes considerable, the movements of the water in the canal, which were hitherto scarcely appreciable, and whose effects were without marked influence, now manifest themselves, and exert a very great influence.

I will point out the most influential of these movements. When a boat goes in a narrow canal with a great velocity, from 13.12 to 16.4 ft., the fluid which it displaces at each instant rises upon its sides, and produces quite a swell there. If we stop the boat suddenly, we see it very distinctly advance forward, and form across the canal a projecting wave, which advances upon the fluid sheet, even to a distance of a mile and one fifth, with a velocity always uniform, independent of that which the boat had, and which would seem to depend solely upon the depth of the water in the canal. Numerous experiments have shown that this velocity is due to a height nearly equal to half the depth of water, increased by the height of the wave: so that if h represent the mean depth, thus increased, the velocity will be $5.6694 \sqrt{h}$. M. Vuigner thinks, that if the water of the canal has a motion of its own, we should add it to or subtract it from the above value, according as we descend or ascend the current.

When the velocity of the boat is small, from 3.28 to 6.56 ft., the displaced fluid, having time to spread over a great surface, will produce only a slight swell; the wave resulting is scarcely appreciable; it advances without marked action, and is soon far away from the body which produced it. We have then the condition treated of in the preceding number; and the resistance, subject to the laws there indicated, increases very nearly in proportion to the square of the velocity.

If the velocity should reach more than one half or two thirds that of the wave, the swell is greatest; the series of waves which it occasions forms a fluid roll, holding itself constantly in front of the boat, and opposing a great resistance to its course. Astern, instead of a slight wake, there is a great depression; the water of the adjacent sides rushes in to fill up the hollow, and there produces tumultuous motions. The lateral currents, starting at the sides of the prow of the boat, in the form of great furrows, diverging and directed downwards, strike upon the banks of the canal, and break against them. As the velocity of the boat increases and approaches that of the wave, the movements which we have been considering increase considerably. Boats have been seen, being raised at the prow by the wave which had just passed them, and which was as high as the prow itself, to incline very sharply to the horizon, to present a great surface to the shock of the fluid, and to experience an insurmountable resistance. Astern, a void is formed, into which the adjacent water rushes violently, and rises again in foam. The lateral currents are borne violently against the banks, and rend them. Could we shoot by this critical position, and by an increase of velocity be placed a little beyond the wave, the circumstances would be changed. The boat would be found as it were descending an inclined plane: gravity lending its help, would diminish the resistance there, or rather, the force of traction; and we should have the singular phenomenon of such a force really diminished in consequence of the increase of velocity.

If the velocity, still increasing, should attain about one quarter more than that of the wave, the state of things would then be entirely different. The swell produced at the sides of the boat by the fluid which it displaces at each step, forms as it were a wave, upon the back of which it seems to be borne; the bow and stern are disengaged. The wave which this swell tends to produce, being left astern, goes quietly to fill up the depression which is made there; the lateral currents merely sweep by the foot of the banks with a regular movement, and in front we only find a plane and tranquil surface. In consequence of this position, and of the immersion of the boat, the resistance is diminished, and it increases in a less ratio than the square of the velocities: we are, then, brought to the condition of the experiments of Macneill (255), where a floating body was moved with a great velocity.

The curve MNOPQ (Fig. 92 *bis*), which Mr. Russell traced, as the result of one of his experiments, shows us the law which the resistances follow in the different cases we have discussed. We there see how, in the branch NO, when the velocity of the boat is nearly that of the wave, the resistance increases, as if the boat had actually to be drawn, to enable it to rise upon an inclined plane; while in the branch OQ, the descending boat requires a less extraneous force to move it, even with a greater velocity.

Ratio to be established between the velocity of a boat and the depth of the canal.

275. From what has been said, the velocity of a boat should always bear an established ratio to that of the wave, and consequently to the depth of the water in the canal, this last velocity depending only upon this depth.

Let us take, for example, the canal of Languedoc, where the depth is generally 6.56 ft. The velocity of the wave, neglecting its small height above the fluid surface, will be then, $5.6694 \sqrt{6.5618} = 14.522$ ft. Thus, the influence which the wave will exert upon the resistances to be experienced by the boats, will only take place between the velocities above one half and below five fourths of 14.522 ft.; that is, between the velocities of 8.20 and 18.05 ft., or even those of 9.84 and 16.40 ft. We must then avoid being found within these limits. The freight boats, which have not even 3.28 ft., are entirely outside the limit. As for the mail boats, which make from ten to eleven thousand metres per hour, and whose velocity is consequently from 9.12 to 10.04 ft., they attain the inferior limit; they may exceed it a little without serious consequences; but they cannot exceed it much, making say three or three and a half leagues per hour, without experiencing a much greater resistance, and without increasing the number of towing horses: even with such increase, they can never accomplish four leagues. It would be necessary for them to exceed five, but with horses, such speed could not be maintained. A steamboat of sufficient power would be required.

So also, the depth of a canal to be established should bear a fixed ratio to the velocity of the motive powers to be used on it. If that of horses, for example, as the greatest speed with relays is 4 leagues or 52494 ft. the the hour, or 14.5671 ft. per second; and as, on the other hand, we must exceed by about a quarter the velocity of the wave, we shall have to fulfil the condition expressed by the equation $14.5671 = 1.25 \times 5.6729 \sqrt{h}$; whence $h = 4.22$ ft. Thus, the depth to be given to the canal will be 4.22 ft. A greater than this would be prejudicial. This assertion may seem paradoxical to those who know, and all watermen know, that, in general, the navigation is easier the deeper the water; but what is true for ordinary boats is not so for those of great speed.

Great velocities damage the banks less.

276. They present also a very extraordinary fact, to which we have already alluded; which is that, with their very great velocity, they occasion much less injury to the banks than with a velocity a little below $5.4339\sqrt{h}$. I have witnessed this fact upon the canal of Ourcq, near Paris, which is 29.528 ft. wide at the surface, by 4.265 ft. in mean depth; the boat used was made in England, such as are in use there, and was 74.476 ft. long and 6.1024 ft. in its greatest width; the velocity of the wave was estimated at 12.139 ft. After having moved the boat with a velocity of 14.764 ft., it was stopped, and a strong, well defined wave was seen to advance in front; when it had reached a certain distance, the boat was put to its former speed; on approaching the wave, the water piled up and bubbled much in front; it was precipitated in the rear, and rose again at different intervals in sharp crests; the lateral currents would strike violently against the banks, and shoot up far above them. But as soon as, with great striving of the horses, we had passed by the wave, we advanced then upon tranquil water; the prow cleft its way easily, and the waves in the rear presented nothing of an extraordinary character. John Russell, the first who has described the existence of this wave, and verified its effects, insists upon this important circumstance; and he expressly declares, that he has seen, in great velocities, the rear wave, so destructive to the sides of the channel, and so dangerous in the navigation of shallow water, disappear entirely.

It is, then, very advantageous to give boats a velocity much greater than that of the wave; but this is not always easy. When we gradually increase the velocity, the waves become more and more strong; they accumulate more and more in front, and the resistance which they oppose to motion becomes so great on approaching the velocity of $5.4339 \sqrt{h}$, that the horses frequently cannot surmount it. To accomplish this end, we should go slowly at first; the waves are then small and are quickly passed; then the velocity should be suddenly increased, by putting the horses to a smart gallop; the waves have not time to enlarge and pile up, and we can get beyond them without great difficulty; beyond this, the towing is easy, provided the horses can maintain the suitable velocity of $6.883 \sqrt{h}$.

I refer to the important memoir of John Russell for other details upon the motion of swift boats, and upon waves in front, as well as those in the rear.* I should observe, that, however interesting may be the facts there reported, and however ingenious or probable may be most of the inferences which the author has drawn from them, this branch of science has but just had its birth, and is a long way from being perfected.

SUB-SECTION II.

HYDRAULIC MACHINES.

Machines divided into two classes.

277. A machine is an assemblage of pieces, levers or wheels, so connected that motion, being impressed upon the first, is transmitted from one to the other, even to the last, which produces a certain effect; that is to say, which accomplishes a certain work, such as raising a body of water or any other body to a given height; such as grinding a certain quantity of grain, spinning a certain quantity of wool, &c. When the machine is

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* Researches in Hydrodynamics. By John Scott Russell. Edinburgh, 1837.

moved by a current of water, it is a *hydraulic machine*. The current is the immediate motor, and gravity, from which it derives its force, whether acting by its weight or by its impulse, is the prime motor.

In all machines, each piece may be regarded as the motor of that immediately following it. It may be considered as a separate machine, receiving movement from that which precedes it, and imparting the same to that which follows it. We shall only consider, in this subsection, the piece which receives directly the action of the motor, and which M. Poncelet has accordingly named the *recipient;* it alone will constitute our hydraulic machine.

There are two kinds of them; the one endowed with a *motion of rotation*, and the other, with an *alternate motion*. *Hydraulic wheels*, comprising *turbines* and *machines* of reäction, form the first class; in the second, we have to consider *water-pressure machines* and the *hydraulic ram*. There are a great many other machines which have been projected, and even executed, but as they are not in common use, they need not be discussed in a manual of this sort. The reason which causes us to pass them by in silence, induces us to dwell more particularly upon those in general use.

Before passing to the determination of the effect which machines, or rather, that which a water-course, through their intermediate action, produces, we will, in the first chapter, fix upon the acceptation to be given to the terms *motor* and *effect*, and establish the expression of their magnitude.

CHAPTER I.

MOTORS AND THEIR EFFECTS.

ARTICLE FIRST.

Motors.

Force of motors.

278. When a motive power, or *continuous* cause of motion, acts upon a machine, it exerts upon the part to which it is applied, a pressure or effort; and while exerting it, it advances in the direction of this effort, or in a direction in which it may be led, and thus runs through a certain space. Suppose, for example, taking the most simple case, that a horse, drawing a heavy weight along a horizontal road, passes over a certain distance with a uniform motion; the effort he exerts, being constant, will be measured by a dynamometer conveniently placed between him and the body. Let K be the number of pounds indicated by the instrument, and L′ the length of the road passed; the product KL′ of the effort by the length of trip, which is but the sum of the efforts made, will manifestly be the *quantity of action* developed by the motive power in this operation; it will be, using the terms adopted by MM. Poncelet and Coriolis, the *mechanical work*, or simply the work done.

Naturally, the more work done in a given time by a motive power, the greater force it has; thus, the greater the body of water raised by a machine the same height in the same time, the greater is its power. Taking the common unit of time, the *dynamic force*, or simply the *force* of a motor, will be, with us, the quantity of action which it shall develop in one second; in fine, *it will be the effort*, multiplied by the velocity; for in $1''$, $L'=v$ and $KL'=Kv$.

279. Let us return to the above example. Suppose the horse detached, and that, in his place, at the end of the trace, (which we will suppose to be without weight, and of a length a little exceeding L',) we fix a weight P equal to K, which, being passed over a pulley, descends into a vertical pit of a depth H equal to L'. The weight, excepting the first instants of motion, exerting an effort like to that of the horse, experiencing a like resistance on the part of the body to be drawn, will descend in the pit with a uniform motion, as the weight of a clock descends, and it will reach the bottom at the same instant that the horse would arrive at the end of the course L'. Thus, in both cases, the body drawn will pass through the same space and with the same velocity. The weight P has then effected all that the horse has done; as a moving power, it is identical, and its quantity of action PH, equal to KL' and developed in the same time, being reduced to the second, will also express the dynamic force of the horse.

In general, for every motive power, we may substitute a weight, which, descending from a certain height, shall measure its force. Its expression, PH or $P^{p}H^{ft}$, and, generally, the product of the number of pounds by a number of feet, occurring often in calculations relating to machines, we will designate them by the exponent p.f or pf; thus we write PH^{pf}.

The expression of the force of motors has been the object of the special consideration of many authors, among others, of Smeaton, Coulomb, Carnot, Navier, of MM. Poncelet and Coriolis. Before confining myself to ideas strictly necessary for understanding what I may have to say upon hydraulic machines, I refer to their works.*

I would merely remark, that the expression PH, last called the *quantity of action*, or *work*, has been named *moving force* by Euler (*Académie de Berlin*, 1751, pp. 272 and 282), *mechanic power* by Smeaton, *momentum of activity* by Carnot. I have thought best to preserve the word *force*, generally used in the arts, where, without any inconvenience, we speak of the force of a current of water, of a machine, of a horse. I have joined the adjective *dynamic*, to distinguish it from the force properly so called, or from the *statical* force, which is simply an effort or a weight. Moreover, "*force dynamique*" is the simple translation of the "*mechanic power*" of Smeaton, and it is synonymous with the "*moving force*" of Euler.

Force of a current of water.

280. The dynamic force of a current will then be expressed by PH^{pf} in 1″. As H^{f} is here independent of the time, P^{p} will be the weight of water carried by the current in 1″. H is the height of the fall of fluid, whether real or fictitious; it is real, when it in reality falls from the height H; it is fictitious, when it acts by virtue of a velocity due to such a height. In estimating the force of a current of water, H is the difference of level between the fluid surface in the upper reach, a little above the fall, and in the lower reach, a little below the same.

The force of the current, depending only upon the magnitude of the product $P \times H$, remains always the same, so long as it undergoes no change, whatever otherwise the respective magnitudes of the two factors P and H may prove to be; thus, we may double one; but we must then diminish the other one half.

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* Smeaton, Experimental Examination of the Quantity of Mechanic Power. Coulomb, Expériences destinées à déterminer la quantité d'action que les hommes peuvent fournir par leur travail journalier. Carnot, Principes de l'equilibre et du mouvement. Navier, Additions à l'Architecture hydraulique de Bélidor, tome I., p. 376. Poncelet, Mécanique industrielle, tome I., p. 46. Coriolis, Du calcul de l'effet des machines.

An example given of a case which refers most directly to our object, will enable us to see the truth of this principle. Let there be, in any place, a quantity of water P, falling from an elevation H; we may there establish two machines, equal in all respects, each having H for the height, and receiving $\frac{1}{2}$ P of the water. Let there be elsewhere a course of water, furnishing $\frac{1}{2}$ P, but with 2 H of fall; we may establish there two machines exactly similar to the two first; they will lie one beneath the other, while, in the preceding locality, they were alongside of each other: this will be the only difference; in other respects, they will be similar, and the two streams will have evidently the same dynamic force; that is to say, each will be capable of moving an equal number of equal machines. This example shows also that the force of a course of water is proportional to H; the greater the height of the fall, the more power will it have to move the machines; it is also evidently proportional to P; whence we conclude, *à priori*, that it is represented by PH.

In the case when the water-course does not present a real fall, the fictitious fall will be $.0155v^2$, v being the mean velocity of the current; and if s is its section, we shall have $P = 62.429sv$; thus, the dynamic force Pv (278), will also be expressed by $.96994sv^3$; it will be proportional to the cube of the velocity.

Animal power.

281. That which a current of water can do for one hour, it may do for an indefinite period. It is not so with animal power, with man, the horse, &c.; they can only exert a certain effort, with a certain velocity, during a limited time; after which, they are obliged to take rest, which enables them to develop anew an equal quantity of action. Their labor being renewed periodically each day of twenty four hours, the duration of action, in this space of time, should be taken into consideration; let T be this duration, expressed in seconds; K being always the effort exerted by the acting force, and v its velocity, KvT will be the *daily quantity of action*.

For example, a common-sized horse, drawing a cart on a horizontal road, or a boat on a canal, exerts an effort of from 99 to 143 pounds, going with the respective velocity of 2.6 ft. and 3.6 ft., and can travel thus for 10 hours, or 36000″ in the 24 hours; so that his mean day's work will be $121^{\text{lbs.}} \times 3.1^{\text{ft.}} \times 36000'' = 13503600^{\text{pf.}}$; in round numbers, $13500000^{\text{pf.}}$

Horse-power of steam engines.

282. The horse being the moving power most commonly known, is usually taken, in the mechanical arts, for the term of comparison, to express the force of motors. That of this animal varies, it is true, according to the muscular power and weight of individuals, and that between very wide limits. But in the series of terms, we may adopt one as a standard of comparison; thus, our constructors of steam engines, like the English, take for the horse-power, that of a horse that, travelling on a level plane, with a velocity of $3.2809^{\text{ft.}}$, shall raise a weight of $165.359^{\text{lbs.}}$ (from the bottom of a shaft, for example, by means of a cord without weight passing over a pulley). Though such a force, the idea of which was furnished by the great horses used in the breweries of England, is far superior to that of our common horses, we will still admit the expression; and we shall understand by horse-power, or, to prevent all equivocation, and employing a term now in common use, we shall understand by *cheval vapeur*, a dynamic force, or a moving power, capable of raising $165.359^{\text{lbs.}}$ a height of $3.2809^{\text{ft.}}$ in 1″; that is, 542.52^{pf} in 1″.

Consequently, to express the force of a current of water in units of this kind, we must divide PH^{pf} by $542.52 = 0.001843PH$, and this quotient will indicate the number of horse-powers which, acting at the same time, will produce an equivalent effect. If Q was the volume of water furnished by the stream in 1″, we should have P=62.429Q, and 0.11507QH, for the number of horse-powers.

ARTICLE SECOND.

Effects.

Effects. Resistances: their kinds.

283. To produce an effect upon a machine is to overcome the resistances which are continually or periodically reproduced in a direction opposed to the motion throughout its duration.

To get a correct idea of resistances and effects, as well as of their ratio to the dynamic force employed to overcome and produce them, let us take the example of a common mill, moved by a wheel with floats.

The whole force PH of the current directed upon the floats, will not be imparted to them. A part of the water P will pass through the interval between the wheel and the sides of the mill-course which conducts it, and it will exert no action. A part of the fall H, that comprised between the centre of percussion of the floats and the lower reach, will be as it were lost. A part of the motive action of the water which strikes the wheel will be destroyed, either by reason of its contraction at its issuing from the reservoir, or by the shock on its meeting the floats, as we shall see in what follows. So that there will only be a portion of the force PH which will be communicated or impressed upon them.

This *impressed force* upon the floats should first act upon the gearing designed to transmit the motion of the horizontal shaft of the wheel to the vertical spindle of the mill-stone. But before reaching it, it will already have experienced two losses against two resistances which it had to surmount, the friction upon the gudgeons of the first of these two turning shafts, and the resistance of the air to the motion of the floats. Arrived at the gearing, it is there subjected to another loss, occasioned both by the friction of the teeth of the wheel against the trundle or wallower bars, and by the shocks of these teeth against the trundles, very small, it is true, but repeated without cessation. Beside that of the wallower, the friction of the pivot of the vertical spindle upon its socket absorbs also a part of its moving action. It is only with what is left that it arrives finally at the mill-stone, and overcomes the resistance which the grain opposes to grinding. To conquer this last resistance is to produce the *useful effect*, that in view of which the machine was built; to it alone can be applied the saying of Montgolfier, "the *vis viva* is that which pays."

As to other resistances, those proceeding from friction, shocks, and the surrounding air, they do but consume, as a clear loss, a large portion of the force which the motor has impressed upon the wheel. They are produced by the various parts of the machinery; they are in some sort inherent in them, and are consequently termed the *passive resistances* of the machine, the useful effect being taken for the active resistance overcome. Though they may have been without a useful result, the force has none the less striven against them; it has none the less conquered the action which they have opposed to motion; and its total effect, which we call the *dynamic effect* of the machine, is composed both of the useful effect produced, and the quantity of action of the passive resistances.

Expression of the dynamic effect.

284. Since the resistances are continually reproduced, the greater the velocity of the machine, the greater will be its dynamic effect produced in a given

time; this effect will be in the compound ratio of the intensity of the resistances and the velocity of the machine.

We observe, moreover, that to produce an effect is only, in result, to move a body resisting motion, and to move it with a certain velocity, in a direction opposed to that of its resistance, or that which it inclines to take: such as, for example, to raise a weight vertically, since it tends to descend vertically, and to raise it with a certain velocity. Moreover, every resistance to motion is but an effort opposed to the action of the motor, and every effort is measured by a weight; so that to overcome the resistance with a determinate velocity is equivalent to raising this weight with this velocity. If, now, we designate by E the dynamic effect, by p_1 the weight representing the active resistance, by u the velocity with which it is overcome, by p_2 the weight expressing a passive resistance, and by u' the velocity with which it is surmounted, p_1u will be the useful effect produced, the sum of p_2u' will be the sum of the quantities of action opposed by the passive resistances, and we shall have

$$\mathrm{E} = p_1u + \text{sum}\ p_2u'.$$

We may admit, as we shall soon see, that all the resistances are exerted upon the same point of the machine, upon that of which u is the velocity, for example. Let then p_3 be the weight representing the sum of the partial weights or efforts, and we have $\mathrm{E} = p_3u$.

We may thus establish $\mathrm{E} = \Pi A$, A being any height, in the same manner as we admitted PH for the dynamic force. But the direct consideration of the velocity is necessary in rotatory machines, such as we shall more particularly discuss.

285. We will say of effects, what we have said of

motors (280), that their magnitude is independent of that of each of their two factors, and that it depends solely upon their product. We may easily, by gearing or by levers, render the weight to be raised a hundred or thousand times greater than it was at first; but then the velocity of raising will be a hundred or thousand times smaller, and consequently a hundred or thousand times more time will be required to raise it to the same height. Whence the adage so well known in mechanics, "that what we gain in force we lose in velocity or in time": the product of this force or weight by its velocity, must always be equal to $p_3 u$.

Since in this expression of effect, we may substitute any velocity, we may put in place of u that of the motor, which is v; but we must then substitute for p_3 a weight p, such that we may have $pv = p_3 u$; it will represent the sum of the resisting efforts, supposed reduced, according to the doctrine of moments, to what they would have had were they applied to the point of the machine where the velocity is v; and we have

$$E = pv.$$

Useful effect.

286. If, as before, we decompose the sum p of resisting efforts into two parts, of which one shall be the effort employed solely to the production of the useful effect, and the other shall comprise all the passive resistances, then, designating by p' the first, and by e the useful effect, we have

$$e = p'v.$$

Total effect equals the force impressed.

287. The motion of every machine, when it is continuous and well established, is either uniform, or, more frequently, periodical. It is periodical when, during a certain period of time, generally quite short, the velocity increases or diminishes by degrees, to diminish or increase equally afterwards, so as to return,

at the end of the period, to what it was at the commencement. We then bring back this movement to that which is entirely uniform, by taking for v, and for each of the other quantities entering into the expression of force and resistances, a mean between the values which they have in this period. Thus the movement of a machine, when it is well established, and it is so generally at the end of one or two minutes, may always be regarded as uniform. Now, uniformity cannot take place and be maintained, save when the resisting action is and remains equal to the moving action. So that when a machine moves with a uniform or periodic motion, the two opposite actions are equal, or, in other words, "*the total effect produced is equal to the force impressed upon the machine.*"

In practice, the force impressed by a motor upon a machine is simply called *force of the machine.*

The force impressed, which is the effort that the motor exerts upon the point of the machine to which it is applied, multiplied by the velocity of this point, is expressed by Kv (278). We have seen (284), that the total dynamical effect was generally expressed by p_3u: we shall, then, have

$$Kv = p_3u \ (= E).$$

For machines in motion, Euler gave the name of *moment of motion* to the product of an effort by its velocity; such are the two which form the above equation: other German authors have called this product the *dynamic moment*. Thus, when motion has become uniform, the dynamic moment of the power is equal to the dynamic moment of the resistances. The statical moments, or product of the efforts by their distances from the axis of rotation, will also be equal, since the velocities are proportional to these distances. The equality of moments is, then, one of the attributes of machines endowed with uniform motion, as well as of machines in equilibrium.

If, in the above equality, we put pv for p_3u, we have

$$K = p;$$

that is to say, that *the effort of the motor is equal to that of the resistances*, both acting at an equal distance from the axis of rotation, but in opposite directions.

Ratio of the real effect to the theoretic effect.

288. When we have to determine the effect E produced by a wheel in motion, observation readily furnishes one of its two factors, the velocity v; but it does not give the other, the sum p of the resistances, except in the particular cases where we make use of a special apparatus for its determination (292).

Nearly all these resistances, as well as the effort of the motor which we introduce in the formulæ, are but the results of calculations based upon the theories of the impulse of fluids, of friction, &c.; but in hydraulics, such results are not admitted until after they have been reduced to those of experiment. Let n be the coefficient of reduction, p_0 the sum of the resistances, or, more exactly, of the efforts of resistances determined by calculation, and K_0 the effort of the motor similarly determined; np_0 will be equal to p, and, after what has been said in the preceding number, $np_0 = nK_0$; we shall then have

$$E = np_0v = nK_0v.$$

In this expression, n is the ratio of the real effect to the theoretic effect, or, what amounts to the same, to the impressed force deduced from theory.

Ratio of the real effect to the force of the motor.

289. We have seen (283), that a motor never impresses the whole of its force PH upon a machine; so that the force really impressed, or the effect E, which is equal to it (287), will always be smaller than PH, and, in the equation

$$E = mPH,$$

m will be a fraction; it will express the ratio of the effect, or of the force impressed, to the entire force of the motor.

It is to these two equations, and principally the last, that we endeavor to reduce, as to form, those which should give the effect of different hydraulic machines.

The coefficients m and n are determined by experiments; their determination is the object of nearly all those which are made upon machines, as we shall see eventually in this chapter.

Limit of effects.

290. We have already observed that the effect E was always smaller than the force PH; it never will be greater; this amounts to the same as saying that it would be absurd to admit that an effect is greater than the cause which produces it. In order that it should be equal, or that we should have $m=1$, it would be necessary that all the action of the motor should be communicated to the machine; which is never the case. Thus, PH is the limit of dynamic effects; they never can attain it; but they may approach it, though generally they are far removed from it; very rarely is m above 0.75, once only have I seen it above 0.90; but very frequently, and in machines in common use, it is only from 0.25 to 0.20, and even below this.

Dynamic unit.

291. The general expression of dynamic effects being pv, their unit will be 1^{km} (or, $1^{\text{kil.}}$ raised $1^{\text{met.}}$ in $1''$); we shall admit no other in this work.

Some authors having judged this to be too small for the industrial arts, have substituted for it one a thousand times greater, 1000^{k} or a cubic metre of water, or a ton* raised 1^{m}, without any account of the time of raising: they have given it the name of "*dynamie*" or of "*dynamode*," and they express the effect of a machine by saying that it is of a certain number of *dynamies* in a certain time. To avoid this double indication, M. Dupin has adopted a unit of another kind, which he calls "*dyname*," and which consists of 1000 French tons raised 1^{m} in 24^{h}; it is equivalent to $11^{km}57$ or 83.748^{pf} in $1''$.

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* In the system of weights and measures, the ton of commerce is 1000^{k}: now, this weight is also designated under the name "*tonne*," especially in the great iron establishments, from analogy to the English ton, which only differs but little from it, it being $1015^{\text{k}}65$.

Most commonly, the dynamic effects are expressed in horse-powers; their number is $0.0133pv$ for an effect pv^{km} (282) or $0.0018426pv$ for an effect pv^{pf}. Quite often, when we wish to give an idea of the useful effect of a machine, we do so by indicating the number of horses which, working together, would accomplish the same results. From many observations which I have made of the modes adopted in the working of the mines, the real useful effect produced by a good draught horse, of common size, supposed to work in the open air, and with two relays, eight hours a day, is about 1200000^{km}; or, admitting the work to be without interruption, 40^{km} or 289.43^{pf} per second.*

We have seen (281) that the quantity of daily action that such a horse can develop, or the total effect which he can produce, drawing a load upon a horizontal road, is $1800000^{km} = 13024000^{pf}$. Thus, about a third of his force is absorbed by the passive resistances of the mode of work, by obliquity in traction, the effect of circular motion, frequent stops, retrograde movements, &c. &c.

Direct measure of effects.

292. It is often necessary to determine immediately, by experiment, the real force of a machine, that is to say, the dynamic

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* See, for the details, my work, "Des mines de Freyberg et de leur exploitation." 1802. Tome I., p. 223, and tome III., p. 123. Annales des Mines. 1830. Tome VII.

The mean term of my observations has been 1116000^{km} per day; but the duration of the work was never more than six hours.

In 1826, one of the officers of the mines at Freyberg, where, perhaps, the mode of working horses is best arranged, published the results of their dynamic work. After a careful examination made by M. Combes, engineer, the mean term of useful effect of fourteen horses, placed at the shafts whose depths exceed one hundred metres, would be per horse working six consecutive hours, 902712^{km}; for the five whose shaft diverged but little from the vertical, and which accordingly gave more readily the useful effect, it would have been 1015882^{km}, or $47^{km} = 270.14^{pf}$ in $1''$; this last number varied between 34 and 61^{km}, or 246.02 and 441.39^{pf}. M. Combes, after having computed the passive resistances, and added them to the useful effect, found for the total dynamic effect, in the five cases above mentioned, as a mean term, 1188117^{km}, or $55^{km} = 397.97^{pf}$ in $1''$.

Such would be the day's work accomplished by a horse working for six hours; but by prolonging the time of labor, I believe that he might reach 1400000^{km}. This effect will, then, be far inferior to that which can be attained by a horse travelling without hindrance and straight forward: we have seen very ordinary horses, drawing boats upon a canal for six consecutive days, accomplish per day 23.484 miles, each exerting an effort of 143.35 lbs., and thus develop a quantity of daily work equal to 17778810^{pf}; usually, the space run through upon a canal, per day of eleven hours, is only 19.79 miles, and the quantity of action is $2070000^{km} = 14978810^{pf}$.

effect which it can produce; whether it be to reduce to reality the results of formulæ, and deduce from them the coefficient of reduction, or to give judgment, with an exact knowledge of the case, in any question which may be raised relating to this effect.

Since an effect is represented by a weight raised vertically a certain height with a certain velocity (284), the idea which first presents itself towards obtaining its measure, consists in raising by a machine a weight up to a certain height, and to observe the time of raising, the space divided by the time giving the velocity. For a hydraulic wheel, by means of a cord wrapped round its axle, and passing over a pulley fixed at a certain height, we can cause to be raised a given weight: but how can we procure an elevation sufficient for a motion whose duration should be at least three minutes, or fifteen turns of the wheel, the motion having to attain uniformity? for it is only from the instant it does that we must start in measuring the elevation. Such a mode has been used with success for small machines, by Deparcieux, Smeaton, Poncelet, etc., and has enabled them to recognise the most remarkable laws of motion in hydraulic wheels; but for great machines used in the arts, it could not be employed.

Dynamometric Brake.

Fig. 52.

Lately, this object has been accomplished. Suppose the wheel whose effect we wish to measure is in full operation; we take off the load which the turning arbor bears, and clasp it with a brake, which we tighten, not to the point of stopping it, but till its velocity shall be reduced to what it was before. It is evident that the resistance opposed to the motion from the friction of the brake is equal to that of the load, that the dynamic effect is the same, and that to have its value, it will be sufficient to determine that of the friction. To ascertain this, instead of holding fast the extremity of the arm of the brake opposite to the arbor, we set it free, and suspend a weight upon it, which we increase until it maintains the arm in a horizontal position; it will then be in equilibrium with the friction, and will measure it. M. de Prony, who first, in 1821, made use, at least to any extent, of this brake, arranged its parts nearly as those represented in Fig. 52. It consists of two pieces of wood, clasping a portion of the arbor, the one above and the other below it, and which are pressed more or less against it by means of strong screws; a weight is suspended at the end of the upper piece. During the motion, we press the brake until this piece is raised, with its weight, and maintained in a horizontal position.

But it frequently happens that this brake cannot be immediately fitted to the arbor without some cutting away, turning, and impairing of the machine. To avoid this serious inconvenience, as well as to adopt one and the same brake for a nearly general use, as well as for a use more convenient in its nature and surer in its results, M. Egen, a German engineer, does not place it immediately upon the arbor, but upon a cast iron collar, which he fixes there at the commencement of the operation. This collar, BBB, is formed of two semicircular halves, one of which is put above and the other below the arbor A, and which are firmly united by means of screws: by means of six other screws, *b*, *b*, *b*, we centre this collar as near as possible to the axis of rotation, and then fasten it to the arbor, by driving forcibly wooden wedges in the space between them. Its lower part is clasped on its rim by a large chain, or friction band, CCC, formed of pieces of strong sheet iron, with hinged joints; its two extremities D and E, also of sheet iron, are sodered to two strong bolts passing through the arm HG of the brake, which bolts are terminated in the form of screws; these pass in the nuts *d*, which we turn with a long key, by means of which we stretch and tighten at will the pressure band. The arm or lever is a piece of wood of .492 to .656 ft. square, and 6.56, 9.84 and 13.12 ft. in length. Near its extremity, at *i*, a hook is passed through it, on which we hang the plate appointed to receive the different weights which we may employ. Between the arms and the cast iron collar is a cushion of hard wood, whose concavity, which rubs against the upper part of the collar, is lined with a strong band of bronze or bell-metal, to insure the frictions of metal against metal. The cushion, as well as the arbor, is traversed by a small funnel, through which runs, during the work, a slender stream of oil, or simply of water, upon the collar, to prevent overheating the rubbing surfaces. Fig. 99.

During the operation, after having loaded the plate with weights which, by a previous trial, are found to be convenient, and after the movement has been well established, the experimenter, laying one hand upon the arm of the brake, in order to judge of the force with which it tends to rise, and holding in the other the key of one of the nuts, loosens or tightens the friction chain, until the arm oscillates but little from the horizontal position. Moreover, there should be previously established checks, to maintain the oscillation between certain limits, and to prevent the accidents which unexpected jerks might occasion. For the details of construction of the brake, and for the mode of use, I would refer to the works of MM. Egen and Morin, who have made frequènt use of them.*

293. From the weight, Π, which maintains the brake in its horizontal position, we deduce the value of the dynamic effect, or the total action of the resistances to motion; the principal of which is the friction which takes place upon the arbor or upon the collar clasped by the brake: we have besides some passive resistances, such as the friction upon the journals of the arbor, and the resistance of the air: designating by Π' the weight representing the effort of friction, by v' the velocity of rotation of the part on which it is exerted, by $\Sigma\omega$ the sum of passive resistances, finally, by v'' the velocity of the point to which we refer them; we shall have $E = \Pi'v' + \Sigma\omega v''$. If the friction of the brake acts on one side as resistance to motion, on the other, it is the force which holds in equilibrium the weight Π borne by the arm of the brake; so that if v''' is the velocity which this weight inclines to take, and which it would take if it were drawn by the movement of rotation, we shall have $\Pi'v' = \Pi v''' = 0.1047 \Pi \mathrm{LN}$, L being the length of the arm of the lever, or the distance of the centre of rotation from the point of suspension of the weight, and N the number of turns of the arbor in a minute; for $v''' = \frac{\pi . 2\mathrm{L} . \mathrm{N}}{60} = 0.1047\mathrm{LN}$. Putting this value of $\Pi'v'$ in the above equation, it becomes

$$E = 0.1047 \Pi \mathrm{LN} + \Sigma\omega v''.$$

In the application, Π represents the weight put in the plate, plus that of the plate, plus that of the lever referred to the point of suspension. The term $\Sigma\omega v''$, the sum of the resistances which the impressed force upon the wheel was obliged to overcome before reaching the part of the machine to which the brake was applied, will be determined by the formulæ of friction, &c.; the greater it is, the more will the results of calculation affect the certainty of the result which we aimed to derive from experiment, and which can only be partially ascertained.

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* Untersuchungen über den Effect einiger in Rheinland-Westphalen bestehenden Wasserwerke. 1831. p. 56 et suiv.

Expériences sur les roues hydrauliques, etc. 1836. pp. 5—11.

If, in a machine composed of many moveable or different pieces, we establish the brake upon the first, the *recipient*, the term $\Sigma \omega v''$ will be small; if it were very small, the brake, by the other term, which properly belongs to it, $0.105 \Pi \mathrm{LN}$ ($= .000193 \Pi \mathrm{LN}$ horse-powers), would give very nearly the *total effect*, or the *force of the machine;* if we could place it upon the last piece, the *operator*, it would give the *useful effect*.

Summary of the chapter.

294. Let us recapitulate our terms and designations. In the investigation of the effects of a hydraulic machine, we shall have to distinguish:

1st. The *entire force of the motive current:* it is PH, P being the weight of water furnished by the current in one second, and H the total fall.

2d. The *force impressed* upon the machine, or simply the *force of the machine:* it is Kv, K representing the effort exerted by the current upon the part or point of this machine which it strikes, and v the velocity of this point. As soon as the motion is well established, this force is equal to the total *dynamic effect* produced, an effect whose expression is pv, p being the weight equivalent to the sum of efforts of all the resistances to motion, after each of these efforts has been reduced to what it would be if it was exerted upon the same point of the wheel as the effort K of the motor, but in an opposite direction.

3d. Finally, the *useful effect* of the machine; it will be $p'v$, being p minus the effort of passive resistances.

These forces and these effects are commonly expressed by a certain number of kilogrammes raised 1^{m} (or pounds raised 1^{f}) in $1''$; or by horse-powers.

The denominations, *force of the motor*, *force of the machine*, *useful effect produced*, seem to me as expressive as exact; they are suited to the genius of our language, as well as to that of science. It does not seem thus with the denominations now frequently used, such as *work of the motor*, *disposable work*, *useful work;* expressions to which different authors who employ them sometimes give different acceptations.

CHAPTER II.

WATER WHEELS, AND MACHINES OF ROTATION IN GENERAL.

Different kinds of wheels.

295. Formerly, in France, except in the southern provinces, there was but little use of other than *vertical wheels*, that is to say, of wheels all of whose points move in vertical planes, and whose axis of rotation is consequently horizontal. We divide them, according to the form of the part which receives immediately the action of the water, into wheels with *floats* or wheels with *buckets.* The first are further subdivided, according to the form of the floats, which may be either *plane* or *curved;* and according as they move in rectilinear or circular water-courses: in the second, wheels with buckets, we distinguish the case where they receive the water at the top from that where they take it below.

Besides these, there are *horizontal wheels*, whose axis of motion is vertical. Some are simple, and preserve the name of wheels, or of "*rouets*" in some places; others are of a more complicated construction, as the *turbines* and *reäction wheels.*

The following synoptic table presents in a body these different kinds of machines of rotation.

- WHEELS
  - Vertical . .
    - with floats, . .
      - plane, in
        - a water-course
          - rectilinear.
          - circular.
        - an indefinite fluid.
      - curved. Wheels of M. Poncelet.
    - with buckets receiving the water,
      - at summit. Overshot wheels.
      - below summit. Breast or undershot.
  - Horizontal
    - with floats, . .
      - struck by an isolated vein.
      - placed in a cylinder. Tub wheels.
      - outside cylinder. Turbines of M. Fourneyron.
    - with conduits. Turbine of M. Burdin.
    - reäction. Wheels of Segner, of Euler, &c.

We proceed to examine in succession these different wheels. But let us bear in mind that this *Treatise on Hydraulics* is not a *treatise on machines*, in which we should have to make known the details of their construction; we can only give such an idea of them as will enable us to appreciate the dynamic effect of a current of water which acts through their intervention.

The Germans, who use no other than vertical wheels, distinguish them by the point of their height where they receive the water, and they are the *Oberschlächtige Wasserräder* (wheels struck at the summit); the *Unterschlächtige Wasserräder* (wheels struck at the bottom); the *Mittelschlächtige Wasserräder* (wheels struck in the middle).

The English admit a similar division and the same denominations; they have the *overshot-wheel*, the *undershot-wheel* and the *breast-wheel;* they very properly subdivide this last into *high-breast* and *low-breast*, according as they are struck above or below the horizontal diameter.

Modes of action of water.

296. Water acts upon wheels either by its weight, or by its impulse, or by its centrifugal force, or by its reäction. We give here some first notions upon each of these modes of action, remarking that it is very rarely the case that one of these alone is exerted upon a water-wheel; most frequently, two, and even three, act simultaneously, and sometimes in nearly

equal degrees. The centrifugal force is found in all wheels with curved floats.

Action of weight.

According to the explanation of the preceding chapter, the effect produced by a current which furnishes P^p of water in 1″, and which acts by its weight entirely, upon a wheel, where it occupies a vertical height of h^f, will be Ph^{pf}. We shall see hereafter (351) a direct demonstration of this.

Action of impulse: general consequences.

297. We have already discussed nearly all the cases of the impulse of water upon solids. We will consider here more especially that relating to our machines, and point out some general consequences which have been deduced from it.

Let us take the most simple case, that of a wheel with floats, exactly set in a rectilinear course. Suppose it receives the impulse of a current of water arriving with a velocity V, and that after the collision it, as well as its floats, has only the velocity v: $V - v$ will be the velocity lost. The effort of impulse upon the wheel will have for its expression (252 and 242) $\frac{62.45\,sV}{32.18}(V - v)$; but $62.45\,sV$ is the weight of water furnished by the current in 1″, a weight which we have designated by P; so that the effort or the hydraulic pressure will be $\frac{P}{g}(V - v)$. In multiplying this by the velocity v of the point on which it is exerted, we shall have for the dynamic action of the impulse, or for the dynamic effect produced by it,

$$\frac{P}{g}(V - v)\,v.$$

This expression may be written as follows:

$$P\left(\frac{V^2}{2g} - \frac{v^2}{2g} - \frac{(V - v)^2}{2g}\right);\ \text{or}\ P\,(h - h' - h''),$$

designating by h, h', h'', the heights due respectively to the three velocities V, v and $V - v$.

The two last terms in the parenthesis manifestly diminish the effect produced. If they were nothing, this effect would be Ph, and consequently the greatest of those which the given current could produce (290); for here h represents the total disposable fall. In order that the first of these last two terms should be nothing, it is requisite that v should also be nothing; now v, in the above expression, refers also to the velocity which the water maintains after the impulse, or upon quitting the wheel: it is, moreover, evident that when the motive fluid leaves a machine, preserving a certain velocity, it still possesses a motive action represented by the product of its weight into the height due to this velocity; in order that it should have impressed all which it had at first, there could be nothing remaining, and consequently its absolute velocity, on quitting the machine, would be zero. In order that the last term may be found to be nothing, it is necessary that $V - v$, or the velocity lost by the impulse, should be nothing; which can never be the case except when there is no impulse on the arrival of the fluid upon the wheel.

$\frac{P}{g}$ is the mass of fluid whose weight is P: if we designate it by M, the expression of the dynamic effect could be put under this other form:

$$Ph - \tfrac{1}{2}Mv^2 - \tfrac{1}{2}M(V - v)^2.$$

But the product of the mass of a body into the square of the velocity with which it is animated is the *vis viva* of this body; we may then say, that the *dynamic effect is equal to the motive force employed to produce it, minus the half of the vis viva possessed by the water on quitting the machine, and minus the half of the vis viva which it loses at its entrance or in its passage through it.*

It is more than a century since Bernoulli pointed out the first of these losses of effect, that arising from the velocity preserved by the fluid on its issuing from the wheel. A short time after, Euler discovered the second; and he observes, that a current of water produces its greatest effect upon certain machines of rotation, when it reaches them with a velocity equal to their own. Finally, Borda, in his important *Mémoire sur les roues hydrauliques*,* expresses himself with more precision and generality: after having called z the velocity with which the water leaves the machine, and $\frac{P}{g}u^2$ the sum of the losses of *vis viva* experienced by it, he gives, as a general corollary of the principles which he has demonstrated in this memoir, the equation

$$pv = P\left(h - \frac{z^2}{2g} - \frac{u^2}{2g}\right),$$

which is exactly what we have established, pv being the total dynamic effect. Borda also remarks, that in the case of the greatest effect, $u = 0$ and $z = 0$; that then $pv = Ph$, and that it would be absurd to admit that we could have a greater.

Carnot, in his profound observations upon the movement of bodies, announces the same results in these terms: "In order that a machine should produce all its effects, it would be necessary, first, that the fluid should absolutely lose all its motion by its action upon it; second, that it should lose all this motion by insensible degrees, and without having any percussion."†

So that, TO PRODUCE ALL ITS EFFECT, THE MOTIVE WATER MUST ARRIVE AND ACT WITHOUT SHOCK UPON THE WHEEL, AND LEAVE IT WITHOUT VELOCITY. This principle will often serve as a basis to the theory of machines in motion.

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* Mémoires de l'Académie des Sciences de Paris. Année 1767.

† Principes généraux du mouvement et de l'équilibre, p. 249.

Action of centrifugal force.

298. When a body moves upon a curved surface, at each instant it tends to pursue its motion in a right line, and consequently to remove from the centre. If m is the mass of the body, u its velocity upon the curve at a given moment, r the radius of curvature, it will tend to depart from the centre with a velocity of $\frac{u^2}{r}$; and with a force of $\frac{mu^2}{r}$ (Poisson, *Mécan.*, § 169); this force, which is the *centrifugal force*, will be the effort or the pressure which the body will exert perpendicularly against the element of the surface upon which it is found. If this element belongs to a machine of rotation of which w is the *angular velocity*, that is to say, the velocity of the molecules placed 1 ft. from the axis, we have $u=wr$, and mw^2r for the effort.

Suppose, now, that the body m moves upon the float de of Fig. 102, for example, and designate by R and R′ the respective radii Od and Oe; the effort at d will be $mw^2\text{R}$. The body m, while exerting this effort, will advance, in the direction of the radii, the space $d\text{R}$, in an infinitely small instant; and the quantity of action impressed, along this elementary space, will be $mw^2\text{R}d\text{R}$. Integrating this expression from d to e, or between the radii R and R′, we have $\frac{1}{2}\,mw^2\,(\text{R}'^2-\text{R}^2)$ for the total quantity of action impressed upon the float by the centrifugal force of the mass m. I shall give elsewhere (391 and 392) a synthetic demonstration of this theorem, and of some others relating to the same force.

Reaction.

299. Every body which acts upon another, which presses upon it, for example, experiences on the part of this last a reäction, equal and directly opposite to the exerted action; this is a general principle of mechanics.

Likewise, when a force, gravity or any other, urging a fluid, causes it to issue through an orifice in a vase, the fluid, or the force which we may consider as a spring interposed between its particles, reäcts against the opposite part of the vase, in a direction exactly contrary to that of the motion impressed, and with an intensity entirely equal, as we shall see in the article on reäcting machines (401).

Notations adopted in the calculations of effects.

300. Before passing to the examination of different wheels, I indicate the meaning of the letters which will enter into the calculation of their effects; the same letter, throughout this chapter, will designate the same thing, whatever may be the kind of wheels in question. Thus, we shall always have,

H = Total fall of the water. This fall, when it is taken to measure the entire force of the current, is the difference of level between the fluid surfaces of the upper and lower reaches (280). But for hydraulic wheels, it is reckoned from the upper level, or that of the reservoir, to the lowest point of the wheel, as this point may be lowered to the level of the lower reach, when this level is constant.

V = Velocity of the fluid on its arrival at the point of the wheel upon which it exerts its action.

v = Velocity of the wheel at the centre of percussion of the fluid. The distance of this centre from the axis of rotation is the *dynamic radius* of the wheel.

h = That portion of the fall comprised between the level of the reservoir and this same centre. It will be the height due to V, if this velocity experiences no loss between the reservoir and its arrival at the wheel.

h_1 = Height really due to V; thus, $h_1=\frac{V^2}{2g}$.

We shall make $h_1=h\ (1-\mu)$, μ being a quantity connected with the before-mentioned losses.

h' = Height due to the velocity v; $h'=\frac{v^2}{2g}$.

h'' = Height due to the velocity $V-v$: $h''=\frac{(V-v)^2}{2g}$ $=\frac{(\sqrt{2gh_1}-v)^2}{2g}$.

P = Weight of water furnished in 1″ by the motive current.

Q = Volume of this same water. $P=62.45Q$.

K = Effort exerted by the motor upon the wheel.

p = Weight representing the sum of all the resistances which the motor has to overcome.

E = Dynamic effect produced by the wheel, or the force impressed upon it by the motor. $E=pv$.

n = Ratio of the real to the theoretic effect, or to the impressed force deduced from calculation.

m = Ratio of the real effect to the force of the motor; $m=\frac{pv}{PH}$.

ARTICLE FIRST.

Vertical Wheels. 1. *With plane floats.*

a. Contained in a rectilinear water-course.

Wheel with floats: its principal parts.

301. We are to treat here of what are strictly termed *float-wheels*. They are the most simple of wheels, and such as were formerly almost wholly in use; they are still in frequent use, principally on small falls, those below five feet.

Figs. 50 and 51. Such a wheel consists, 1st, of a *revolving shaft;* 2d, of two rims or shroudings, and even of three, in very large wheels; 3d, of arms, which connect each rim to the arbor, and which are arranged in different ways, as we see by the figures; 4th, of supports, strong wooden pins, imbedded and held fast upon the shroudings; 5th, of floats nailed or bolted upon the supports; 6th, and quite often of counter-floats or planks, fixed flat against the rims, and enclosing a part of the interval between the floats.

The motive water is led to the wheel by a water-course whose sides nearly touch the floats, leaving them only the play necessary for motion. It is delivered to the course through a gate-way, whose board is raised to a greater or less height, as we wish to deliver more or less water.

I shall not here enter into the details of construction. I shall only, and with the view of furnishing proper directions for the engineer charged with the establishment of such a wheel, make some observations upon the best disposition, and upon the principal dimensions to be given to parts which have an immediate influence upon the effect of the machine, to wit, the sluice, the course, and the floats; these observations are applicable to several of the wheels which we shall hereafter discuss.

Gates. 302. The fluid mass, on its issuing from the gate, experiences a contraction; then dilating, it meets the sides of the water-course and follows them. Even should it have, when at the section of greatest contraction, a velocity due to the height of the reservoir, yet a notable portion is afterwards lost by the effect of this dilation, and that of the friction against the course, if it has any length; so that quite often it arrives at the floats with only three quarters of this velocity.

We prevent this loss of velocity, and consequently of force, 1st. by establishing the gate as near as possible to the wheel; we thus render the resistance of the course nearly insensible; 2d, by disposing the sluice so as to reduce the contraction as much as may be; for this purpose, we prolong its bottom and lateral sides (above the opening), into the bottom and sides of the water-course; and we widen its entrance, or that of the canal which precedes it, so that the horizontal section of this entrance may have the form represented by Fig. 4; 3d, we incline the gate-board and all the front part of the gate-way; this inclination amounts to carrying the orifice nearer the floats, and nearly approaches the openings of pyramidal troughs, where the contraction is almost nothing (51). Experiments made by M. Poncelet place beyond a doubt the good effect of this inclination; a gate inclined 63° to the horizon (1 base to 2 height), gave him 0.75 for the coefficient of contraction, and he had 0.80 with an angle of 45° (1 base to 1 height); an upright gate, in the same circumstances, gave about 0.70.* By disposing his sluices in the manner above indicated, this philosopher accomplished the end of bringing the motive current upon the floats of the wheel with a velocity but little differing from that due to the height of the reservoir; it is true that the opening of the gate was great, and the diminution of the velocity is as much the less as the opening is more considerable.

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* Mémoires sur les roues hydrauliques verticales à aubes courbes. 1827. p. 78 and following. In these experiments, the opening of the gate-board was made perpendicular to the bottom of the course.

If, without loss of fall, we might direct the water immediately upon the floats, in causing it to issue through an orifice in a thin plate, or through a pyramidal trough, the velocity would experience only a few hundredths of diminution.

The water-course.

303. Immediately past the gate, the water-course is directed, with a slight inclination, towards the wheel; it passes beneath, and then continues in a right line (Fig. 50). Fig. 50.

Its size is determined by the volume of water which it is to conduct; the thickness of the fluid sheet in the water-course (supposing for an instant the wheel to be raised up) should never be above 0.82 ft. nor below 0.49 ft. If it were less, the quantity of water escaping between the flooring and the lower edges of the floats, without exerting any action upon them, would be proportionally too great; and the force of its current would be notably diminished. That this diminution may be as slight as possible, we should not give to the space necessary to be left between the sides of the water-course and the edge of the floats more than from .0328 to .0656 ft.

If ever so little attention is given to careful constructions, we do not make the water-courses entirely rectilinear. Their bottom or flooring should arrive at the level of the lower edge of the second float above the vertical diameter; there, it curves concentric with the wheel, as far as the plumb line of this diameter; then it falls suddenly a decimetre, (.328 ft.) at least, and finally pursues its course with the slope permitted by the locality (Fig. 51). Fig. 51. Its breadth, immediately before reaching the floats, is a little less than theirs; it then increases and encloses the floats beyond the vertical diameter. By these dispositions, the water, on its arrival at the wheel, impinges upon it with all its mass, without experiencing a loss through the intervals; after that, the lowering and enlargement of the wheel-course favors the clearing of the water, and does not obstruct its motion. We refer to the above-cited work of M. Poncelet for the good establishment of courses.

Floats: dimensions.

304. After what has just been said, the breadth of the floats is fixed by that of the course, and by the size of the intervals. Their height, in the direction of the arm of the wheel, ought to be such that in the greatest rising of the water against the first float struck by it, a portion of the fluid, which tends to run past its upper edge, although retained by the counter-float, shall not lose a part of its action: we prevent this loss by giving to the height of the floats about three times the thickness of the sheet of water in the course, without, however, exceeding 2.13 ft. The distance from float to float, measured upon the exterior circumference of the wheel, should be a little less than their height.

Diameter of the wheel.

305. Their number, then, will depend upon the extent of the circumference or of the diameter, and this dimension is nearly arbitrary.

The dynamic effect of the wheel is proportionate to the velocity of the floats: it requires only this velocity, which is independent of the diameter. When the diameter is required, we usually determine it by the number of turns which it is proper the wheel should make in a certain time, in order that the transmission of motion to that part of the machine which does the useful work, and which should consequently have a certain velocity, should be effected with the greatest simplicity and with the least gearing possible. This is accomplished in such a way that the wheel shall have a velocity and dimensions adapting it to fulfil the office of a fly-wheel, so as to maintain a suitable uniformity of motion. If u is the velocity at the extremity of the floats, N the number of turns wished in a minute, the diameter will be $\frac{60u}{\pi N}$, or $19.1\,\frac{u}{N}$. For the case of good effect, we shall have nearly $u = 3.08\sqrt{H}$; and consequently, the diameter will be $\frac{58.8}{N}\sqrt{H}$. Finally, in practice, we never make it less than 13.12 ft., nor more than 26.25 ft.

Number of floats.

306. According to the adopted size of the diameter, we shall give to the wheel the number of floats indicated opposite. This number is divisible by 4; from the fact that constructors are in the habit of putting an integral number of floats in each of the four quarters of the wheel. We may besides, without any disadvantage, increase by 4 each of the numbers of the table.

DIAMETER.	FLOATS.
ft.	No.
13.12	28
16.40	32
19.68	36
22.97	40
26.25	44

Bossut, in raising the same weight by a small wheel of 3.346 ft. diameter, sometimes with 48, at other times with 24 floats, obtained effects which were in the ratio of 4 to 3, whence he concluded that it would be better to give a greater number of floats to wheels than is usually done. (*Hydrodyn.*, § 1029 and following.) But his water-course was rectilinear, and in such a course, the wheel takes positions in which the spaces between the flooring and the edge of the floats shall be the greater as their number is the smaller; whence it follows that a great quantity of water is lost without exerting any action. Smeaton, to whom this fact was well known, remarked that this no longer occurs, and that the effect is not necessarily diminished, by lessening the number of floats, when we curve the flooring concentrically with the wheel, and that it was sufficient to give such a length to the curved part, as that one float might enter it before the other left.*

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* Recherches expérimentales sur l'eau et le vent, p. 24.

307. Some mechanists have supposed that the dynamic effect is increased by inclining the floats upon the direction of the arm, and they have given them such an inclination. But what may be advantageous for a wheel plunging in an indefinite fluid (329) is no longer so for one established in a mill-race. Bossut having compared the effects obtained with floats inclined 0°, 8°, 12° and 16°, found that they were respectively as the numbers 1, 0.949, 0.956 and 0.998 (*Hydrodyn.* § 1043): so that in these experiments, the only ones with which I am acquainted, the inclination has been a disadvantage. Inclination.

In the case only where a wheel might casually be plunged in the race of a canal (for we cannot admit that it is usual, inasmuch as its establishment then would be faulty, and would have to be changed), the inclination of the floats would favor their clearance; or rather, it would prevent the floats, after they had passed the vertical, from taking up and raising a certain quantity of water, which, acting in a direction opposite to the motion, would diminish the effect.

This inconvenience is obviated in large wheels established upon the arms of a river, where the fall is very small, and where the floats are composed of different pieces, by giving them a slight inclination, but more and more as they approach the exterior circumference of the wheel.

308. Attempts have been made to increase the dynamic force, by means of lining the floats with borders, or side pieces, like those which have already been discussed (239 and 258). But we have observed that their action was inconsiderable in the case where the paddles which receive the impulse of the fluid are placed in a water-course. It will be still less upon the floats Side pieces.

of a wheel; and in the experiments of M. Poncelet, made at a powder-mill in Metz, these flanges have augmented the effect but a fifteenth.

We produce, and with more certainty, an analogous effect, by fixing and enclosing the floats between two circular plates, similar to those which form the crown or shrouding of bucket-wheels.

In narrow wheels, cast iron floats, slightly cylindrical, the axis of the cylinder being in the direction of the radius, produce the effect of these side enclosures.

Putting the wheel in motion.

309. When we put in motion a machine at rest, and for this purpose open the gate, the fluid is precipitated forcibly against the float which is opposite to it, rises and flows over all its parts; continually pressed by that which arrives without interruption, it exerts a greater effort than when the motion is established. A portion of this effort is put in equilibrium with that of the resistances to be overcome; the remaining portion acts, in the first moment, to break the adhesion contracted during the repose by the pieces of the machine which should move upon each other; and then, striving against the inertia of the masses, it accelerates more and more its motion. As the velocity of the wheel increases, its action becomes more feeble, (since this action is proportional to the relative velocity); soon the acceleration, diminishing gradually, becomes insensible and as nothing; and the wheel, after a few turns, in consequence of the velocity impressed upon it, and in virtue of its inertia, continues to move, as it were, of itself, either with an entirely uniform motion, or with a velocity which, oscillating between near limits, may be reduced to a mean and continuous velocity.

Analytical expression of effect.

310. We have already seen (252 and 297) that the action of an impulse, or the dynamic effect produced

by it upon the floats of a wheel, or, more exactly, upon a paddle well set in a water-course, and which yields perpendicularly before the fluid, was

$$\frac{P}{g}(V-v)v.$$

Is it the same for a series of floats presented in succession to the current, or two or three at a time, and under different angles of inclination? Experience alone can afford us just ideas upon this subject; meanwhile, we assume that the action of the impulse upon the wheel is not equal, but of the same nature, and having the same form of expression as the above.

Velocity, load and effect in the case of a maximum.

311. In this expression of effect, when the wheel is moved by the same current, v is the only variable. If $v=0$, the effect will be nothing; a machine which does not move cannot produce any. It will still be nothing when $v=V$; a wheel which goes as fast as the current cannot receive action from it. It is moreover evident that v can never exceed V. So that the effect will increase according as the velocity of the wheel, starting at zero, shall increase; but only up to a certain point, beyond which this effect will decrease, returning to nothing when the velocity shall be equal to V; between these two extremes there will, then, be a *maximum* of effect. Differentiating the variable part of the expression, $(V-v)v$, and making this equal to zero, we have $Vdv-2vdv=0$; whence $v=\frac{1}{2}V$; that is to say, that *a wheel with floats produces its greatest effect, when its velocity is half that of the current.*

The effort of the water upon the floats is $\frac{P}{g}(V-v)$ (297); this will also be the value of the load of the machine, that is to say, of the sum of resistances which it can overcome, these quantities being equal (287).

For the case of *maximum* of effect, where $v=\frac{1}{2}V$, this load will be

$$\frac{PV}{2g}.$$

For the same case, the dynamic effect, being equal to this load multiplied by its corresponding velocity $\frac{1}{2}V$, will be equal to $\frac{PV^2}{4g}$, or, observing that $\frac{V^2}{2g}=h_1$ (300),

$$\tfrac{1}{2}Ph_1.$$

The greatest effect of which a current arriving at a machine is susceptible, with P of water, and a velocity due to h_1, is Ph_1; that of a wheel with floats will therefore be only half of this.

If the entire fall H had been made available, and experienced no loss of velocity, either at the gate or in the course, we should have $h_1=H$, and for the *maximum* effect

$$\tfrac{1}{2}PH.$$

Whence we conclude, *that the greatest effect which can be produced by a current of water acting by its impulse upon a wheel with floats, and upon a hydraulic wheel in general, is but half of the greatest effect of which it is capable.* And yet we could never have arrived even to this half, but through suppositions which are not realised; it is a limit which we cannot attain, and from which we are usually far removed, as we shall soon see.

Experiments of Smeaton. 312. We pass to the modifications which experience must make in the results of a theory, which, moreover, we have only admitted with reserve.

We shall devote some time to this subject, both because we are dealing with nearly the only wheel that is moved solely by the impulse of water, and because the field of experiment has been successfully explored by a man of superior merit, Smeaton, one of the most celebrated engineers of England. His observations were made, it is true, on a small scale, the model of the wheel being only two feet in diameter; but they were so well directed towards the principal points of the problem to be solved, and executed with so much skill, that they enable us to recognise the principal circumstances of the motion of wheels with floats. It was only after Smeaton had satisfied himself that their results were conformable with those observed by him on large wheels, that he published them.*

Upon the axle of a wheel, a cord was wound, which passed over a pulley on the top of the machine, and which bore at its end a basin, in which were placed at pleasure various weights. The water was furnished to the wheel by a reservoir, which was constantly kept at the desired height.

The experiments were divided into classes and series: those of the same class all have the same opening of the sluice-gates; and in those of the same series, they moreover had the same height of reservoir, and consequently the same quantity and the same velocity of motive water, or the same dynamic force.

The velocity of the fluid, at the moment of striking the wheel, as well as the passive resistances, were determined previously and directly by experiments of a very ingenious character, which may be found in the memoir of the author.

These preliminaries having been established, a small weight was at first put in the basin; when the motion was well established and had become uniform, they counted the number of turns made by the wheel in 1′ or 60″, and thence deduced the velocity of the elevation of the weight: this was the first experiment of the series. Then the basin was lowered, and a heavier weight placed in it, and the time of raising it was taken. So, in succession, for a third, fourth, &c., weight, up to the weight which was so heavy as to arrest the motion: the series of experiments was then completed. That term in which the product of the weight raised (adding to it the weight representing the passive resistances) into the respective ascensional velocity, was found to be greatest, was the term of *maximum effect* of the series.

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* His Memoir, read before the Royal Society in 1759, made a part of the Experimental Researches upon water and wind, translated from the English by M. Girard.

Smeaton in this manner made twenty-seven series of experiments, and he published a table presenting the circumstances relating to the experiment of *maximum* of effect in each series. The following table, containing eighteen of these experiments, is an extract from it. The dotted transverse lines to be seen in it, separate the six classes of experiments; from one class to the other, the opening of the sluice-gate was gradually enlarged. The titles of the columns indicate their contents sufficiently well. I shall confine myself to the remark, that, for each experiment, $h_1 = \frac{V^2}{2g}$, $H = h_1 \frac{\alpha}{6}$, α being the number of the experiment or of the horizontal line noted in the eighth column, and 6 the number in the ninth; H is the height of the water above the gate sill; $\psi = p\gamma$, γ is the corresponding number of the tenth column, and ψ represents, for each series, the weight which, put in the basin, would arrest the wheel.

Water expend'd in 1″. P	Velocity of Current. V	Velocity of Wheel. v	Weight raised. (Resist.) p	Effect. pv	Coeffic'nt conclud'd n	RATIOS.			
						$\frac{v}{V}$	$\frac{pv}{Ph_1}$	$\frac{pv}{PH}$	$\frac{\psi}{p}$
lbs.	ft.	ft.		lbs. ft.					
4.583	9.166	3.125	.6336	1.98	0.74	0.34	0.32	0.16	1.30
4.05	8.541	2.916	.4972	1.45	0.71	0.34	0.32	0.17	1.33
3.566	7.812	2.698	.3784	1.021	0.67	0.35	0.30	0.16	1.37
2.975	6.77	2.437	.2519	.614	0.64	0.36	0.29	0.17	1.20
2.233	5.416	1.979	.1495	.296	0.63	0.37	0.29	0.18	1.11
1.9	4.375	1.666	.0972	.1620	0.61	0.38	0.28	0.16	1.08
5.7	8.75	3.203	.6525	2.090	0.66	0.37	0.31	0.18	1.27
4.75	7.50	2.708	.5000	1.354	0.72	0.36	0.33	0.19	1.15
3.9	6.56	2.604	.2922	.761	0.61	0.40	0.29	0.20	1.15
2.79	4.79	2.187	.1344	.294	0.60	0.45	0.30	0.21	1.11
5.95	7.499	3.02	.5579	1.685	0.68	0.40	0.32	0.23	1.25
5.50	6.874	2.786	.4379	1.219	0.63	0.41	0.31	0.22	1.24
3.80	4.999	2.447	.1798	0.440	0.62	0.49	0.30	0.23	1.04
5.981	7.08	2.812	.4967	1.397	0.63	0.40	0.30	0.24	1.09
4.366	4.999	2.551	.2093	0.534	0.63	0.51	0.31	0.24	1.08
5.916	6.249	2.843	.3823	1.087	0.61	0.46	0.30	0.24	1.06
5.116	5.208	2.562	·2439	.625	0.58	0.49	0.29	0.24	1.06
6.00	5.208	2.708	.2736	.741	0.64	0.52	0.30	0.25	1.08
1	2	3	4	5	6	7	8	9	10

The four first columns of the table present the data of the experiment; the six last, the results deduced from them.

Let us sum up these results.

Consequences of the experiments for the case of *maximum* of effect.

313. A glance at the sixth column shows that the coefficient of reduction of the theoretic effect to the real effect is not constant, and consequently, that the admitted theory does not adapt itself to all the circumstances of the movement of wheels with floats.

Its results, as to effect, are so much further from those of experiment, as the velocity is more considerable, as we may see in the table opposite, which answers to the only entire series of experiments which Smeaton has given us. The quantity of motive water used there was 4.46 lbs., and its velocity 9.222 ft.

v	pv	n
ft.	lbs. ft.	
4.691	1.512	0.52
4.363	1.671	0.57
3.773	1.671	0.59
3.510	1.765	0.64
3.117	1.751	0.67
2.756	1.714	0.69
2.296	1.967	0.71
1.706	1.280	0.71

The coefficient n does not present so great varieties in the experiments of the great table, which answer to the *maximum* of effect of each series: and even, making abstraction of some anomalous numbers, we have for the mean of each class (one only excepted) very nearly $n = 0.64$; and consequently,

$$\text{E or } pv = 0.64 \cdot \tfrac{1}{2}Ph_1 = 0.32Ph_1.$$

This ratio of pv to Ph_1, immediately given by each experiment, is noted in the eighth column of the table; it only varies from 0.28 to 0.32; and the mean term has no where exceeded 0.30. Nevertheless, Smeaton thought he had good cause to raise it as high as ⅓ for great wheels; that is to say, to admit their effect to be ⅓ of the force which the current possesses on its arrival at the floats.

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314. The ratio of this same effect to the entire force of the motor, or m (300), indicated in the ninth column, is not so constant in its character as the preceding; it gradually increased from one class to the other, from 0.167 up to 0.25. So that, in the experiments of Smeaton, *the greatest dynamic effect was only from a sixth to a quarter of the entire force of the motor.* I doubt if in great machines, even supposing them well arranged, it attains this last value; though theory indicates it as double, or $\frac{1}{2}$PH.

315. The ratio of the velocity of the wheel to that of the current gradually increased from one class to the other, that is to say, in proportion as the opening of the sluice-gate was greater, from 0.36 up to 0.52; it was, as a mean, 0.44. Smeaton does not admit over 0.40. Bossut, after a series of some experiments, also adopted this same number; but as the velocity of the current was measured at the surface, they have given too small a result; it would approach 0.50 in taking the mean velocity. I believe, that in machines well arranged and well conducted, we may very nearly attain this theoretic limit; and, with some authors, I shall adopt $v=0.45V$, always for the case of *maximum* of effect.

316. Finally, the last column shows that the load which arrests the wheel is only from one to two tenths greater than the load for the *maximum* of effect. But according to theory, it should be double; indeed, the load ψ, corresponding to the velocity $v=0$, is $\frac{PV}{g}$ (310); and that which corresponds to the *maximum* is $\frac{PV}{2g}$ (311).

General Formula.

317. The results we have just given refer to the case where the velocity of the wheel is found to be in

the most advantageous ratio to that of the current at the moment of striking the floats. But usually, this is not the case; the effect is less, and its coefficient n, experiencing great variations, as we have seen in the small table of Sec. 313, can never be expressed by a general formula.

However, when the velocity of the wheel does not exceed certain limits, one third to two thirds that of the current, without the risk of any notable error, especially in excess, we may take .60 for the coefficient, and admit

$$E=0.60\frac{P}{g}(V-v)v=.01864P(V-v)v=1.1640Q(V-v)v.$$

318. The velocity V, with which the water arrives at the floats, is always difficult to determine. It meets, as we have seen (302), with losses between the sluice-gate and the wheel; without them, we should have $V=\sqrt{2gh}$; and h, the difference in level between the surface of the reservoir and the centre of percussion of the floats, would be easily measured.

Smeaton, who made observations upon the losses really experienced, and who has sometimes seen them as high as one fifth of the velocity, has also remarked that they diminish, when the height of the opening of the gate increases; so much, says he, that in mill-sluices, when great volumes of water are discharged, under moderate heads, V will be very nearly equal to $\sqrt{2gh}$. M. Poncelet has also observed that the loss of velocity is less in great openings; and that through an opening .7217 ft. in height, and even with a head of 4.593 ft., he found $V=0.99\sqrt{2gh}$. Still, to prevent mistakes, and supposing that the sluice-way is otherwise suitably arranged, we will admit $V=0.95\sqrt{2gh}$

$= 7.6215\sqrt{h}$; and consequently, we shall have generally

$$E = 1.1642Q\,(7.6215\sqrt{h} - v)v.$$

When v is very near to $\frac{1}{2}V$, this expression will be reduced to

$$E = 16.907Qh.$$

319. The ratio between the effect and the entire force of the motor will be established in a manner still less sure. Smeaton, even in the case of *maximum* effect, found it vary from 0.16 to 0.25. So that we shall have nearly always

$$E < 0.25PH \text{ or } < 15.612QH.$$

Finally, we but little regret our inability to give a more precise expression of the effect of wheels with floats moved by the impulse of water, inasmuch as this kind of wheel is nearly out of use.

Blast-Engine.

320. Notwithstanding this remark, suppose we are to establish a wheel to put in action a blast-engine, appointed to throw into a high furnace for melting iron by means of coal or of coke, three quarters of a cubic metre or 26.487 cubic ft. of air in a second, with a velocity of 426.51 ft.; and that we have upon a small river a fall of 5.4134 ft. We wish to determine the volume of water required to move the machine.

That we may have three quarters of a cubic metre of air in the furnace, in view of the inevitable losses, we must count upon a cubic metre. At the level of the sea, and at zero of the thermometric temperature, it would weigh 2.8671 lbs.; at the site of the mill, it will weigh only 2.6906 lbs.; we will admit 2.7568 lbs. The height due to the velocity of 426.51 ft. is 2821.57 ft. Thus the useful effect to be produced is equivalent to raising 2.7568 lbs. to a height of 2821.57 ft., or $1075^{\text{km}} = 7778.59^{\text{lbs. ft.}}$ in one second. By reason of the passive resistances of the wheel, of the machine and air pipe, we will augment this number by a third, and we shall have for the dynamic effect, $10371.45^{\text{lbs. ft.}} = E$.

On the fall of 5.4134 ft., we will take .98427 ft. for the distance between the centre of percussion of the floats and the lower level; and there will remain but 4.4292 ft. for the value of h. Thus the equation will be $10371.45 = 16.907Q \times 4.4292$; whence $Q = 138.49$ cub. ft.

We will reckon upon 141.266 cub. ft. This water, having to run in a water-course with a velocity of $16.04^{\text{ft.}} = 7.6215\sqrt{4.4292}$, the section of the fluid sheet in itwill be 8.888 sq. ft., and as its thickness should not exceed .6562 ft., its breadth must be 13.418 ft.; let us put it at 13.45 ft. Leaving a space .0492 each side between the course and the wheel, we shall have for the breadth of the latter, that is to say for the breadth of the floats, 13.353 ft. Their height will be 2.132 ft.; for under the wheel, the water will rise 1.97 ft. and more: they will therefore be furnished with counter-floats ("*contre-aubes*"). Their number will be forty, the diameter to be given to the wheel being 20.34 ft.; each will be formed of four planks, .574 ft. wide, and inclined gradually upon the radius 0°, 10°, 20° and 30°; the three iron supports to hold them will have three bends or angles of 170°. The wheel will make about seven turns per minute, and its motion will be communicated without gearing to the pistons of the blast cylinder, either by means of cranks, winches, balance-beams, or by cams, in the form of eccentric wheels, which will accompany them in their ascent and descent.

The float-wheel just described, exceeding 13 ft. in width, consuming 141.26 cub. ft. of water per second, with a fall of 5.413 ft., having thus a force equivalent to 89 horse-power, will be one of the most efficient which we can have.

If charcoal were used in the furnace, we should not require over 17.66 cub. ft. of air per second, with a velocity of 328 ft. A volume of water of 44.14 cub. ft. would be sufficient to move the wheel. We should give it a width of only 4.92 ft.; its floats might be plane and 1.968 ft. deep.

2. *Wheels established in a circular water-course or curb.*

Form and disposition of their parts.

321. We have seen (Sec. 303) that the most advantageous disposition of the course for float-wheels is in curving it under the lower part of the wheel and concentric with it, for a short length, (one or two of the

float spaces,) and consequently a very small height. The advantage increases as the height or versed sine of the curved part is greater; so much so, that now they are made as great as possible compared to the fall; and we give them two thirds, three quarters, and even a greater proportion of its value. In this way, we obtain wheels of very good effect, perhaps the best that can be had with small falls, those of eight feet and less. Figure 61 gives a good idea of their disposition. Fig. 61.

322. Manifestly, the circular course or curb should be constructed with great care, and of masonry, if possible; its apron, or cylindrical surface, should be very smooth, well centred, and so that its axis shall be exactly the axis of rotation of the wheel which the *curb* or *mantle* encloses.

The space to be left between its surface, that of the bottom as well as its sides, and the edges of the floats, should be from 0.0328 ft. to 0.049 ft. We should never make them less; in the best suspended and best made wheels, after a while, some portions yield or wear out, some joints begin to play; and if the space is too small, the floats will soon rub and scrape against the curb. This consideration should induce us to establish very solidly the walls or pillars upon which the *gudgeons* are supported.

The breadth of the course, as well as that of the wheel, should be such that the water, running freely over its bed, might not have a depth of over 0.656 ft., nor under 0.049 ft.

The diameter of the wheel will be determined in the manner and according to the considerations shown in Sec. 305; generally, it is from 16.4 ft. to 23 ft.

The number of the floats will be such as above described (306). Their height should never be less than three times the thickness of the fluid sheet of water in the course. They should be placed in the direction of the radius. Still, good millwrights give them a slight inclination; quite often they incline them to the radius with an angle $90° + a$, a being given by the equation $\cos. a = 1 - \frac{2H}{D}$. Sometimes they give the forms indicated in Fig. 61 by a b c, or a' b' c'.

The sluice-gate should be made and disposed with all the precautions indicated in Sec. 302, and in such a manner that the water should fall very nearly perpendicular upon the float receiving the impulse. Better still, if it can be done, when we cause the water to fall by simply flowing over a sill established at the top of the curved apron.

323. Water acts upon wheels established in such a course, both by its impulse and by its weight. **Theoretic effect.**

If from the point e, taken at the surface of the reservoir, we drop the perpendicular ef, upon the horizontal line passing through the bottom of the wheel, and let k be a point taken at the level of the one where the water arrives at the first float struck; ef will represent the total fall H; and ek the portion h of this fall employed in the generation of the velocity with which the impulse is made. After this has taken place, the water spreads out upon the float, descends with it pressing upon its upper surface; so that the fluid which is in the course, throughout the whole height kf, presses upon all the floats found there, and urges them in the direction of motion; this action of the weight will be expressed (296) by $P \times kf$ or $P(H - h)$. The action of the impulse is expressed by $\frac{P}{g}(V - v)v$, or $P(h - h' - h'')$ (297); or, more exactly still, with the nota- **Fig. 61.**

tions of Sec. 300, and according to what we shall hereafter see (352), by P $(h - \mu h - h' - h'')$, μh referring to losses experienced in the velocity of the current between the gate and the wheel. Uniting these two partial actions, the total action, or the effect pv which results from them, will be

$$P\ \{(H - h) + h - \mu h - h' - h''\}.$$

We have two corrections to be made for this expression.

First, even when all the water P expended shall have acted by its impulse upon the first float it meets; beyond that, when it descends in the course, pressing upon the succeeding floats, the part of the fluid which is found in the intervals between the edges of the floats and the sides of the course exerts no pressure, and has no effect; and consequently, it should be subtracted from P in the expression P $(H - h)$. The amount of this part cannot be rigorously determined. Still, if we consider, 1st, that the resistance experienced by this water against the sides of the course diminishes the velocity which gravity tends to give it, more and more during its descent upon the bed of the curb; 2d, that this velocity is still more diminished by the continual obstructions which the water meets in its passage through the spaces, varying at each instant, for a wheel is never perfectly centred; 3d, finally and especially, that the velocity is altered by a continual mingling of the water in the spaces with that resting upon the floats, we may conceive that in nearly every case, the velocity of one will be that of the other, and consequently equal to that of the floats. In such a case, if we designate by A the section of the fluid sheet in the course, and by a that which answers to the spaces, P $\frac{a}{A}$

will be the portion of the fluid which produces no effect; we must deduct this from P in the expression of effect, which will become $P\left(1-\frac{a}{A}\right)(H-h)$.

Secondly, the portion of the bottom of the wheel which plunges in the water of the course, there loses a part of its weight equal to the weight of the fluid which it displaces. In consequence of this loss, there does not exist an equal distribution of the weight of the wheel around the axis of rotation; and the wheel tends to turn against the current; let p' be the weight representing the effort of this tendency; this will be a new resistance which the motor must overcome, and it should be added to the other efforts or resistances of which the sum is p.

We have then, n being the coefficient of reduction of the results of calculation to those of observation,

$$(p+p')v=nP\{(H-h)\left(1-\frac{a}{A}\right)+h-\mu h-h'-h''\}.$$

The example which we shall shortly give will show us the manner of applying this formula.

324. For common use, it may be simplified. The quantities p' and $1-\frac{a}{A}$, supposing the constructions equally well made, will be very nearly proportional to the force of the machine, or to P; and they may consequently be comprised in the value of n. Moreover, we shall see (354), that the quantity $\mu h+h'+h''$ always exceeds $\frac{1}{2}h$, and that it is very nearly $\frac{2}{3}h$. So that the equation is simply

$$E=nP\left(H-\tfrac{2}{3}h\right).$$

325. Let us determine the coefficient n.

Real effect.

Let us see its value in a machine, perhaps the most perfect of the kind we have discussed; it is a wheel established at the chrystal ware manufactory of Baccarat, near Lunéville, by two good English constructors, and similar to those in use in their country. It is 13.14 ft. in diameter, with a breadth very nearly the same; it has 32 floats, 1.312 ft. deep; and it is hung in a circular course, 6.037 ft. versed sine; upon a fall of 6.758 ft.; the space between the sides of this course and the edges of the floats is reduced to some millimetres, says M. Morin.* The motive water was let upon the wheel over a weir 12.79 ft. long, with the head h_0 above the lip noted in the following table. According to the experiments of M. Castel, the volume of water discharged will be $3.5567 \times 12.79 h_0 \sqrt{h_0}$; whence we have the values of P. The fall was 6.037 ft.$+h$, and I have represented by H′ the factor $H - \frac{2}{3}h$. As for p, the sum of the resistances to motion, it is the result of experiments made by M. Morin, by means of a dynamic brake; to the effort immediately indicated by the brake, this author has added the passive resistances, which he determined by calculation; finally, as they do not reach to $\frac{1}{12}$ of p, a little uncertainty respecting them would be but of small consequence.

v	p	P	h_0	$\frac{pv}{PH'}$	$\frac{pv}{PH}$
ft.	lbs.	lbs.	ft.		
7.64	108.04	1726.8	.7185	0.762	0.707
3.805	227.10	1740.1	.7217	0.792	0.734
3.182	269.06	1740.1	.7217	0.783	0.726
2.723	306.50	1715.8	.7152	0.777	0.720
2.395	348.40	1715.8	.7152	0.773	0.716
2.132	385.90	1726.8	.7185	0.755	0.700
Mean	. . . .	1727.5	.7184	0.772	0.717

Thus, for the machine at Baccarat, n would be, as a mean, 0.772.

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* Expériences sur les roues hydrauliques, etc., par M. Arthur Morin, Capitaine d'artillerie. 1836.

But we rarely meet a wheel with so small a play as this, and it will only be for machines very carefully constructed and maintained that we can admit

$$E = 0.75P\,(H - 0.7h).$$

326. The above experiments give 0.717 for the ratio of pv to PH. But where, as for the wheel upon which they were made, shall we find the height of the circular curb so great as $\frac{9}{10}$ of the fall? Most frequently, this height, or more exactly, that upon which the water only acts by its weight, is not over one third, and we generally have from

$$0.60PH \text{ to } 0.65PH.$$

In the application, we shall not use these expressions, but the preceding, $0.75P\,(H - 0.7h)$; diminishing the numeric coefficient a little, if the machine is in an ordinary condition.

327. Upon a canal fed by a river, we have an iron-mill, to which we wish to add a rolling-mill of thirty horse-power. The available fall at low water is 8.202 ft.: we will employ a wheel moved by the weight of the water. It is required to indicate the volume of water necessary to put it into action, and the principal dimensions to be given it. **Example.**

We require for the working of the rollers that the wheel should make six turns per minute, with a velocity of 7.38 ft. Accordingly, its dynamic radius should be 11.745 ft. (305), and we will make the whole diameter 24.278 ft.

It shall be a wheel with floats, of which there shall be forty-eight, and formed of two planks; the small one will be placed in the direction of the radius, and will be .722 ft. in height; the greater will make with it an angle of 160° ($= 90° + 70°$; $1 - \frac{2H}{D} = 1 - \frac{15.748}{24.278} = \cos.\ 69° 27'$), and we will give it a height of 1.397 ft., so that the two united shall make 1.968 ft. in the direction of radius. The counter-floats will be 1.148 ft. in breadth.

We will sacrifice .328 ft. of the total fall for lowering the apron immediately below the wheel. The height H will then be 7.874 ft. We take from this 6.562 ft. for the height of the curve to be given to the circular part of the course, and there remains 1.312 ft. for h: thus $H - h = 6.562$ ft. We have seen that $\mu h + h' + h''$ was greater than $0.5h$, and we have made it $0.7h$; consequently, $h - \mu h - h' - h'' = 0.3h = .3936$ ft.

After this, the equation will be

$$(p + p')\ 7.382 = 0.90\text{P}\ \left[6.562\left(1 - \frac{a}{\text{A}}\right) + .3936\right].$$

Let us determine the unknown quantities.

The weight p, representing the sum of resistances to the motion of the wheel, is given by the conditions of the problem; the dynamic effect pv being equal to the action of thirty horse-power, or to $16280.7^{\text{lbs.ft.}}$, and v being equal to 7.382 ft., we shall have $p = 2{,}205.4$ lbs.

To determine p', A and a, we must have the dimensions of the sheet of water which descends upon the curved bed, and consequently know P, which is precisely the quantity sought. Let us take at first an approximate value: for this purpose, let us make $p' = 132.32$ lbs., and $\frac{a}{\text{A}} = 0.1$; these quantities, substituted in the equation, give $\text{P} = 3043.5$ lbs., or $\text{Q} = 48.736$ cub. ft. Since the velocity of the fluid sheet should be 7.382 ft., its section, or A, will be 6.6021 sq. ft. $\left(= \frac{48.736}{7.382}\right)$. We will admit 0.6562 ft. for the thickness of this sheet; its width, or that of the course, will be 10.061 ft. Leaving .0656 ft. of space between the sides of the course and the edges of the floats, we shall have

$$a = .0656\ [10.061 + 2\ (.6562 - .0656)] = .7377^{\text{s.ft.}}:\ \text{thus}\ \frac{a}{\text{A}} = .11173.$$

To get p', we will observe that eight floats at least plunge continually in the water of the course, and that they are submerged for a depth of .5906 ft. in the direction of the radius, or .6299 ft. in reality, by reason of their inclination of 160° to the radius. Since the width of the floats is $10.061^{\text{ft.}} - 0.131^{\text{ft.}}$, or 9.930 ft., and their thickness .0984 ft., the weight of the fluid displaced by each of them will be 38.491 lbs. $(= 9.9411 \times .6299 \times .09842 \times 62.45)$: we will carry it up to 41.9026 lbs., on account of the ends of the supports, which also plunge into the water. This weight is as a force tending to lift the floats vertically: if we estimate it in the direction of the motion of rotation, it will be 41.9026 sin. i, i being the angle made by the radius of the wheel with the vertical, at the centre of immersion of the floats: this radius being 11.844 ft., and the dynamic radius being 11.745 ft., this force referred to the extremity of this last, or augmented in the ratio of these two numbers, will be 42.255 sin. i. For the eight floats, we must multiply 42.255 by the sum of the eight values of the sin. i, which will be 4.52049, the angles being, as a mean, 10°, 17½°, 25°, 32½°, 40°, 47½°, 55° and 62½°. Thus we shall have $p = 191.01$ lbs.

Substituting these values in the equation, it will become $(2{,}205.4 + 191.01)\ 7.382 = 0.90P\ [6.562\ (1 - .11173) + .3936]$, and it will give for the second value of P, 3158.8 lbs.; then $A = 6.8523$ sq. ft., $a = .7627$ sq. ft., $p' = 201.13$ lbs. For the third value of P, we have 3169.2 lbs., and 10.466 ft. for the width of the course.

It will be well to augment this width when the water arrives in greater quantity; we may carry it to 10.63 ft., and the width of the floats will consequently be 10.508 ft.

The force of the motor, 3169.2 lbs., falling 8.202 ft., is equivalent to forty-eight horse-powers; the dynamic effect is but two thirds of this.

The rolling-mill of which we have been speaking, and whose effect is but that of thirty horses, is of an ordinary kind: there are those which, with great velocity, produce the effect of fifty horses and upwards.

328. In the commencement of our observations upon wheels contained in a circular course, we remarked that it was best to increase the height of the course, so as to reduce as much as possible the distance between the float-board, which receives the first impulse of the fluid, and the reservoir. This is, in fact, the method of obtaining the greatest dynamic effect, with the least consumption of water; but this condition, though worthy of great consideration, is not the only one which determines the choice and disposition of the wheel to be used. For example, where we may have an abundance of water, we should consider less its economy, and rather regard the expense required in a construction made according to the rules which we have given: thus, instead of a small distance between the float-board impinged upon and the reservoir, we may sometimes have a very great one. This is the case

with the iron mills of the Pyrenees, where there are great falls and large streams; the wheels established there are otherwise remarkable for their simplicity and the solidity of their construction. I will give a brief description of them.

They are from 8.20 to 9.84 ft. diameter, including the floats; their circumference is formed by four segments or felloes of oak, extending from one arm to the other; these arms consist of two strong timbers, crossing the shaft, with a thickness of 0.49 ft. and a width of 1.148 ft. The floats, 24 in number, are 1.148 ft. deep and 0.2296 ft. thick: the middle is hollowed out to half the thickness. Upon this hollow, as upon the rimmed plates of Morosi (239), falls a great fluid vein, issuing from a nearly vertical trough, whose mean length is 9.84 ft. Above, there is a wooden reservoir, commonly with a depth of 6.56 ft., and as much in breadth. A little below the orifice of issue of the trough, the water strikes the floats; beyond this, it, as well as the wheel, is contained in a circular curb or sweep, whose sides are 0.98 ft. distant from the edge of the floats.

Thus, upon a fall of 24.60 ft., or rather, of 21.325 ft. real fall, admitting as a mean 3.2809 ft. of water in the reservoir, about 14.764 ft. will serve for the impulse, and there remains but 6.562 ft. for the weight to act. The orifice of the trough being usually 0.885 ft. by .722 ft., the head being 13.124 ft., and taking 0.97 for the coefficient of contraction (51), the discharge or consumption of water will be 18.01 cub. ft.

Generally, in the forges of the Pyrenees, it is computed that, with a fall of from 22.96 to 26.25 ft., there is required 17.658 cubic feet of water per second to move a hammer of from 1323 to 1543 lbs. a height of from 0.984 to 1.476 ft., which strikes from one hundred to one hundred and twenty blows per minute.

A bucket-wheel of 19.68 ft. diameter will produce a like effect with but 11.654 ft. of water only: the economy would be great, and advantage should be taken of it in a place where there is a scarcity of water; but where there is an abundance, it is possible that it may be better to establish one of the float-wheels just described, than to employ a wheel of double the height, nearly eight times the width, and whose construction, establishment and maintenance will require a much greater expense.

3. *Wheels moving in an indefinite fluid.*

Principal dimensions.

329. These wheels are principally used in boat-mills, or mills upon barges moored in the middle of rivers.

We suppose, in this case, that there is no water-course or other construction to increase the natural velocity of the current, on its arrival at the wheel.

The diameter of these wheels never exceeds from 13 to 16.4 ft. The floats are usually twelve in number; it is thought, however, there may be an advantage in increasing this number to 18 and even to 24. According to Fabre, who has given particular attention to this kind of machine,[*] the height of the floats should not exceed $\frac{28}{100}$ of the radius of the wheel, measured to the centre of percussion; it will thus be at most a quarter of the entire radius; quite often it is but a fifth. This author made them to plunge entirely in the water; which may be an advantage in deep streams, when, by reason of some peculiar circumstance, the greatest velocity is below the surface of the current; but generally, their force is greater when a portion of the float (in its vertical position) is elevated above the surface, the portion below remaining the same. Their width varies from 8 ft. to 16.4 ft.

Inclination of floats.

330. Deparcieux, after having made the very important observation, that water produced its greatest effect when acting by its weight (for it was before supposed that it exerted its greatest action by its impulse), having remarked that the water rose upon the floats, as upon an inclined plane, as soon as their edges reached the surface of the current, and that it acted then by its weight, supposed he could increase this action by giving the floats a greater inclination. To verify this conjecture, he made a small wheel, 2.85 ft. in diameter, carrying twelve floats 0.72 ft.

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[*] Essai sur la construction des roues hydrauliques, etc., par M. Fabre. 1783. p. 297.

in height by .656 in width, and to which, by means of an ingenious mechanism, he gave such an inclination as he deemed best. This wheel raised different weights, by means of a cord passed over a pulley fixed above it. It was placed upon the small river Bièvre, near Paris, in a place where the velocity of the current was 1.148 ft., and it there served for many series of experiments. I confine myself to citing the results of one of them. The arc plunged in the water was 96°, and the weight elevated was 2.85 lbs. The angle of inclination of the floats referred to the radius drawn to their interior edge, is noted in the first column opposite; and the time of one revolution of the wheel, corresponding to this angle, is in the second column. The angle of 30° was that of the greatest effect; it increased it in the ratio of 18 to 39.[*]

ANGLE of inclination.	TIME of one revolution.
0°	39″
10°	25″
15°	19″
30°	18″
40°	20″

Bossut, with nearly the same apparatus, also made a series of experiments. In one of them, the inclination of the floats being successively 0°, 15°, 30° and 37°, the effects obtained were found in succession to be as the numbers 1000, 1081, 1083, 1037. Here, also, the angle of 30° was found to be the most advantageous, though the increase was much less than in the experiment of Deparcieux.

Even if there should be some exaggeration in the results given by the last philosopher, it is none the less positive that the inclination of the floats increases the effect of these wheels. The best method of effecting this inclination appears to me to be that already mentioned (307 and 320), which consists in inclining gradually the cross-pieces which form the floats.

I also believe, and for reasons given elsewhere (252), that borders or rims fixed upon the two sides of each float will produce a marked increase in the effect.

Theoretic effect.

331. Wheels with floats moving in an indefinite water-course having been the object of the first theory

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[*] Mémoires de l'Académie des Sciences de Paris. 1759.

given upon wheels in motion, I shall dwell for a while upon this matter.

Before the eighteenth century, machines had only been considered as in a state of equilibrium. Suppose it had been a hydraulic machine; after having estimated the effort of the current upon it, a subject to which Galileo and Descartes had made some contributions, they calculated the weight which, placed at the extremity of a lever, for example, should put it in equilibrium. If, then, it was necessary to move this weight, they either diminished it, or the length of the lever, until they attained the desired velocity. But to what point should the weight be diminished, or the velocity increased, that is to say, the velocity of the wheel, compared to the velocity of the current, to obtain the greatest effect? As to this, they were in complete ignorance.

Parent, of the Academy of Sciences in Paris, directed his attention to this object, and, after long researches, remarked that the increase of velocity should have a limit, beyond which the effect, in place of increasing, would go on decreasing; and consequently, that there was a maximum, the knowledge of which would be of great importance in the establishment of machines: he sought for it, and published the result of his calculations in a memoir, quite remarkable for the period in which it was written.* After having unfolded some new principles upon the action of gravity, upon that of motors, and upon its measurement, he shows that in a hydraulic wheel established on a current, the effort of the water against the floats is only due to the excess of its velocity over theirs; and he makes it proportional to the square of this excess: he furthermore admits that it is equal to the weight of a prism which has for its base the part of the float struck by the fluid, and for its height, the simple height due to the difference of these two velocities, so that we have

$$E = 62.45s\,\frac{(V - v)^2}{2g}\,v.$$

In the case of *maximum* of effect, the variable factor $(V - v)^2 v$, being differentiated and made equal to zero, gives $r = \frac{1}{3}V$; that is to say, that for the greatest effect, *the velocity of the floats should be one third that of the current.* This value of v, substituted in the expression of the effort, changes it to $62.45s\,\frac{4}{9}\,\frac{V^2}{2g} = \frac{4}{9}\,62.45sh = \frac{4}{9}\,\Pi$, making $62.45sh = \Pi$; thus the *effort will be* $\frac{4}{9}$ *of the weight of equilibrium* Π, employing the expression of Parent. Multiplying this effort by the velocity, $\frac{1}{3}V$, which answers to it, we have $62.45\,\frac{4}{27}\,shV = \frac{4}{27}\,Ph$; that is to say, that the *dynamic effect of such a wheel will be* $\frac{4}{27}$ *of the force of the current*, ("*of the natural effect of the current*,") in the words of the author.

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* Mémoires de l'Académie Royale des Sciences. Année 1704.

Such is the theory of Parent, regarded as a great step made in the science of mechanics, and, in fact, it was the first. It was adopted by all the savans of Europe, and applied to all wheels with floats.

Nevertheless, in 1766, Borda, in the important memoir which we have already cited (297), showed that it could not be applied to wheels with floats established in a course; that here all the particles which pass, with a velocity V, with a section s of fluid running in the course, arrive upon the wheels and impinge against them; that their number or volume is sV, and their mass $\left(\frac{62.45sV}{g} =\right) 1.9404sV$; that, in the impulse, they lose $V - v$ of velocity, and consequently, $1.9404sV\,(V - v)$ in quantity of motion; now, the quantity of motion lost by a fluid vein against a plate measures the force or effort of the impulse (234); thus the effort of the current against the floats will be $1.9404sV\,(V - v)$. This theory of Borda, for wheels contained in a course, is universally admitted; it has been so in this Treatise.

It seems to me that it is applicable also to wheels moving in an indefinite fluid. Here, also, all the particles which pass with a velocity V, with a section s of current equal to that of the float, excepting some partial deviations, which we shall hereafter notice, arrive with an impulse; their volume is also sV; and they lose, in the collision, a quantity of motion expressed by $1.9404sV\,(V - v)$.

For wheels established upon an indefinite water-course, as well as for those contained in a course, we have

$$E = n\ 1.9404.sV\,(V - v)v.$$

The section s will be that of the vertical portion of the float which plunges in the water, and n will be a coefficient comprising the corrections due to the deviations of the fluid fillets on their approaching the wheel, to the non-pressure at the back of the floats, &c.

Real effect.

332. The experiments of Bossut, made upon a small wheel, afford us this coefficient. It was 3.198 ft. in diameter; it had twenty-four floats, 0.442 ft. in breadth, and plunging 0.354 ft. in a current having a velocity of 6.081 ft. By means of a cord, wound round its axle, it was made to raise weights, gradually increased, which naturally reduced more and more the velocities. These weights and their respective velocities are noted in the adjoining table. I remark, that the passive resistances of the machine are not comprised in the weight p'; so that $p'v$ represents only the useful effect, and not the total effect or force impressed upon the wheel. Consequently, the values of n, calculated by the formula $p'v = n\ 1.9404 s V\ (V - v)\ v$, will indicate too small coefficients or ratios between the real and theoretic effect; and the coefficient, which was 0.84 for good velocities, would probably have been about 0.90, if regard had been paid, as it should have been, to the passive resistances.

p'	v	n	
		Borda.	Parent.
lbs. 3.149	ft. 3.687	0.706	1.79
4.044	3.271	0.773	1.67
4.951	2.851	0.825	1.55
5.129	2.772	0.832	1.53
5.308	2.680	0.838	1.51
5.398	2.641	0.842	1.50
5.487	2.595	0.845	1.47
5.668	2.486	0.846	1.43
5.848	2.345	0.847	1.37

On the other hand, M. Poncelet, who made observations upon the wheels of some boat-mills established upon the Rhine, at Lyons, and who has remarked that the theory of Borda expressed the results of experiments better than that of Parent, has only had 0.80 for the coefficient. Taking the mean term 0.85, though I believe it to be rather small, we have

$$E = 1.6493\, sV\, (V - v)\, v.$$

I have also given, in the above table, the coefficients derived from the formula of Parent. They present more variations, especially in the neighborhood of the *maximum*, than those of the formula of Borda; which disposes me to favor the latter. Furthermore, his coefficients are less than 1; the others, on the contrary, are greater: now, in machines, there are so many causes of loss in the effect, causes which theory cannot take into account, that usually the results of calculation exceed those of experiment, and consequently, the coefficient of reduction must be a fraction; I am so accustomed to this order of facts, that it would be repugnant to me to admit the contrary.

333. In the experiments above cited, the *maximum* of effect corresponds to the velocity of 2.641 ft., which is to that of V, or to 6.081, as 0.434 is to 1; making, then, $v = 0.434V$, the above expression of effect becomes $0.405sV^3$ (in ft. and lbs.); let us set it at

$$.400sV^3,$$

a very simple value of the total effect which this wheel can produce.

This is equivalent to $.4122Ph$ (considering that $P = 62.45sV$ and $h = .01553V^2$). We have said (313) that the effect of wheels with floats, placed in a rectilinear course, was but $0.32Ph$; that of wheels moving in an indefinite water-course would be about a third greater. But how much more considerable is the volume of water that has been used!

Wheels of steamboats.

334. The paddle-wheels which steamboats carry on each of their sides, and which, like oars, produce a progressive movement, are also similar to these wheels. Consequently, the theory which we have given can be applied to them. The determination of

their effect, however, becomes involved with a new velocity, that of the boat. Moreover, it requires the determination of two coefficients by experiment; one, relative to the resistance of the boat, would be analogous to those already mentioned in Secs. 259—270: the second regards the action of the fluid upon the wheels; they are placed in circumstances so different from those of boat-mills, that the coefficients determined for the latter cannot serve for the former, without verification and some modifications. M. Poncelet, it is true, has made some experiments, by means of the dynamometer, upon the effort exerted by the wheels of a boat made fast in stagnant water: but these are not wheels of a boat in motion; and the experiments do not seem to me to be varied enough.

Until we have some experiments entirely satisfactory, profiting by those for which we are already indebted to the philosopher just named, and applying here the theory of Parent, which leads to a more simple expression, we will give, but provisionally, for the expression of the dynamic effect of a steamboat, and consequently for the expression of the force required to be impressed on it,

$$.1142\,S\left(\sqrt{\frac{S}{s}}+3\right)(\pm V \mp u)^2;$$

S being the immersed section of midships of the boat, s the surface of that portion of the paddles which is immersed, (that of two paddles supposed to be in a vertical position), V the velocity of the fluid, u the absolute velocity of the boat. The upper signs refer to the case where the boat ascends, and the lower signs to that where it descends the stream.

The expression just given shows that the moving force to be employed will be so much smaller, as the impelled surface of the paddles is greater. But the trouble from large wheels upon boats causes us to give these paddles a width but two or three times their height, which is from a third to a fourth of the radius.

4. *Wheels with curved floats.*

335. Although undershot wheels with plane floats are not impressed with over a fourth or a fifth of the motive force applied to them, they have still some

advantages, which lead to their frequent use; their establishment, even when well made, is attended with small expense, and they may receive quite a great velocity without any notable loss of their effect. M. Poncelet has undertaken, with a full preservation of these advantages, to avoid their enormous loss of force, and has accomplished his purpose, in a most satisfactory manner, by substituting curved floats for the plane. He gave a description of his important machine in a Memoir, (for which a prize was awarded by *the Institute* in 1825,) to which he afterwards made some additions, and which is in the hands of all engaged upon hydraulic machines; I shall confine myself to a succinct exposition of the theoretic principle of this wheel, and of the effect of which it is capable.

Principle. 336. Let us suppose a wheel with curved floats, and so disposed that when a float has arrived at the bottom of the wheel, the inferior element of its curvature is horizontal and its superior element vertical. We will at first admit that it is in a state of rest, and that a fluid fillet, animated with a velocity V, arrives horizontally upon its inferior element. Continuing to advance, it will rise up along the curve; during its elevation, gravity will by insensible degrees deprive it of its velocity V; and it will be entirely lost, conformably to the general laws of the ascent of heavy bodies, when it shall have attained the height $.015536V^2$; then it will descend; it will rejoin the float, if it had passed it; it will follow it, pressing again upon it; gravity, during its descent, will restore the velocity of which it had deprived it during the ascent, and it will quit the float with the velocity V which it possessed on its arrival. Suppose, now, that the wheel turns with the velocity v at its periphery. As soon as the fillet, having always the velocity V, attains the inferior element of the lowest float, it will have, relatively to it, the velocity $V-v$; it is with this relative velocity that it commences advancing and ascending upon the curve; it will rise nearly to the height .0155 ft. $(V-v)^2$; and after descending, and on quitting the inferior element, it will have then in relation to it the velocity $V-v$. But this element moves itself, with a velocity v, in a direction exactly opposite; consequently, the absolute velocity of the fluid at its issue will be $V-v-v=V-2v$. If $v=\frac{1}{2}V$, it will be $V-V$, or zero; that is to say, if the velocity of the wheel is half of that which the fluid had on its arrival, its absolute velocity on quitting the floats will be nothing. We have here, then, a motive current, which experiences neither shock nor loss of velocity at the instant it joins the wheel, and which possesses none at the moment of quitting it; it has then expended upon it all its motion, and has communicated to it all its force; the two conditions for the production of the greatest possible effect (297) are thus fulfilled in the wheel of M. Poncelet, such as we have represented it. Thus, if P is always the weight of the fluid furnished by the current in 1″, and h_1 the height due to the velocity V, the effect will be expressed by Ph_1.

But what is true for a simple fillet is no longer so for a mass or sheet of water of a certain thickness. Its molecules strike the floats, making an angle more or less great with the elements impressed, and so lose both velocity and force. This mass, at the moment of its quitting the floats, no longer moves in a direction exactly opposite to them. Moreover, as in all wheels which turn in a mill-course, a part of the motive water escapes, without exerting any useful action. So that the real effect will no longer be Ph_1; it will be but a portion of it.

Experiments of M. Poncelet. Consequences.

337. M. Poncelet has also determined the amount of this portion, that is to say, the ratio between the effect really produced, and the force employed to produce it; he has deduced it from many series of experiments.

He first made use of a small model of a wheel, having a diameter of 1.64 ft., and of the form indicated in Fig. 63; and he made thirteen series of observations analogous to those made by Smeaton upon a wheel with plane floats (312). I give in the following table what relates to the determination of the *maximum* effect, in eight of these series.

HEIGHT			Water expend'd per second.	Weight raised and resistances	Velocity of wheel.	RATIOS.		
of opening of gate.	of total fall. H	due to velocity V. h_1	P	p	v	$\frac{v}{V}$	$\frac{pv}{Ph_1}$	$\frac{pv}{PH}$
ft.	ft.	ft.	lbs.	lbs.	ft.			
.0328	.456	.298	2.073	.183	2.165	0.50	0.65	0.42
	.797	.548	2.823	.376	3.477	0.59	0.63	0.44
.0656	.357	.256	3.572	.346	2.001	0.49	0.75	0.54
	.521	.397	4.366	.503	2.296	0.46	0.67	0.51
	.797	.633	5.844	.635	3.540	0.56?	0.61	0.49
.0984	.357	.239	5.315	.635	1.935	0.49	0.76	0.51
	.521	.381	6.550	.796	2.329	0.47	0.74	0.54
	.797	.617	8.580	1.168	3.248	0.52	0.72	0.56

M. Poncelet also operated, on a larger scale, upon a wheel 11.745 ft. in diameter, comprising, between two circular plates like those of bucket-wheels, thirty floats, 1.246 ft. high in the direction of radius, and 2.493 ft. wide. I give below the result of seven observations, remarking, 1st, that it was admitted, after some preliminary experiments, that the velocity V of the fluid, on its arrival at the wheel, was in the mean equal to the velocity due to the head h, and consequently, that $h_1 = h$; 2d, that p' represents solely the weight really raised by the friction brake, by means of which the experiments were made; thus $p'v$ is only the useful effect; while, in the preceding table, p including the passive resistances, pv was the dynamical effect.

Opening of the gate.	H	h	P	p'	v	$\frac{v}{V}$	$\frac{p'v}{Ph}$	$\frac{p'v}{PH}$
ft.	ft.	ft.	lbs.	lbs.	ft.			
.328	5.216	4.691	615.3	183	8.00	0.46	0.51	0.46
.688	4.002	2.657	974.8	264	6.79	0.52	0.70	0.56
.722	4.168	3.444	1160	302	8.92	0.60	0.68	0.56
.656	4.986	4.297	1182	352	8.63	0.52	0.60	0.52
.997	2.657	1.804	1157	227	7.41	0.69	0.81	0.55
	3.969	3.117	1438	383	8.63	0.61	0.74	0.55
	4.986	4.143	1784	476	9.61	0.59	0.63	0.52

It will be observed, in these two tables, that the small openings of the gate rendered an effect much less than the others.

From these experiments and observations, M. Poncelet concludes,

1st. That the velocity of the wheel which gives the *maximum* of effect is 0.55 of the velocity of the current. It may, however, vary from 0.50 to 0.60 without notable disadvantage.

2d. That the dynamic effect is not below $0.75Ph$ for small falls with great openings of the gate, nor below 0.65 for small openings and great falls.

3d. That this same effect, compared to the entire force of the motor, or PH, will be 0.60 of it; and it may descend to 0.50 in very small openings.

Expression of effect.

338. For the cases usually presented in practice, and for wheels well arranged, with velocities which do not differ considerably from 0.55 of that of the current, we shall admit, having regard to the passive resistances,

$$E = 0.75Ph \text{ and } E = 0.60PH.$$

We have seen (313 and 314), that for wheels with plane floats, the numerical coefficients of these two expressions of the dynamic effect were but 0.32 and 0.25; so that the effect of wheels with curved floats is more than double that of wheels with plane floats. This conclusion, to which we have arrived in such a manner as to combine the experiments which have been made on both, would lead us, in good constructions, to avoid entirely wheels with plane floats, and to use instead those with curved floats.

Rules relative to the floats.

339. I refer here to the Memoirs of M. Poncelet,* for the rules to be followed in the establishment of wheels with curved floats, and I make here only a few observations upon their characteristic part, the floats.

1st. Their number should be double that which we have indicated for wheels with plane floats (306).

2d. Their height in the direction of the radius, or the distance between the exterior and interior circumference of the wheel, should always be more than a fourth of the effective fall; we should give it a third in falls of 4.593 ft.; and one half in those which are below this.

3d. The inferior element of the curve, which we have seen to make no angle, or nearly none, with the exterior circumference, when the sheet of motive water was extremely thin, will make one of 24°, 30°, and, generally, greater according as the sheet is thicker. We give this element its proper direction, and to the floats the curve which they should have, by means of the following draft; from the point A, where the surface of the current BA meets the exterior circumference, raise the perpendicular AK, and from the point C, where it intersects the interior circumference, with CA for radius, describe the arc AE; it will fix the form of the floats. They should be made of narrow planks, united like the staves of a cask, or of single large planks curved by fire, or of strong iron plates. Fig. 64.

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* Mémoires sur les roues hydrauliques à aubes courbes, mues par dessous. 1827.

4th. A little beyond the vertical diameter of the wheel, we lower by a sudden step the floor of the tail race, so that the water may experience no obstacle in issuing from the floats; otherwise, the effect would be subjected to a considerable diminution. Thus, M. Poncelet, who, in the last experiment of the above table, had $p'v = 0.63Ph$, with a step of 0.984 ft., had but $0.54Ph$, the step being 0.262 ft.

Example. Saw-Mill.

340. In a place where the current presents a fall of 5.249 ft., we wish to establish a mill for sawing timber, which is to saw 129.168 sq. ft. per hour; that is, to make a cut 3.2809 ft. wide and 39.371 ft. in length. The wheel, or prime mover, is to have curved floats, and it is required to indicate its dimensions, as well as the quantity of water necessary to put and keep the mill in action.

We know that a saw moved by a force equivalent to a horse-power, will saw, as a mean, 53.820 sq. ft. of timber in an hour; or, more generally, that the sawing of 10.764 sq. ft. is equal to a useful effect of from 325615 to $434154^{\text{lbs.-ft.}}$, according to the quality of the timber to be sawed.* Let us adopt, to prevent misconception, the last of these two numbers: the 129.168 sq. ft. to be sawed per hour, or 3600″, will be equivalent to a useful effect of $1447.1^{\text{lbs.-ft.}}$ per second. The resistances of the carriage, and of other parts of the machinery, will absorb nearly an equal quantity of action: so that the dynamic effect to be produced will be $2894.3^{\text{lbs.-ft.}}$ $(= E)$.

Upon a fall of 5.249 ft., we will take 0.4921 ft. for dispositions relating to the mill-course, and 0.3937 ft. for half the opening of the gate; there will remain, then, for the head, but 4.3632 ft. $(= h)$.

With these numerical values of E and h, the formula $E = 0.75Ph$ gives $P = 884.7$ lbs. By the formula $E = 0.60PH$, we have 1105.8 lbs. We will adopt this last value, and, making a small increase, we will count upon a consumption of 15.892 cub. ft.

The head being 4.3632 ft., the velocity due to it will be 16.758.

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* Architecture hydraulique de Bélidor et M. Navier. Tome I., p. 509.
Calcul de l'effet des Machines, par M. Coriolis. p. 246.

For .95 of this, or for the velocity of the fluid in the course upon its arrival at the wheel, we shall have 15.92 ft.; the wheel will take nearly .55 of this: thus the velocity at its periphery will be 8.756 ft.

It corresponds to the mechanism adopted, to have the wheels make eight turns per minute. Consequently, we give it a diameter of 21.3258 ft.; the floats, in number sixty-eight, will be 1.968 ft. deep, in the direction of the radius, and their breadth between the shroudings 2.296 ft. I will observe, in relation to this last dimension, that the thickness of the sheet of water in the course having to be nearly 0.5249 ft., it would be proper to give it a width above the wheel of 1.9027 ft. $\left(=\frac{15.892}{15.912 \times .5249}\right)$.

5. *Bucket-Wheels.*

341. These are the most powerful of wheels. Of all hydraulic motors, there are none which combine in the same degree, force and simplicity, or, at least, economy in the expense of establishment and maintenance. They are also very frequently used, and should be used, to the exclusion of all other wheels, for falls between 9.84 and 39.37 ft. The importance of the subject leads me to treat of it to some extent; moreover, it is the kind of wheel with which I have been most occupied, and almost my favorite machine.

a. Wheels receiving water at the summit.

Principal parts of a wheel with wooden buckets.

342. These wheels, called also *overshot wheels*, are made of wood or iron. The state of our forests, notwithstanding their decay, will still admit of our using the first for a long time, and they are generally by far the most economical.

Such a wheel consists, 1st, of an axle or shaft, with its gudgeons; 2d, of different arms, with their auxiliaries; 3d, of a crown or shrouding, with its lining and its buckets.

343. The shaft is formed of a piece of sound oak. Its length depends upon the breadth of the wheel; and it is made from 1.64 to 2.62 ft. square. Its two ends are rounded, slightly conical in form, so as to receive the great iron rings, whose purpose is to consolidate these extremities, weakened by the notches which have been made for the reception of the gudgeons.

These journals are of iron, and they turn upon *plummers* of the same material, or, in machines carefully constructed, of brass. Their diameter, for great wheels weighing from 26465 lbs. to 32082 lbs., is from 0.65 ft. to 0.82 ft.

According to a rule given by the English engineer Tredgold, calling ϖ the weight which a gudgeon should support, its diameter should be $.0.00287 \sqrt{\varpi}$; prudence requires us to carry it to $0.0038 \sqrt{\varpi}$. The same engineer gives to the length of his gudgeons but five sixths of their diameter.

The gudgeons make a part of the cranks, which are very often used to transmit the motion. I remark, on the subject of this transmission, that when a wheel is required to communicate a movement of vibration to a beam, for example, or a reciprocating motion to rods, the crank, notwithstanding the irregularity of velocity which it occasions, is still the best means to be used; it is preferable to cams, and even to eccentric wheels: it accompanies better the pieces to be moved, and it also prevents shocks and jarring, great causes of the depreciation and ruin of machines.

344. The arms should never pass through the axle, as they would weaken this principal part. They are united to it in pairs; and the two couples, disposed in the form of a cross, leave between them a square embracing the axle, and upon which it is fixed by means of wooden wedges, driven forcibly home, among which are also inserted some small iron wedges; there is not a more convenient or solid connection to be had. Each of

these crosses with four arms sustains one of the two cheeks of the crown; there are thus required eight principal arms. They are strengthened by cross-pieces, in wheels exposed to severe shocks, as those which set in motion forge hammers. In great wheels, the increased strength is effected more conveniently by eight *auxiliary arms*, placed and disposed as we see in Fig. 54, (the crown of which, in proportion to the arms, should have only one half of the size represented).

When the wood (oak or fir) is sound, and no particular considerations require a great weight of wheel, we may dispense with giving to the arms such large dimensions as are usually adopted. In the greatest and most elegant of those which I have seen, which had a diameter of 42.65 ft., and served for draining the water of the mine of Huelgoat in Brittany, the arms were only 0.62 ft. square near the axle, and 0.46 ft. square at their extremity.

In modern constructions, in place of uniting the arms in the manner just indicated, they are disposed as radii, such as we see in Figs. 51 and 63. They are maintained at their extremity opposite the crown by great cast iron rings, previously fastened to the axle by means of wedges.

345. The crown is formed of two cheeks, or circular plates, composed of planks from 0.196 to 0.262 ft. thick, dressed and connected like the felloes of a wagon wheel. They are lined upon the exterior face with planks having half their thickness; sometimes this lining only covers the joints of the felloes and the part of the crown which receives the extremity of the arms. Formerly, they gave to the cheeks a width of 1.31 ft. and even 1.64 ft.; then it was reduced to 1.06 ft.; and in the beautiful English wheels, we have but from 0.85

to 0.88 ft. We may adopt 0.98 ft. for wheels of all sizes, even for those which receive a great volume of water, and move with a great velocity; in this case, we increase the width of the wheel, that is to say, the distance between the cheeks; we shall soon see (349) on what considerations it is determined.

When it is fixed, and the two circular plates have been placed accordingly, we nail against their inner edge, that which is toward the axle, cross planks from .098 to .131 ft. thick, and which stretch from one plate to the other; we join them as close as possible, like the staves of a barrel; the cylinder thus presented is the *bottom* or *lining* of the crown.

Between these two plates, and in the mortises cut for this purpose upon their interior face, we place the two planks or plates, which, with the lining, constitute the bucket. As this is the essential part of the wheel in regard to the dynamic effect which it may produce, I dwell a while upon its form, that is to say, upon the position to be given to the plates.

Draft of the buckets.

346. I indicate, first, the design that experience has recognised as that uniting the greatest advantages with the most simplicity, in the countries where the construction of bucket-wheels has received the best care and consideration.

Fig. 55.

The height or the diameter of the wheel being once fixed upon, with its half, as a radius, we describe the *exterior* circumference; the arc AS forms a part of it: then, with a radius less by AB, the width of the crown, we trace the *interior* circumference BQ; AB, which we have generally fixed at 0.984 ft., is the *depth of the buckets*. Finally, from the common centre, with a radius ending at C, CB being the third of AB, we describe the third circumference, CDE:

the centre of gravity of the water contained in the buckets being usually found upon it, or very near it, its radius will be the *dynamic radius* of the wheel. The distance from one bucket to the other, measured upon this last circumference, is, as a mean, 1.049 ft.; but as constructors commonly divide the wheel into quarters, and place in each an integral number of buckets, this distance experiences a small variation, according to the size of the wheels. Consequently, the number of buckets will not be exactly proportional to the diameter; it will be that indicated in the adjoining column. The diameter designated in the first column is that of the exterior circumference; this is the diameter of the wheel, properly speaking, and we designate it by D.

DIAMETER.	NO. OF BUCKETS.
ft.	
9.842	24
13.123	36
16.404	44
19.685	56
26.247	76
32.809	96
39.371	108

The circumference being divided into as many equidistant points, C, D, E, &c., as it is to have buckets, from each of these points, we draw, towards the centre, the lines CB, DF, EG, &c.: they fix the position of the small plates. That of the great, AD, HE, &c., is determined by the following consideration: it is necessary that the angle HEG, comprised between the two pieces of the same bucket, should have the least possible opening, so that the bucket may retain the water a longer time; but, at the same time, it is necessary that it should be sufficiently open, so that the space D*b* may not be contracted to such a degree as to cause any difficulty in the arrival of the water upon the bottom or *shoulder* of the bucket, and so that there shall be no rebounding and tendency to throw the water outwards, before it has reached it. Consequently, it is necessary that D*b* should be somewhat greater than the thickness of the fluid sheet which falls upon the wheel: by widening this sheet, as well as the buckets, it is true, we can diminish this thickness nearly at will: still, we should not give to D*b* less than from 0.36 ft. to 0.39 ft. For this purpose, we make the angle HEG from 110° to 118°, according as the wheels are from 13.1 ft. to 39.37 ft. in diameter; so that the inclination of the great plate or arm of the bucket referred to the exterior circumference of the wheel, that is to say, to its tangent at the point of contact, will be about 31°: it never should be over 33°. In my practice, I obtain these advantages of a good construction, and very simply, by producing the extremity H of the arm of the bucket from .098 to 0.131 ft. beyond the point *d*, which is the extremity of the radius passing through the point F.

347. Another form, quite common in France, consists in making $IK = \frac{1}{2} IP$, and in extending the edge of the arm only to L, the extremity of the prolongation EL of the small plate of the anterior bucket.

Sometimes the form seen in NOP is given to the great plate: the water is then retained a longer time. But the workmen are reluctant in taking hold of constructions which are not, either in execution or in repairs, extremely easy; consequently, in many cases, they place the small plate perpendicular to the great, simply because the joint of a right angle is the most simple and easy of execution. Without any such obstacle, it would be best to curve the great plate, as indicated by the arc RS, whose extremity S makes a very small if any angle with the circumference: in this method, which is recommended, and easily executed with iron-plate arcs, the buckets hold the water even to the lowest point of the revolution. Finally, if these last dispositions, which we have just indicated, are more favorable for holding the water upon the wheels, they are a little less for its entrance into the buckets.

Frequently, the small plate or *shoulder* of the buckets is pierced with two or three holes of 0.13 ft. in diameter. When, after having passed the lowest point of the wheel, a bucket rises, the exterior air, entering through these holes, prevents the vacuum tending to be formed there, and which would be formed if it were exactly closed, and in consequence of which the bucket would bear with it a mass of water acting in an opposite direction to the motion of the machine. Moreover, it is through these holes that the buckets are emptied when the wheel is stopped.

Mode of conveying water to the buckets.

348. As to the mode of bringing water upon overshot wheels, there are two cases to be distinguished: that where the level of the reservoir is nearly constant, and that where it presents great variations.

In the first case, at one or two decimetres (or from 0.328 to 0.656 ft.) below this level, we establish the mill-course, to which we give a width nearly equal to that of the wheel. At its origin, it is widened, so as to avoid the contraction of the fluid; and, at the extremity of the widening, we establish the gate, which serves to regulate the quantity of water to be let upon the wheel. Beyond this, the mill-course is directed in a right line towards the wheel, with a slope of about $\frac{1}{10}$: it passes to some centimetres (centim. = 0.39 in.) only above its summit; it continues for a length of from 1.64 to 1.97 ft., gradually contracting, in such a manner that its width, at the extremity, shall be about a decimetre (.328 ft.) smaller than that of the buckets. The sheet of water which it conducts, arriving at the end, falls freely in the second or third bucket, reckoning from the summit.

If the level of the reservoir is subject to frequent rise and fall, we adapt to the bottom of this reservoir a course closed at the top, and whose upper face is inclined, as we see in Fig. 54. It is terminated by a trough, like the frustum of a pyramid, whose faces make with the axis an angle of 6° or 7°; and this axis is inclined and directed towards the upper part of the small plate of the bucket, which faces against it in such a manner, that it may be struck as directly and perpendicularly as possible; the action of gravity, as well as the movement of the wheel, approaching the direction of impulse on the inferior plate. The width of the orifice, or its horizontal dimensions, should be a little smaller than that of the buckets, and its height should not exceed 0.328 ft.; nearly always, it is much less. These course-troughs, much used at the forges of Dordogne and of Lot, bear the very significant name of *duck-bills.*

In many places, these troughs are fitted to boxes or small reservoirs, established inside the mills and above the wheels; they are called "*water-chambers.*" Their water is brought from the great reservoir, by tunnels or conduits. The resistance of the sides of these conduits, diminishing the force of the current, the water is always at a lower level in the chamber than in the reservoir (182); here is a loss of fall upon the wheel, and consequently a loss of moving force, without being compensated by any special advantage.

Width of wheel.

349. The width of the buckets, that of the wheel between its two shrouds, is determined by the volume of water it is to receive and carry.

Let Q be the volume issuing from the mill-course in 1″, d the distance between the buckets, reckoned upon the exterior circumference, and v' the velocity of the points of this circumference; it is evident that there will pass in one second, at the mouth which discharges the water, a number of buckets equal to $\frac{v'}{d}$, and consequently, that each of them will receive a volume of water equal to Q divided by $\frac{v'}{d}$, or to $Q\frac{d}{v'}$. It is necessary that the bucket should contain not only this quantity, but also a quantity three times as great; otherwise, the water would be discharged too quickly. If l represents the width of a bucket, S the area of its transverse section, or rather, the section of the fluid mass which it can contain the instant that it is immediately under the jet issuing from the mill-course, Sl will be its capacity, and it is requisite that $Sl = 3\frac{Qd}{v'} = 180\frac{Q}{MN}$, M being the number of buckets of the wheel, and N the number of turns which it makes in a minute, since $d = \frac{\pi D}{M}$ and $v' = \frac{\pi DN}{60}$. Thus ,$l = 180\frac{Q}{MNS}$; such will be the width, in practice, to give the wheel. The quantity Q, introduced into this expression, is what the wheel should consume, to produce its total effect, and we shall see hereafter (366) what this quantity is; N will be the number of turns made in 1′, under the same circumstances, and S will be determined upon the section of the bucket.

In my method of disposing a wheel and its buckets (346), we have, with slight variations, S = 0.775 sq. ft., and $M = \frac{\pi D}{1.115}$, and consequently, $l = 82.29\frac{Q}{ND}$.

Theoretic Effect.

Theoretic effect, or the force impressed upon the wheel.

350. The force of a current which conveys a quantity P of water in one second, and which falls from a height H, is PH (280). If, in its fall upon a machine, it there exerts all the motive action of which it is capable, it will impress this same force upon it; but this is rarely the case in practice, and the force really impressed is less than PH (283). In a bucket-wheel, all the fluid of the current acts upon it, from the point where it first meets it, to that where it leaves it (for we may always admit, and we shall admit, that it quits it instantly); thus the factor P undergoes no diminution. It will then be experienced by the other factor H, and consequently a part of the total fall will be as it were lost. *So that, to have the force effectively impressed upon a bucket-wheel, we have only to determine the different losses of fall which have occurred between the level of the reservoir and the bottom of the wheel, and to subtract them from* H, *in the expression* PH.

Fig. 54.

Let there be a wheel ERF, receiving its water from the reservoir M. Take AB, the vertical distance between the level of M and the extremity F of the vertical diameter, to represent the total fall H. Divide this distance into three parts; AC ($=h$), comprised between the surface of the reservoir and the point where the water strikes the wheel; CD, equal to fg, *the height of the arc charged with water;* and DB, the distance between the point where the water is supposed to issue from the buckets, and the bottom of the wheel. Let us point out and estimate the value of these losses, upon each of these parts.

351. It is at once manifest, that there can be no loss

Force impressed by the water contained in the buckets.

upon the height CD of the arc charged with water; the fluid P, acting constantly and with all its weight upon the portion of the wheel corresponding to this height, will impress upon it a force P . CD.

Though this assertion may be regarded as evident, and though it is an immediate consequence of what has been said upon motors and their effects, I proceed to give a direct and synthetic demonstration.

In the first place, the effort made to produce the motion of rotation, by the weight of the fluid borne by the arc charged with water, that is to say, by the weight of the fluid contained in the buckets, from the level of the point C to that of the point D, is equal to the effort which would be exerted by the weight of a prism of water GH, placed at the extremity R of the dynamic radius OR, of which $GH = CD$ will be the height, and which will have for its base the section of the fluid arc (the water of the buckets being supposed to be uniformly distributed upon this arc). To demonstrate this, it is sufficient to show that the momenta of these two efforts are equal. For this purpose, let us suppose that the arc charged with water is divided into an infinity of small elementary arcs, such as *mn;* designate by σ the section of the fluid arc, and by φ the specific weight of the water; $\sigma \,.\, mn \,.\, \varphi$ will be the weight of the small arc *mn;* since it acts vertically, the distance between the direction of its effort and the centre of rotation will be the horizontal *rs:* thus we shall have for its momentum, $\sigma \,.\, mn \,.\, \varphi \,.\, rs$; as the similar triangles, *mnt* and *rOs*, give $mn \,.\, rs = Or \,.\, pq$, we shall also have $\sigma \,.\, \varphi \,.\, Or \,.\, pq$. The sum of all these partial momenta, or the momentum of the entire arc, will be, then, the common factor $(\sigma \,.\, \varphi \,.\, Or)$, multiplied by the sum of the small heights *pq* of the elementary arcs; now, this sum is evidently *fg* or CD; its momentum will consequently be $\sigma \,.\, \varphi \,.\, Or \,.\, CD$. That of the prism GH is manifestly $\sigma \,.\, \varphi \,.\, GH \,.\, OR$. And since $GH = CD$, and $Or = OR$, the two momenta are equal.

In the second place, the effort exerted at R, and in the direction of motion, being $\sigma \,.\, \varphi \,.\, CD$, and the velocity of the point R being v, the force impressed will be $\sigma \,.\, \varphi \,.\, CD \,.\, v$ (278). Since Q is the volume of water passing in a second, and since in this time this volume should pass through the section σ with the velocity

v, which is likewise that of the fluid after it has reached the wheel, we shall have $Q = \sigma v$: moreover, we have $P = \varphi Q$. Taking the values of σ and of φ, in these two equations, and substituting them in the above expression of the force impressed, it becomes $\frac{Q}{v} \cdot \frac{P}{Q} \cdot CD \cdot v = P \cdot CD$, as we had already determined it.

Losses of head above the arc charged with water.

352. We come now to the losses of head which take place above the arc charged with water, that is to say, upon the height AC.

AC or h is the height due to the velocity with which the fluid arrives at the wheel, and with which it gives the impulse, provided it has suffered no diminution between the reservoir and the buckets: but this is not usually the case, and when the fluid meets the wheel, the height due its velocity is only h_1, a quantity smaller than h by μh, since $h_1 = h\ (1 - \mu) = h - \mu h$ (300). The value of the coefficient μ depends, 1st, upon the loss of velocity occasioned by the contraction which the fluid sheet undergoes in its passage through the orifice of the reservoir; 2d, from the resistance which it meets against the sides of the mill-course which conducts it to the buckets; 3d, from the dispersing of the fluid fillets, many of which strike upon the edges and sides of the bucket, losing a portion of their velocity before reaching the small plate, or the water which covers it; 4th, from the oblique direction with which the sheet arrives upon this plate; this obliquity is often as much as 30°, whence results then a diminution of 0.14 in the value of h, and consequently of the force of the impulse. These causes combined may bring μ up to 0.1, to 0.2, and even to 0.3, according to local circumstances, such as a more or less perfect arrangement of the gate-way. We may determine its value according to these circumstances: however, we shall have only an approximation; for a rigorous determination is an impossibility. Let Aa be the portion of the fall AC representing this value, the remaining part, aC, will be h_1, or the height really due to the velocity V of the fluid, and consequently equal to $.01554V^2$.

353. According to what has already been said (297), this height will still be subject to two other diminutions or losses, ab and bd; the one h' ($=.01554v^2$), which is the height due to the velocity v of the wheel, and increases with this velocity; the other h'', or $.01554(V-v)^2$, which is the height due the velocity lost by the shock, and which, on the contrary, is diminished when this same velocity v increases. The sum of these two losses will be the smallest possible, or $.01554\{v^2+(V-v)^2\}$ will be a *minimum*, where $v=\frac{1}{2}V$; they will be equal to each other, each one will be $\frac{1}{4}0.01554V^2=\frac{1}{4}h_1$, and the two combined ($ad$) will be equal $\frac{1}{2}h_1$. In this case, that of the *minimum* of loss, the remaining part, dC, which alone remains the *effective fall* (inasmuch as we have for the impressed force but $P \times d$C), will be equal to $\frac{1}{2}h_1$ ($=\frac{1}{2}a$C), and it will consequently be smaller than $\frac{1}{2}h$ ($=\frac{1}{2}$AC).

If the sum of the two losses h' and h'' are not less than $\frac{1}{2}h_1$, it may be, and most generally will be, much greater; it will be so much the greater, according as the inequality between h' and h'' is the more considerable; it will be at its *maximum*, if one of these two quantities, h'', for example, becomes nothing, which would be the case if we had $V=v$; in this case, we should have $h'=0.01554V^2=h_1$, or $ab=a$C; and there would remain nothing for the effective fall. But, in reality, this will not happen: V, which cannot be less than v, will predominate; h'' will have a real value, and there will always be an effective fall, though it may be exceedingly small.

354. Recapitulating, we say, that this fall is essentially smaller than $0.5h$; that, in very good dispositions of the wheel, it may be $0.4h$; and that, in ordinarily good dispositions, it will not be over $0.33h$. So that, *in hydraulic wheels*, *about two thirds of the height comprised between the level of the reservoir and the point where the fluid strikes the wheel*, *is lost for all effective work.*

The effect due to this height h, and which is generally $P\ (h - \mu h - h' - h'')$, will then be, most commonly, $P\ (h - \frac{2}{3}h) = \frac{1}{3}Ph$. This result is analogous to that which experience has given for the best wheels, moved solely by the impulse of water, as we have seen in Sec. 313.

355. Since a third only of the part of the fall above the point where the water strikes the wheel is effective, while the part which is below, even to the point of discharge of the buckets,—that is to say, the height of the arc charged with water,—is entirely so, there is a manifest advantage in increasing the latter at the expense of the former; that is to say, *that in establishing a bucket-wheel so as to receive from the motive current all the force which it can impress upon it*, *we should make the distance between its summit and the level of the reservoir as small as possible.* There would, however, be no benefit in diminishing it to such a point, that the fluid would arrive at the wheel with a velocity less than that of the buckets; it would not act upon them, until, in continuing to descend, it had acquired a velocity equal to theirs. (Whence it happens, that at the point where the fluid exerts its action, we should not have $v > V$.)

Finally, this distance between the point of action and the level of the reservoir will always have, in reality, a notable value. We should never establish the summit of the wheel at less than 0.984 ft. below this level; and between the summit and the point where the fluid may act directly or indirectly upon the plates, there will always be about 0.984 ft.; so that, usually, h will be 1.968 ft., at least. Let us admit such a value; make $\mu = 0.2$, and consequently $\mu h = .3936$ ft.; there will remain $h_1 = 1.574$ ft. Taking the quarter of this quantity for h', we shall have $v = 5.0294$ ft.: this will be the velocity with which the wheel will render its greatest effect, (deducting, however, a slight diminution, produced by the velocity, in the height of the arc charged with water).

If we diminish the velocity 5.0329 ft., for example, to 3.2809 ft., we shall diminish the effect; in place of the effective head of .7874 ft. ($= 1.9685 - .3937 - .3937 - .3937$ or $h - \mu h - h' - h''$), we shall only have one of .4594 ft. ($= 1.9685 - .3937 - .1640 - .9514$); and the two effects will be to each other as $a + .7874$ is to $a + .4594$, a being the height of the arc charged with water.

In case we give the wheel all the velocity it can have with $h = 1.9685$ ft., which would be the velocity of the fluid $V = \sqrt{2gh_1} = 10.072$ ft., there would no longer be an effective head, and the effect, compared to the preceding, would not be greater than a.

Loss of fall below the arc charged with water.

356. Let us examine now what takes place below the arc charged with water.

The portion of the fall DB found there is evidently entirely lost. It is composed of two parts; the one eB is lost by reason of the form of the buckets, that is to say, by reason of the inclination of their great plate; and the part De is lost by an effect of the velocity of the wheel, or rather, by the centrifugal force resulting from it.

Leaving out of the account the action of this force, the surface of the water contained in the buckets is horizontal. According as, by reason of the revolution of the wheel, the buckets descend, this surface approaches gradually the edge of the great plate or arm;

the instant after having attained it, and having assumed in consequence the position *hi*, the pouring of the water from the buckets begins; and it ends when this plate has arrived at the horizontal position *kl*. The arc F*h*, which measures the distance from the bottom of the wheel to the point where the water begins to pour out, will be the arc for *the commencement of discharge*, and F*k* will be that for *the end of the discharge*.

This last is equal to the angle *ukl*, which the great plate makes with the tangent at the circumference, an angle which is known from the rules adopted in the tracing of the buckets, and which we shall designate by a. The arc F*h* is equal to F*k* + *kh*, and *kh* is equal to the angle *xhi* which the great plate makes with the surface of the water, at the commencement of the discharge, an angle which we shall call z; thus, $Fh = a + z$.

Whatever may be the magnitude of these two arcs, or the law by which the volume of water discharged at each instant by the same bucket is diminished, from the beginning to the end of the discharge, we may always admit *a mean arc of discharge;* such that the quantity of motive action due to the water borne by the wheel remains the same, whether all the water P is entirely preserved by the buckets even to the extremity of this mean arc, where it may be discharged suddenly, or whether the discharge is effected gradually, from the end of the first arc to that of the second. Usually, the distance between these two extremities is inconsiderable, and we may, without sensible error, take the arithmetical mean; the mean arc will then be $a + \frac{1}{2}z$, or F*e*′, the point *e*′ being at an equal distance from *h* and *k*.

If upon AB we take *e* at the level of *e*′, B*e* will be the loss of fall sought. Now, B*e* is equal to the versed sine of the mean arc Fe', an arc whose radius is the semi-diameter of the wheel; thus, D being the diameter, we shall have

$$\mathrm{B}e = \tfrac{1}{2}\mathrm{D}\ \{1 - \cos.\ (a + \tfrac{1}{2}z)\}.$$

357. The angle z which the fluid surface makes, at the commencement of the discharge, with the arm or great plate, depends upon the volume of water received by the buckets, as well as upon their form and dimensions; form and dimensions which will be known either by the rules followed in the construction of the wheel, or by measurements directly made. The determination of this angle being an operation of pure geometry, I propose to indicate it: the examination of the figure will, moreover, satisfy us as to the reasoning adopted.

Fig. 56. Let ABa or ABCa' be a section of a portion of the bucket containing the water at the moment when the discharge commences, and let us make $\mathrm{AB} = \alpha$, $\mathrm{BC} = 6$, $\mathrm{AC} = \gamma$; the angle $\mathrm{ACB} = \alpha'$; $\mathrm{BAC} = 6'$, $\mathrm{ABC} = \gamma'$; the surface $\mathrm{ABC} = s'$. From what has been said (349), s being the area of the section of the fluid mass contained in the bucket, $s = \frac{\mathrm{Q}d}{lv}$.

We have two cases to distinguish: that where the fluid surface Aa is below AC, then $s < s'$; and that where this surface is above at Aa', then $s > s'$. In the first, by making $\frac{2s}{\alpha} = \varepsilon$ (this is the line ab of the figure), we have $\text{tang.}\ z = \frac{s}{\alpha + \varepsilon \cot.\,(180 - \gamma')}$; in the second, $z = 6' + \omega$, and making $\frac{2\,(s - s')}{\gamma} = \varepsilon'$ (this is the line $a'c$), we have, without sensible error, $\text{tang.}\ \omega = \frac{s'}{\gamma - \varepsilon'\ \text{tang.}\ \alpha'}$.

Loss due to the centrifugal force. 358. It remains now to determine the loss arising from the centrifugal force; a loss sometimes considerable, and which, notwithstanding, has not yet been taken into consideration. A short time since, M. Poncelet, having devoted his attention to this object, determined a theorem as remarkable for its simplicity as happy in its consequences, and he made it serve in a complete demonstration of this important point in the theory of bucket-wheels. He has had the kindness to communicate with me, and I proceed to expose the principal results of his labors.

But first, I call to mind, that when a body participates in a rotatory motion, each one of its particles is animated with a centrifugal force. If m is the mass of one of them, u its velocity, and r its distance from the centre of rotation, its centrifugal force will be $\frac{mu^2}{r}$ (298); it will also be expressed by mrw^2, if w is the angular velocity of the body, that is to say, the velocity of the particles situated at one foot from the same centre, since $u = wr$.

Thus, each particle of fluid contained in the buckets of a wheel in motion is subject to the action of two forces, gravity and the centrifugal force. Let e be one of these particles; let us take ep to represent the first force mg, and, upon the direction of the radius Ce, eq for the second mw^2r; the diagonal er of the parallelogram will be their resultant; and it will be the same as if the particle was subjected to the sole action of the force which er represents in intensity and direction. If we prolong er up to the vertical drawn through the centre C of the wheel, it will meet it in the point O, such that $\text{CO} = \frac{g}{w^2}$; since $\text{CO} : \text{C}e\ (= r) :: ep\ (= mg) : pr\ (= mw^2r)$. Now, this distance CO, not depending in any wise upon the position of the particles, will be the same for all; all the directions of forces will coincide then towards O, and this point will be, as it were, the centre of action where they are directed. Fig. 57.

The surface of a fluid being always perpendicular to the direction of the force acting upon its particles, that of the water contained in the buckets will then be perpendicular to the lines drawn from O to its different points, and consequently, the section of this surface *st* will always be the arc of a circle having its centre in O.

In the revolution of the wheel, the extremity *s* of this arc will approach gradually the edge of the great plate or arm of the bucket, and it will reach it when the bucket shall have arrived at the position ABI; then, or immediately after, the discharge commences. It will cease when the bucket shall have descended into the position A′B′I′, so that the arc, the limit of the fluid, shall have passed under the plate A′B′.

Such are the grounds admitted by M. Poncelet.

(Ordinary cases.) 359. In most cases, those where the diameter of the wheel is not below 13.12 ft., and where the velocity at the periphery does not exceed 9.84 ft., we may regard the surface of the water in the buckets as plane; and consequently, it will be perpendicular to a line drawn from the point O to the centre of its figure.

With this supposition, I determine the two arcs of discharge AE and A′E. The first, or the angle ACE, for which it serves as a measure, is equal to $\mathrm{GAF} = \mathrm{GAB} + \mathrm{BAD} + \mathrm{DAF} = a + z + y$, calling y the angle DAF or its equal aOC, the point a being the middle of AD. Draw ag perpendicular to aC, and call b the angle which the first of these lines makes with the tangent AG, (supposing them prolonged), an angle which is equal to ACa; the angle OaC or its equal $ga\mathrm{D} = \mathrm{GAD} - b$; moreover, the triangle OaC gives sin. aOC ($=$ sin. y) : aC ($= r'$) :: sin. OaC ($= a + z - b$): OC $\left(= \frac{g}{w^2} = \frac{gr^2}{v^2}\right)$, whence the sin. $y = \frac{r' v^2 \sin.(a+z-b)}{gr^2}$; r' is the semi-diameter of the wheel diminished by one half the depth of the buckets. The angle b will generally be very small; it will have but a small influence upon the value of y, and consequently but little upon that of the arc of discharge; we may neglect it without sensible error; but, by way of compensation, we will substitute for r' the dynamic radius r, which is somewhat smaller, and then the sin. $y=\frac{v^2}{gr}.\sin.(a+z)$. The arc A′E is equal to the angle $a+y'$, calling y' the angle a'OE; and in a manner analogous to that employed for y, we shall find sin. $y'=\frac{v^2}{gr}.\sin.\ a$.

Here also we may admit a mean arc of discharge, and without error for application, taking an arithmetical mean between the two determined arcs, we shall have for its value $a+\frac{1}{2}z+\frac{1}{2}y+\frac{1}{2}y'$.

Its versed sine, being equal to BD (Fig. 54), will be the loss of fall arising both from the form of the buckets and from the centrifugal force; calling it h''', we shall have

$$h'''=\tfrac{1}{2}D\{1-\cos.\ (a+\tfrac{1}{2}z+\tfrac{1}{2}y+\tfrac{1}{2}y')\}.$$

I show, by way of an example, the mode of calculating this loss of fall.

The wheel is 37.303 ft. in diameter; it has ninety-two buckets, with a width of 3.5499 ft. and 1.0652 ft. deep, and the following dimensions (Fig. 56): AB $=\alpha=$ 1.5223 ft., AC $=\gamma=$ 1.6831 ft.; the angle GAB, which AB makes with the tangent AG at the circumference, where $a=31°\ 37'$, BAC $=\delta'=9°\ 08'$, ACB $=\alpha'=53°\ 10'$; the surface of the triangle ABC $=s'=$ 0.20344 sq. ft.; finally, the distance between the buckets or $d=$ 1.2247 ft.

At the time of the observation which gave rise to the present example, the wheel received in 1″, 5.2974 cub. ft. of water (=Q); its velocity, at the extremity of the dynamic radius of 17.939 ft. $(=r)$, was 8.2022 ft. $(=v)$.

The section of the water in the bucket before it commenced the outpour or $s=\frac{Qd}{lv}=\frac{5.2974\times1.2247}{3.5499\times8.2022}=0.22282$ sq. ft. This section being greater than 0.20344 sq. ft. $= s'$, to get the angle z (357), we shall have $s' = \frac{2(s - s')}{\gamma} = \frac{2 \times .01938}{1.6831} = .023028$ ft.; and consequently, the tang. $\omega = \frac{.023028}{1.6831 - .023028 \text{ tang. } 53^\circ\ 10'}$, which gives $\omega = 0^\circ\ 47'\ 54''$; thus $z = \delta' + \omega = 9^\circ\ 08' + 47'\ 54'' = 9^\circ\ 55'\ 54''$. For y and y', sin. $y = \frac{(8.2022)^2 \text{ sin. } (31^\circ\ 37' + 9^\circ\ 55'\ 54'')}{32.182 \times 17.939}$ gives $y = 4^\circ\ 39'$, and sin. $y' = \frac{(8.2022)^2 \text{ sin. } 31^\circ\ 37'}{32.182 \times 17.939}$ gives $y' = 3^\circ\ 30'$. Whence $a + \frac{1}{2}z + \frac{1}{2}y + \frac{1}{2}y' = 40^\circ\ 39'\ 27''$, and $h''' = \frac{1}{2}37.303$ $(1 - \text{cos. } 40^\circ\ 39'\ 27'') = 4.499$ ft.

Decompose this double loss. That which proceeds from the form of the buckets, or $Be = \frac{1}{2}D\,[1 - \text{cos. } (a + \frac{1}{2}z)] = \frac{1}{2}37.303$ $(1 - \text{cos. } 36^\circ\ 34'\ 57'') = 3.672$ ft. There remains, then, for eD, or for the loss due to the centrifugal force, 0.827 ft: this force has, then, increased the loss of fall, below the arc charged with water, in the ratio of 100 to 122.

360. It is thus that we should calculate this loss, when we wish to determine the dynamic effect of a wheel. Still, when it is intended to establish a wheel, and we wish to have at sight the loss of effect resulting from the discharge of the water, we may have recourse to the following table, where the values of h''' are expressed in fractions of the diameter.

DIAMETER of the wheel.	LOSS OF FALL h''', THE VELOCITY BEING 0ft.	3.2809ft.	6.562ft.	9.843ft.	13.124ft.	16.404ft.
feet.						
9.843	0.15 D	0.16 D	0.23 D	0.36 D		
13.124	0.15 "	0.16 "	0.21 "	0.25 "	0.46 D	
16.404	0.14 "	0.15 "	0.20 "	0.25 "	0.36 "	0.46 D
19.685	0.14 "	0.15 "	0.18 "	0.23 "	0.32 "	0.45 "
26.247	0.14 "	0.15 "	0.17 "	0.20 "	0.26 "	0.34 "
32.809	0.14 "	0.14 "	0.16 "	0.18 "	0.23 "	0.29 "
39.371	0.13 "	0.13 "	0.14 "	0.16 "	0.20 "	0.24 "

This table has been calculated under the supposition, 1st, that the buckets are of the number and form indicated in Sec. 346; 2d, that they carry one half of the water which the bucket that first arrives under the current can receive. This last supposition is the occasion of the loss here noted being nearly always superior to what we shall have in reality. For example, if, as usual, the buckets should have but a third of the water which the first can contain, the six multipliers of D, for the wheel of 19.685 ft., would be 0.12, 0.13, 0.15, 0.195, 0.27 and 0.38. The small anomalies in the members of the same column arise from the number of the buckets not being exactly proportional to the diameter. This table affords evidence of the effect of velocity: thus, for the wheel of 19.685 ft., the velocity being 3.2809 ft., the loss of fall below the arc charged with water has been but 2.952 ft., and it will be 6.299 ft., more than double, with a velocity of 13.124 ft.

Case of very great angular velocities.

361. The mode of determining the effects of the centrifugal force given in Sec. 359, will apply to nearly all cases occurring in practice; but we should not employ it for small wheels, which move with great velocities, such as those which put in play the hammers of iron mills; there are some which are not over 8.2 ft. in diameter, which make thirty-five turns per minute, and which consequently have a velocity 15.026 ft. at the periphery. The centre O of forces descends, then, below the crown of the wheel; from this it results, that the upper buckets cannot receive the water, or but a very little of it; those which follow will contain more; but the quantity diminishes rapidly, and the discharge is soon finished. Fig. 58, where the dotted lines represent the surface of the water in each bucket, shows this state of things. Fig. 58.

To have the force impressed on such a wheel, we will divide, mentally, the arc charged with water into a certain number of parts, ten or twenty; we will suppose a bucket placed successively in the position corresponding to each of these divisions; from the point O, CO being always $\frac{gr^2}{v^2}$, we will describe in the bucket, and for each of its positions, the limiting arc of the fluid, and we will calculate, by the rules of geometry, the section of the mass of water which is below it. Let q_1, q_2, q_3, &c., be these sections, and q that whose value is $\frac{Qd}{lv}$ (349); let, then, h_1, h_2, h_3, &c., be parts of the vertical diameter, or the falls from one position to the other; the weight of water passing in 1″ in the bucket, considered at each of its successive positions, will be $P\,\frac{q_1}{q}$, $P\,\frac{q_2}{q}$, &c., and consequently, the force impressed upon the wheel will be equal to

$$\frac{P}{q}(q_1\,h_1+q_2\,h_2+\&c.).$$

Analytic expression of effect.

362. Conformably to the principle of our theory (350), subtracting from the total fall H the four losses whose value we have assigned, we shall have, for the force impressed, or total effect, the product,

$$P\,(H-\mu h-h'-h''-h''').$$

But this expression is deduced from theoretic considerations; and consequently will not be employed in practice, until after having been put in accordance with the results of experiment. Let, as above (288), n be the coefficient of reduction, we shall have

$$E=nP\,(H-\mu h-h'-h''-h''').$$

The effect of a bucket-wheel will then be so much the greater, as the five quantities μ, h, h', h'', and h''', are smaller, or, according to what we have said in Secs. 352—355 and 359, in proportion as,

1st. The gate-fixtures and mill-course are disposed in a more perfect manner;

2d. The diameter of the wheel is the greater, relatively to the fall;

3d and 4th. For a difference between the fall and the diameter, or rather for a given value of h, h' and h'' approach the nearer to equality; this condition will be the better satisfied, in proportion as the velocity of the wheel shall approach nearer to the half of that of the fluid, on its arrival upon the plates;

5th. Finally, by reason of a good disposition of the buckets, and of a small velocity, they will hold the water at a greater height.

Real Effect.

We pass to experiments which should give us the value of coefficients, and make known the principal circumstances of the motion of our wheels.

Experiments of Smeaton.

363. Smeaton had already made, in 1759, upon a small wheel 2 ft. in diameter, a series of experiments, similar to those which he had executed upon the wheel with floats, and which we have already discussed (312). But various details, especially upon the dimensions of the buckets, which would be necessary for an application of the formulæ, are wanting. Consequently, I shall confine myself to giving the principal results at which that author arrived.

1st. Tracing, in his experiments, the ratio between the force employed and the effect produced, he saw that, relatively to this effect, the fall could be divided into two parts; the one would be the diameter, and the other would be found above it; this latter would produce a much less effect in proportion to its magnitude; and he concluded that it was best to give to the wheel the greatest possible height.

2d. In view of the small action of the upper part of the fall, he sought a ratio between the effect and the lower part, that is to say, the diameter of the wheel, and had quite constantly $pv = 0.80\text{PD}$. (Some German authors, using a ratio of the same kind, admit $pv = \frac{5}{6}PD$; that is to say, according to them, the effect would be $\frac{5}{6}$ of PD).

3d. Smeaton moreover found, that when the fall did not exceed the diameter but by a small quantity, he had $pv = 0.72PH$.

4th. He concluded then, from his various observations, especially from those made upon mill-wheels, that the velocity of a bucket-wheel ought to be from three to six feet. He was governed, moreover, by the established principle, that a bucket-wheel is the more effective, the slower it turns. What we have already seen (353, 355 and 362), enables us to appreciate this assertion according to its proper value; and we have established a limit of velocity, below which we cannot safely go without diminishing the effect. Finally, we may subject the velocity of the wheel to nearly all the variations that we may judge proper; we may go as high as 8.2 ft. and more, provided that the height of the arc charged with water, that great element of the force of bucket-wheels, does not experience any marked diminution.

I shall not stop to consider the experiments which Bossut made with a small bucket-wheel, and which may, moreover, be seen in his *Hydrodynamique* (§§ 1048—1051); he has taken no account of the passive resistances; and therefore, we cannot draw any conclusion from them.

Experiments made at Poullaouen.

364. I myself made, in 1805, at the mine of Poullaouen in Brittany, in concert with its director, M. Duchesne, some series of experiments upon a very large wheel used for the draining of the mine. The heavy loads which we put upon it were weighed, as it were, by means of a strong dynamometer, which we

graduated ourselves, by loading it successively with different weights, up to 19849 lbs. Our experiments were given in detail in vol. XXI. of the "*Journal des Mines*," to which I refer the reader, and I confine myself to citing some of them, to show the coefficients *n* and *m* which they indicate; but first, I give an idea of the machine.

The wheel, which was 37.303 ft. in diameter, and whose principal dimensions we have given in Sec. 359, carried, at each extremity of the revolving shaft, a great crank, which, through the intervention of a horizontal rod 121 ft. long, and of a bent lever, communicated a reciprocating motion to a vertical rod 321.5 ft. long, which descended into one of the pits of the mine: it there put in action seven pumps, placed one under the other, and which were in the mean 1.066 ft. in diameter and 31.1 ft. in height; their piston was hooked to an iron arm fixed and braced to the rod. Thus the machine, working fourteen pumps, could elevate two columns of water, weighing together 24,260 lbs.; usually, it did not raise more than from 11,000 to 15,000 lbs.

When we commenced our operations, M. Duchesne first caused all the pumps to be detached from the two rods, and we examined the circumstances of motion in this case, where the load consisted only of these two rods, which were in equilibrium with each other. Then to one of the rods we fastened at first four pistons, (without raising water); then, and leaving them as they were, we put in operation the first pump; then, and in succession, we increased the load of this same rod by a second pump, a third, a fourth, a fifth, and finally a sixth. The load remaining the same, we sometimes varied the velocity. The dynamometer, which was suspended at one end upon the extremity of the horizontal arm of the lever, and which at the other bore the rod, indicated in each experiment the weight of the load raised.

This load, which represents the active resistance, being referred to the extremity of the dynamic radius of the wheel, equalled the .0426 of weight indicated by the dynamometer. The passive resistances arising from the friction of the gudgeons of the wheel, as well as from the supports of the horizontal rods and of the bent lever, were calculated at different times. Still, I must not conceal the fact, that there was some uncertainty as to their true value, as well as to that of the quantity of water discharged; but as both can only be erroneous by excess, and as one occupies the place of numerator while the other is in the denominator of the values of the coefficients, I do not believe any error of consequence can exist in these last values.

The entire height H, from the level of the reservoir to the bottom of the wheel, was 38.944 ft.; and the part h, comprised between this same level and the point where the water struck the buckets, was 2.952 ft. I made $\mu=0.3$, and consequently, $\mu h=0.8856$: the other losses of fall, h', h'' and h''', were calculated by the methods above mentioned. These four losses being subtracted from H, gave the *effective head* H′, that which, multiplied by P, expresses the force impressed upon the wheel.

The following table presents the results of six of our experiments; the four last, where the loads were better proportioned to the size of the machine, deserve our chief consideration.

	Load indicated by the dynamometer.	RESISTANCES		Velocity of buckets.	Dynamic effect.	Water expended in 1″.	LOSSES of FALL.			Effective fall.	RATIO of effect to the force	
		0.0426 C	passive.								Impressed $\frac{pv}{PH'}=n$	of motor. $\frac{pv}{PH}=m$
		p		v	pv	P	h'	h''	h'''	H′		
	lbs.	lbs.	lbs.	ft.	lbs. ft.	lbs.	ft.	ft.	ft.	ft.		
1	374.9	15.8	185.8	11.28	2279	78.0	1.96		4.26	31.82	0.917	0.749
2	2194.4	93.5	192.9	7.03	2301	74.9	1.01	.196	3.64	32.20	0.874	0.745
3	3987.2	173.8	200.2	8.53	3118	106.5	1.14	.131	3.83	32.94	0.909	0.769
4	6004.2	255.8	210.3	8.39	3900	131.6	1.08	.164	3.93	32.87	0.901	0.760
5	12132.	517.7	240.7	7.51	5694	191.6	0.88	.229	4.06	32.87	0.904	0.763
6	12132.	517.7	240.7	8.20	6215	213.4	1.05	.164	4.23	32.61	0.893	0.748

The mean of the four last values of n is, then, 0.902
And that of the values of m 0.760

In reality, I believe these coefficients rather too small than too great, for the wheel at Poullaouen.

Real effect deduced from theoretic effect.

365. Notwithstanding this remark, and in default of other determinations, I shall admit for n the value just found, and in view of $P=62.45Q$, we shall have

$$E=56.203Q(H-\mu h-h'-h''-h''').$$

Such is the formula to which we have been led, and which we shall use, when we wish all the exactness afforded by science in its actual state.

366. But it may be simplified so as to make its application easier. The three quantities μh, h' and h'', taken together, are equivalent to $\frac{2}{3}h$ (354); moreover, except in extraordinary cases, h''' varies but from $\frac{1}{7}$D to $\frac{1}{5}$D (360); so that the effect will be expressed by $n'Q\,(H - \frac{2}{3}h - \frac{1}{6}D)$. Taking the coefficient given, for this new form, by the experiments at Poullaouen, we have

$$E = 59.325Q\,(H - \tfrac{2}{3}h - \tfrac{1}{6}D).$$

This expression will also serve to determine the quantity of water necessary to produce a given effect.

Ratio of effect to the motive force.

367. In bucket-wheels, the effect can also be deduced from the motive force.

Our experiments have indicated that at Poullaouen it was 0.76 of this force.

Smeaton, it is true, never found it above 0.73, in his observations upon the small wheel of 2 ft. diameter; but he himself felt but little satisfaction with the ratios which he found between the effect and PH; and the expression 0.80PD, which he adopted in preference, corresponds nearly to 0.75PH, for great wheels, in which the fall exceeds the diameter but by a few decimetres.

Lately, M. Morin has made several experiments, by means of the friction brake, upon two small bucket-wheels, well established; that of 11.22 ft. diameter has given him for effect, comprising the passive resistances, 0.71PH as a mean, and sometimes 0.80PH; for the other, having only 7.48 ft. diameter, he had, as a mean term, 0.81PH; the coefficient varied, however, from 0.71 to 0.90.

Finally, M. Egen, who made a great number of observations upon wheels of different kinds, admits for good bucket-wheels from 0.75 to 0.80.*

Adopting the smaller of these two numbers, we have

$$E = 0.75PH = 46.837QH.$$

I will observe, that in bucket-wheels, the real effect, the force really impressed upon the wheel by a given current, is obtained by the simple measure of the height of the arc charged with water, in a much surer and easier manner than by the dynamometric brake, and even that of the direct elevation of a weight; for these two means not giving all the passive resistances, there remains some one which always must be determined by calculation, which throws some uncertainty into the results, as I have proved and mentioned while on the subject of my experiments at Poullaouen. When the buckets take all the water of the current, and preserve it to the point of discharge, which is the case with all well arranged wheels, their total effect is to the force of the motor at least as the height of the arc charged with water is to the total fall; I say *at least*, for there always remains something, for useful effect, in the portion of the fall which is above the arc. In most cases, the height of this arc will equal five sixths of the diameter, minus two or three decimetres.

Machine for extracting the products of a mine.

368. Let us show, by an example, the mode of applying the formula which we have established.

Near a coal mine, we have a water-course, from which we can procure a fall of 22.966 ft.; we wish to use it by establishing there an overshot wheel, for the purpose of raising one thousand hectolitres (or 3531.6 cub. ft.) of coal from a depth of 984 ft., in twenty-four hours. We require the volume of water necessary as a motor, and the principal dimensions to be given to the wheel.

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* Untersuchungen über den Effekt einiger in Rheinland-Westphalen bestehenden Wasserwerke, p. 91 et passim.

The hectolitre of coal drawn from the mine weighs 198.49 lbs. The twenty-four hours of work, considering the time lost in emptying and filling the butts which carry the coal, will be reduced to eighteen hours, or 64800″. Thus the effect to be produced will amount to the raising of 198491 lbs. a height of 984.27 ft. in 64800″, or 3014 lbs. raised 1 ft. in 1″: this is the useful effect. We should increase it at least a quarter, on account of the passive resistances of the machine: it will consist of a great drum, in two compartments, mounted upon the shaft of the wheel; upon each compartment is wound a cable, passing over one or two great pullies ("*molettes*"), and carrying at its extremity a coal bin; one is raised while the other descends. We shall therefore estimate the dynamic effect at $3907.3^{\text{lbs. ft.}} = E$.

The fall being 22.966 ft., we may establish a wheel 21.654 ft. in diameter. We give it 64 buckets, having a depth of 0.984 ft., and whose two plates make an angle of 114° with each other; the breadth of the small one will be 0.328 ft.; with this disposition, it follows that the distance between the buckets will be $d = 0.99853$ ft. $\left(= \frac{\pi\,(21.654 - 1.312)}{64}\right)$, and that the angle a, which the arm or great plate makes with the circumference of the wheel is 30° 53′ $\left(\cos.\ a = \sin.\ 114^\circ\ \frac{21.654 - 1.312}{21.654}\right)$. The water will reach the wheel at about 2.296 ft. below the surface of the reservoir; thus we shall have $h = 2.296$ ft., and we will adopt $\mu = 0.2$.

Consequently, the height h_1, due the velocity of arrival, will be 1.836 ft. ($= 2.296 - 0.2 \times 2.296$), and this velocity will be $V = \sqrt{2gh_1} = 10.873$ ft. We will cause the wheel to turn five times per minute, whence results a velocity $v = 5.3255$ ft. $\left(= \frac{5 \,.\, \pi\,(21.654 - 1.312)}{60}\right)$. Furthermore, we have $H = 22.966$ ft.

The formula $E = 46.837QH$, gives here $Q = \frac{3907.3}{46.837 \times 22.966}$ $= 3.6325$ cub. ft. for the volume of water to be expended.

Let us see now what that of Sec. 365 gives. For the diminutions which H undergoes, we have

1st. $\mu h = 0.2 \times 2.296$ . . . . .4592 ft.
2d. $h' = .01553v^2$ 4406
3d. $h'' = .01553\,(V - v)^2$ 4782

For h''', according to what we have said as to the size and disposition of the buckets, we have $S = Q\,\frac{d}{v} = \frac{3.6325 \times .99853}{5.3255} = .6811$ sq. ft.; admitting that they carry one third of the water which they contain when full, we shall have $s = .25522$ sq. ft. By the methods given (357 and 359), we find $z = 12^\circ\, 59'$, $y = 3^\circ\, 26'$, and $y' = 2^\circ\, 33'$: the half sum of these three angles being $9^\circ\, 29'$, we have,

4th. $h''' = 10{,}827\,\{1 - \cos.\,(30^\circ\, 53' + 9^\circ\, 29')\}$ . . 2.577

Sum of losses of fall, ... 3.9557

Thus $Q = \frac{3907.3}{56.203\,(22.966 - 3.9557)} = 3.657$ cub. ft.; a value nearly identical with the preceding.

There will be nearly the same quantity given by the formula of Sec. 366, viz., $Q = \frac{3{,}907.3}{59.326\,(22.966 - 5.1411)} = 3.6916$ cub. ft.

Notwithstanding this agreement, to have at least all the effect desired, we will raise the volume of water to be expended up to 4.2379 cub. ft.

With such a consumption of water, the width of the wheel should be (349) $\frac{180 \times 4.2379}{64 \times 5 \times .6811} = 3.5$ ft., and its exterior width will be 4.156 ft.

Bucket-wheels, with a great height of water above the summit.

369. The bucket-wheels which we have been discussing are generally of great diameter, always above 9.84 ft., and their summit elevated within a short distance from the level of the reservoir. But there are cases where the destination of the wheel does not admit of such a plan, and we are compelled, as in the case of wheels bedded in a circular course (328), to depart from a disposition the most favorable for the production

of a great effect, and to leave a great height between the reservoir and the point where the water strikes the wheel, which then has a diameter much inferior to the fall.

These cases usually occur for those bucket-wheels which put in play the hammers of iron mills. The great shocks to which the different parts of the mechanism are exposed, do not admit the use of gearing, and as we require rapidity of motion, we are constrained to give to the wheels thirty or more turns per minute, and to impress them with velocities from 13 to 16 ft.; to obtain this, we require heads from 2.95 to 3.94 ft., at least. Moreover, the action of the hammers is intermittent, and it is rarely the case that they are in action twelve hours in the twenty-four; in order not to lose the force which the current constantly imparts, it is stored, during the interval, in a reservoir or basin established a short way above the forge; during the work, it imparts a force double, and at certain periods, more than triple that of the natural current; then it expends more water than it receives, and its level is lowered, so that to have, towards the end of the hammering, a head of 1.96 ft. for example, we should require, at the commencement, one of 6.56 ft. Whence it happens, that in many places we see wheels of 6.56 ft. height only, with falls of 13 ft. In such cases, a forge-wheel would be quite well disposed, if it had a height of water of 4.92 ft. upon the sill of the penstock, 0.98 ft. for the slope of the course, and 7.22 ft. for the diameter.

370. Naturally, the dynamic effect of such a wheel would be determined, as we have already remarked, by the method due to M. Poncelet, and which has been shown in Sec. 361.

The formula 56.203Q $(\mathrm{H}-\mu h-h'-h''-h''')$ however, is not applicable here; Q experiences losses, and h''' cannot be calculated by the rules of Sec. 359.

Still, an endeavor has been made to learn the ratio between the effect and the force of the motor; M. Egen, who has devoted much time in this research, has seen it vary from 0.37 to 0.57.

Water falling from a great height, and impetuously, upon a wheel moving very quick, does not enter entirely into the buckets; a portion, especially that which strikes the edge of the plates and crowns, is thrown back and borne away by the centrifugal force: this force, moreover, hastens the emptying of the buckets. We can prevent a part of these losses, and lessen their bad effects, by covering the front of the wheel, from the part which receives the fluid to its lower extremity, with a wooden curb, similar to a circular course. This enclosure, whose good effects have been tested by experience, returns back into the buckets the water which was at first expelled; if it cannot rest there, it is always retained, and, descending along its concavity, a portion presses the plates of the buckets, nearly the same as it presses the floats of wheels contained in curved courses.

371. One of the data most frequently needed by the millwright is the knowledge of the quantity of water necessary for a wheel to put in play a given hammer. The rational determination of this quantity would require a theory of hammers: but notwithstanding the outline of such a theory which M. Poncelet has given us, we have not, as yet, a complete one; meanwhile, experiment must supply the want, and I indicate here some results which it has given: the three first are due to M. Egen, and the two last I have observed myself.

PLACE of observation.	HAMMER.			DIAM. of Wheel.	Fall.	Water expend'd in 1″.	Force of Motor.
	Weight.	Lift.	Blows in 1′.				
	lbs.	ft.		ft.	ft.	cub. ft.	hrs. p'rs.
Westphalia,	66	0.59	313	8.82	16.27	10.806	20.1
"	154	1.54	224	7.48	14.04	22.249	35.5
"	496	2.88	103	7.74	12.40	21.542	31.5
Sweden, . .	705		90	9.18	11.74	15.892	21.5
Agennais, .	1323	2.13	85	6.72	11.15	15.185	19.5
"	705	2.29	110	6.86	11.81	15.892	21.6

I will remark, that the quantity of water needed for a hammer increases in a much greater ratio than the velocity to be given it, (nearly as the cube of this velocity,) as is shown from the observations made upon the first of the machines noted in the above table, and the results of which are seen in the adjoining table. During these observations, the wheel made from twenty to thirty-seven turns per minute; with this last velocity, each bucket received but one cubic foot of water, and could contain 2.82 cubic feet.

Water in 1″.	Velocity of wheel.	SERIES OF		
		Waters	VELOCITIES.	
			simple.	cube.
cub. ft.	ft.			
1.55	9.12	1.00	1.00	1.00
2.47	11.08	1.59	1.21	1.78
4.41	14.14	2.84	1.54	3.67
6.46	15.81	4.16	1.73	5.16
10.27	17.19	6.58	1.88	6.63

b. Breast-Wheels.

Character and advantages of these wheels.

372. In overshot wheels, the water-leader, after having passed their summit, delivers the water in the second or third bucket in front of it. Their lower part moves in a direction opposite to that of the current in the tail-race; so that if, by any accidental cause, such as frequently occurs in mills, there should be any swell of the water or backwater, the wheel plunges in a fluid endowed with motion in a direction opposite to its own; its velocity is retarded, and the effect undergoes sometimes a notable diminution. We remedy this evil by delivering the water upon the back of the wheel; its lower part, moving then in the same direction with the tail-race current, can be submerged two or three decimetres, or from .65 to 0.98 ft., without its velocity being sensibly changed; which enables us to lower it as much, and in consequence to increase the useful fall; a real and sometimes an important advantage.

Breast-wheels, or "*roues par derriere*," as they are called in certain localities, (*Rückenschlächtige Räder*, wheels struck in the rear), have also another advantage. Since they receive the water below their summit, we may raise it, and it is usually raised, above the level of the reservoir; their diameter is then greater than the height of the fall. This excess of elevation is profitable in certain respects, in small falls, those from 8 to 16.5 ft.; the wheel being a third, a fourth, or a fifth higher than an overshot wheel, in the same place, has greater force, or rather, maintains better that with which it has been impressed. It is necessary, however, to take care lest this advantage may not be more than compensated by a great loss of head below the arc charged with water; a loss which we shall soon consider.

Manner of letting on the water.

373. In the wheels we are now discussing, the water is either let on the buckets immediately by a leader, which is then open at its extremity, — but in this case, the velocity of the fluid should be small; or it is delivered through a trough, analogous to that of which we have spoken (348), and which is represented in Fig. 59.

When the wheels are well constructed and kept in careful repair, like the great cast iron wheels, the water is let on by simply letting it flow smoothly over a sill placed immediately above the buckets. The plate AB, instead of being raised, as in ordinary sluices, is lowered, and so much the more, as a greater supply is required. When it is lowered, its upper edge A constitutes the sill of the overfall; after passing it, the fluid falls into a sort of rack, or system of tunnels, which direct it into the buckets; and for this purpose, we dispose the great plates or arms so that when they arrive opposite the rack enclosures, they shall have the same direction, which is generally vertical. These sluice-gates, as well as the iron wheels to which they are

Fig. 60.

adapted, are much used in England, whence they passed over into France many years since.

I shall not enter upon a description of these iron wheels; it may be found in many works on industrial mechanics, in the "*Traité des Machines de M. Hachette*" (p. 127), &c. I shall merely observe, that the shaft of these wheels, as well as of some of our wooden wheels, is a cylinder, or a prism of six or eight faces, of good cast iron, hollow, and often swelled in the middle. Its diameter (that of the circle inscribed in the polygon presented by the section made at one end) depends upon its length, and upon the weight it has to support; designating by λ this length, and by ϖ this weight, it will be, according to Tredgold, $0.01634^{\text{ft.}}\sqrt[4]{\varpi\lambda^3}$. We should give to the interior diameter three fourths of this value. The buckets are generally made of strong iron plates; and their width, which is that of the wheel, may be as great as from 16.5 to 19.68 ft.

374. The dynamic effect of wheels receiving water below their summit is also given by

Dynamic effect.

$$0.90P(H-\tfrac{2}{3}h-h''').$$

In this expression, h''' will be greater than in overshot wheels, since it is proportional to the diameter, which is here greater compared to H. So that, as a result of this, the effect would be less, if we did not give to H a greater height, and we have seen that this can usually be done.

Let us see now what will be the effect of a good wheel of this kind, of the best wheel, after the English pattern, now in France, and perhaps the best on the continent, and which operates the spinning mill of MM. Schlumberger, at Guebwiller, on the upper Rhine. It is 29.85 ft. in diameter, 10.33 ft. in width; it carries ninety-six buckets, held between two crowns, having a width of 0.984 ft.; it is made of wrought and cast iron, and weighs 55.136 lbs. The water is let on at about 50°

from the summit, by a sluice-gate, like that mentioned in the preceding number. The fall varies from 25.26 ft. to 25.59 ft. Among the experiments made by M. Morin upon this wheel, I give those which seem to have been executed under the most favorable circumstances. The moving force employed, with a discharge of 12.007 cubic ft., equalled 19001 lbs. ft., or thirty-five horse-powers; the velocity was 5.052 ft., or 3.23 turns per minute; the effect produced, given by a dynamometric brake, if we add to it 1165 lbs. ft. for the passive resistances, would be as high as 14978 lbs. ft.; it would be the 0.788 of PH.

Such a result is a *maximum* but rarely attained; even in great wheels, very well established, we usually have below 0.75; and in general, I think we should admit only

$$E = 0.70PH.$$

375. The loss of head h''', being proportional to the diameter, there would be an advantage in making it as small as possible, always keeping it at least equal to the fall, since the nature of the wheels in question requires that their summits should not be below the level of the reservoir; in other words, the dynamic effect will be so much the greater, as the letting on of the water is less distant from the summit of the wheel. In most cases, this distance, measured upon the exterior circumference, would be 30°, and even less, for wheels of 19.68 ft. and more; in small wheels, it is necessary to increase it to 40°. English constructors go as far as 52° 45′; it is a rule which they have adopted, and for which I find it difficult to assign a reason.

Can this rule have been governed by the condition of keeping vertical the rack enclosures of the penstock? But such a condition would cause a variation in this arc, according to the size of the diameter, and the number and form of the buckets. As to the effect, there would be a loss, rather than an advantage, in following it.

If letting on the water to the buckets at 52° from the summit of the wheel diminishes the effect, it would be decreased much more, should we deliver it at 90°, that is to say, upon the middle of the wheel, as has often been done.

It would be worse still, if we descend below the middle, as has been the practice of some countries; scarcely will the fluid have entered the buckets, when it will quit them; the height of the arc charged with water, on which the effect almost wholly depends, will be too small; it would be better, in this case, to have a wheel of less diameter.

Finally, when the fall is below 8 ft., and we wish to profit by the advantage gained in causing water to act by its weight, in place of carrying it in buckets, we let it into a course, enclosing very nearly the part of the wheel which is below the level of the reservoir, and we then find ourselves in the case or condition of float-wheels established in a circular course, of which we have already treated (321 — 327). By proceeding thus, we obtain two advantages; that of causing the motive water to exert its action upon the wheel, even to the lowest point of its revolution; and that of freeing the wheel from the weight of this water, which is now transferred to the course. But, on the other hand, we experience two disadvantages; a portion of the water which would have entered the buckets escapes through the space left for the play of the wheel in the course; 2d, the portion of the floats which plunges in the water of the course, loses a part of its weight there; and this loss is as a new resistance to motion (323). Finally, with the falls in question, those below 8 ft., the advantages outweigh the disadvantages.

ARTICLE SECOND.

Horizontal Wheels.

Different kinds.

376. If vertical wheels are most generally used in the north of Europe, horizontal wheels are most in use in the south; they operate nearly all the mills in the southern departments of France. We must admit that they are eminently adapted to this kind of mills; they require the most simple mechanism, and dispense with all gearing and transfer of motion; the same axle which receives the wheel upon its lower part, carries the moveable millstone at its upper extremity. In the usual construction, it turns upon a pivot in a socket sunk in the middle of a piece of wood or step (*palier*), which is raised or depressed at will, according as we wish to enlarge or diminish the space between the runner and the bed-stone.

The wheels of our ancient mills consist of a simple nave, on the circumference of which are embedded floats, nearly always curved, and of different forms, as we may see in the *Architecture Hydraulique* of Bélidor. On some, the motive water is injected in an isolated vein, through a trough; others, placed at the bottom of a tub open beneath, are impelled by the whirl of water cast upon them.

Towards the middle of the last century, and still later, rotating machines were designed and executed, where the water operated principally by reäction, and which have attracted the attention of mathematicians.

More lately, in 1825, M. Burdin, engineer of mines, after having distinguished himself by his writings upon the theory of machines, and by different inventions, introduced into mechanics a new kind of machine, possessing a movement of rapid rotation around a vertical shaft, and to which he gave the name of *turbine;* he has disposed them so as to fulfil the conditions of greatest effect. A short time afterwards, M. Fourneyron, one of his pupils, designed one of a new kind, which he has established in many places; with a quite simple arrangement of its parts, it is every where recommended by the greatness of its effects, and by its peculiar advantages; from the first, it has ranked among the best hydraulic machines, and is now, if I may so express myself, the order of the day among mechanicians.

Let us treat of these different wheels, following the order of the synoptic table given in Sec. 295.

1. *Wheels moved by the impulse of an isolated vein.*

377. These are very common in mountainous regions; in the Alps and Pyrenees, they work the mills there called "*à trompe*" or "*à canelle,*" &c.; because the water is cast upon the floats, either through a "*trompe,*" (a sort of pyramidal trough somewhat inclined, and analogous to that mentioned in Sec. 51,) or through a trough inclined from 20 to 45°, called "*canelle.*" Their form.

The wheel of one of the first kind of mills, represented in Fig. 53, is 5.249 ft. in diameter and 0.656 ft. in height. The floats, or "*cuillers*" (ladles), as they are called in the provincial phrase, eighteen in number, have a length of 1.312 ft. in the direction of the radius. In each, the part which receives the action of the impulse is concave, and with an oblique surface; its intersec- Fig. 53.

tions, by a series of vertical planes, perpendicular to the direction of the radius, form curves, whose curvature increases in proportion to their distance from the centre; their superior element is vertical, and the inferior, from what has been said, is more inclined the nearer we approach the end, where it is nearly horizontal, and where, consequently, the curve is very nearly a quarter of a circle.

Effect. 378. The motive water, being cast upon the floats with velocities from 23 to 26 ft., and frequently more, acts almost wholly by impulse. Let us first admit that it acts only in this manner; and to reduce the question to its most simple expression, let us suppose that a fluid filament strikes an elementary plane of the floats. If we designate by i the angle which the filament makes with this plane, and by j the angle which this same element makes with the direction of motion, which is horizontal, the effect of impulse will be expressed (243) by $\frac{P'}{g}(V \sin. i - v \sin. j) \sin. j$; P' being the weight of fluid which falls in 1″ upon the element. Multiplying by the velocity v of this element, we shall have for the dynamic effect,

$$\frac{P'}{g} v \sin. j\,(V \sin. i - v \sin. j).$$

The sum of all these elementary effects will be the total effect.

For the *maximum* of effect, we find, by the usual method, $i = 90°$ and $v = \frac{V}{2 \sin. j} = \frac{\sqrt{2gh_1}}{2 \sin. j}$. These values, substituted in the above equation, will change it to $\frac{1}{2}P'h_1$.

So that, if the float was entirely plane, P being the sum of the values of P′, we should have

$$E = \frac{1}{2}Ph_1,$$

an expression similar to that which we had for vertical float-wheels (311).

379. The real effect will naturally be below the theoretic effect. A great number of fluid filaments, being scattered, will not touch the floats, the height of which is very small; most of the others will not impel perpendicularly the surfaces which they strike, a condition required by the expression $i = 90°$; and these surfaces will not have the velocity required by the second condition. On the other hand, and compared with other float-wheels, the number of fluid particles which exert no action is very great; those which do, impel with a velocity due nearly to the entire fall H; and the concavity which receives them performs the office of the rim enclosures of Morosi (239). Moreover, the particles which descend upon the curved part of the floats, act there both by their weight and their centrifugal force. Thus, we very often see the dynamic effect exceed a third of the force of the motive current, while for vertical wheels with plane floats, it is rarely a fourth of it. (314.) Notwithstanding, I shall generally admit only

$$E = \tfrac{1}{3}PH.$$

Let us see what experience teaches us upon this subject.

In 1822, MM. Tardy and Piobert, officers of artillery, then residing at Toulouse, made, by means of the dynamometric brake, experiments upon the various mills of the environs of this town; among others, upon that "*des Minimes*," whose wheel, represented in Fig. 53, has been described above (377). Unfortunately, they left Toulouse without completing their labor. Still, the results which they obtained, and for the knowledge of which I am indebted to a courteous communication from them, will give at least a good idea of this important matter, upon which we have hardly any positive knowledge.

The brake which they used was fitted immediately upon the vertical shaft; a dynamometer indicated the effort with which it tended to be drawn along with it. We will remember that Π being this effort, L the length of the arm at the extremity of which it is exerted, and N the number of turns of the wheel in a minute, the effect is given by the expression $0.105\Pi LN^{\text{lbs. ft.}}$ (292).

I limit myself to presenting, in the following table, the results of five of the eighteen experiments. Previous observations had given 0.95 for the value of m in the expression of the expenditure of water $mS\sqrt{2gH}$. I will remark, that the effect, and consequently the ratio of the effect to the force, is a little greater than that shown in the table, because the resistance of friction of the pivot against the socket is not given by the brake.

FORCE.		EFFECT.			RATIO	
Fall.	Water in $1''$.	Effort.	Arm of Brake.	Turns of wheel in $1'$.	of effect to force.	of v to V.
ft.	cub. ft.	lbs.	ft.			
13.97	10.69	67.7	3.74	110	0.312	0.862
13.87	10.66	73.2	3.70	104	0.320	0.818
13.68	10.56	77.6	3.67	102	0.330	0.811
13.25	10.38	99.6	3.67	86	0.403	0.694
12.99	10.31	83.1	4.16	90	0.382	0.736

We see from the two last columns, that, with an equal force, the effect increases as the velocity of the wheel diminishes, compared to that of the current; and perhaps, if the velocity had been lowered to a certain term, the ratio of effect to the force might have attained its theoretic limit, 0.50.

The simplicity of the wheels just discussed, and consequently the small expense required in their establishment and maintenance, bring them into frequent use; and it may not always be best to replace them by others, even having a superior dynamic effect. They are among the number of wheels to be established with advantage in certain localities.

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2. *Wheels placed in a Well or Pit.*

Principal dispositions.

380. The percussion wheels which we have just considered are principally in use on small water-courses with great falls; but upon rivers, for example, upon the Garonne, the Aude, the Tarn, the Aveyron, the Lot, &c., where there is much water and little fall, instead of these trough-mills, we use tub-mills, (*moulins à cuve*). In fact, the wheel is there placed in a well or cylinder of masonry and sometimes of carpentry, open at both ends. This kind of mill is well known, from the description given by Bélidor, over a hundred years since, of the *Mill of Bazacle*, at Toulouse, which he regarded as the most simple and ingenious of water mills (*Architecture Hydraulique*, tome I., § 669). Moreover, nearly all those existing upon the rivers we have named, as well as on their tributaries, are disposed very nearly in the same manner.

Fig. 65.

The wheel is usually but 3.28 ft. in diameter, with a height of 0.656 ft. It carries nine floats, very nearly in form like those of the trough-mills (377); each half is made of one piece of elm, cut by the miller himself, and these two halves are united and held by two iron bands.

The well is generally 3.34 ft. in diameter and 6.56 ft. in depth; the wheel is placed very near the bottom. The mass of masonry, in the middle of which it is placed, is pierced, for its whole height above the level of the wheel, with a channel serving as a water lead; this contracts towards the well, and on emptying into it, it is only 0.722 ft. wide. One of its sides is tangent to the interior side of the well, as is seen in the figure.

Mode of action of the water.

381. The motive water, after having passed under the gate at the entrance of the course, is borne with

rapidity upon the adjacent part of the cylindric wall of the pit; in striking, it is at first considerably raised; then following its circuit, it descends and reaches the floats, upon which it acts by impact and its weight; it bears them along in its whirl.

On account of the circular motion, the centrifugal force urges and presses the water against the interior face of the well, upon which it consequently forms a lining of some thickness, so that if it finds in its descent a space between this face and the wheel, (and such must be the case to allow for the necessary play of the wheel,) a great portion passes through it without any action upon the floats. This statement of itself shows how prejudicial is every interval, however small it may be; so that, in new constructions, those made within a few years, it has been suppressed. We place the wheel immediately under the well, and give it a diameter somewhat greater than its own; so that nearly all the motive water arrives upon the floats. Though it acts there after losing a part of its velocity, and with some disadvantage, I have still found, that by this disposition solely, there has been a saving of over a third. In these constructions, instead of the long course of 6, 10 and 13 ft., we have made them in castings not over a foot long; we have also reduced to a trifling amount the mass of masonry, hitherto so very considerable.

Theory of wheels with curved floats.

382. Before examining the effect really produced, we give the theory of wheels with curved floats in general, and deduce it from the principle already mentioned (297): in order that a fluid should impress all its dynamic action upon a wheel, it is necessary that it should enter and act without shock, and that it should issue without velocity.

Suppose (though it may seem difficult to realize for a wheel in motion) that a particle, or a series of fluid particles A, arrives in the direction AB upon the curve BC, in a vertical plane perpendicular to the part B, of the float of which BC would be the section made by this plane. Fig. 100.

Let $BD = V = \sqrt{2g \,.\, BH} = \sqrt{2gh}$ be the velocity of the particle in its direction AB.

$BE = v$ the horizontal velocity of the point B of the wheel.

$ABE' = i$ the acute angle made by AB with the horizon.

Take $BE' = BE$, and construct the parallelogram BDFE′; the diagonal BF, the resultant of the two velocities V and v, or rather V and $-v$, will represent, in direction and magnitude, the relative velocity of the fluid the moment it meets the curve. That there should be no shock at the point of meeting, it is requisite that the first element of this curve should have the direction BF.

In order that the fluid may still act without shock in descending from B to C, so as to lose none of its velocity, it is sufficient that BC should be a curve, free from uneven and salient points.

That the particles may issue without velocity, that is to say, in order that in quitting the extremity C of the curve, they may have no motion, but may fall vertically by their weight alone, it is necessary that the velocity they have, upon the last element of the curve, should be equal and directly opposite the velocity v of this element. Since this velocity is horizontal, it is necessary that that of the particles upon the last element should be so also; and consequently, it will be required, that this element, which directs them, should be itself horizontal. The velocity of the fluid at B, upon the top of the float, being the diagonal BF of the parallelogram of which V and $-v$ are the sides, will be equal $\sqrt{V^2+v^2-2\,V\,v\cos.\,i}$. This initial velocity, if we neglect the friction on the float, will not experience a loss from B to C; on the contrary, it will be increased by the velocity which gravity will impress upon the particles during this descent, of which the height IC is that due to the increase of velocity; so that the total velocity will be (38)

$$\sqrt{V^2+v^2-2\,V\,v\cos.\,i+2g\,.\,IC}=$$

$$\sqrt{2gh+v^2-2v\cos.\,i\sqrt{2gh}+2g.IC}=\sqrt{2gH+v^2-2v\cos.\,i\sqrt{2gh}}$$

observing that h or BH plus IC equals the entire fall H. Such is the velocity with which the fluid will tend to quit the point C, and with which it would quit it, in the direction CG, if this point were immovable. This velocity, we have just said, should be equal to v, thus

$$v=\sqrt{2gH+v^2-2v\cos.\,i\sqrt{2gh}};$$

whence $v=\dfrac{gH}{\cos.\,i\sqrt{2gh}}$.

Recapitulating the conditions of greatest effect, it will be requisite, 1st, that the first element of the float should be in the direction of the resultant of the two velocities V and v, the latter being taken in a contrary direction; 2d, that the concavity of the float present a continuous curve (without salient points); 3d, that the velocity of the float, or v, should be $\dfrac{gH}{\cos.\,i\sqrt{2gh}}$. These conditions being fulfilled, we shall have

$$E=PH.$$

383. From what has just been said, the determination of a general expression for dynamic effect will be rendered easy.

General expression of effect.

The absolute velocity which the fluid possesses on quitting the wheel, being estimated in the direction of motion, or from G to C, will be the velocity v of the float on which it is borne, minus that which it has in a contrary direction, and consequently it will be

$$v - \sqrt{2gH + v^2 - 2v \cos. i \sqrt{2gh}}.$$

That which it had on its arrival at the wheel, also estimated in the direction of motion, was the horizontal component of V, and consequently, $V \cos. i = \cos. i \sqrt{2gh}$. It has, then, lost $\cos. i \sqrt{2gh} - v + \sqrt{2gH + v^2 - 2v \cos. i \sqrt{2gh}}$. Multiplying this value by the mass $\frac{P}{g}$ of fluid impinging on the wheel, we shall have the quantity of motion lost, and consequently, the effort exerted by the fluid against the wheel (242). This effort, multiplied by the velocity v, will give the effect E, and we shall have the equation established by Borda in 1767,

$$E = \frac{Pv}{g} \left\{ \cos. i \sqrt{2gh} - v + \sqrt{2gH + v^2 - 2v \cos. i \sqrt{2gh}} \right\}$$

For the *maximum* of effect, we have $v = \frac{gH}{\cos. i \sqrt{2gh}}$, and this value, put in the above equation, reduces it to $E = PH$, which, we have already seen, answers to this *maximum*.

Real effect.

Thus, a horizontal wheel with curved floats, according to the suppositions we have made, would produce an effect equal to the entire force of the motor; this would be a perfect machine.

384. But what is true for a filament, for a thin fluid sheet, properly directed upon a float suitably disposed, is no longer so for the great volumes of water arriving *en masse* upon the wheels of our pit mills. A portion escapes through spaces, without exerting any action upon the wheels; and the other portion is far from exerting it in the most advantageous manner, it impinges against and impels the floats often under great angles, and in issuing from them preserves a marked velocity. The lower edge of the floats is not horizon-

tal, as the theory of the fluid filament indicated that they should be; if it were, the mass of water would not free itself with sufficient ease. Thus this kind of wheel, perfect in theory, is one of the most imperfect in reality, and even in favorable circumstances, its dynamic effect will be nearly always below $\frac{1}{8}$PH. It can only be in mills which have adopted all the new dispositions spoken of above (381), that we can have generally

$$E = 0.25PH.$$

Here, also, the experiments of MM. Tardy and Piobert show us what it really is.

They made some series upon a wheel of the same mill of Bazacle, a wheel having the dimensions above mentioned (380). The fall was 7.81 ft.; the volume of water passing through an orifice of the gate was determined by the common formula, with 0.66 for the coefficient: finally, the volumes of water noted in the annexed table can only be regarded as approximate, and we may admit that they are sensibly the same in each of the three series; — series arranged according to the ratio of their velocities. A brake, applied immediately to the axle, indicated the effect, exclusive of the friction upon the pivot: in consequence of this last circumstance, the effect recorded in the table is somewhat too small. The mean term of its ratio to the force, or of the numbers of the last column, in the twenty-two experiments of MM. Tardy and Piobert (here we have given but twelve), was 0.125; that is to

Water expend'd in 1″.	EFFECT.			Ratio of effect to force.
	Turns of wheel in 1′.	Weight at lever.	Arm of lever.	
cub. ft.		lbs.	ft.	
18.47	60	31.53	3.57	0.188
17.89	70	87.55	2.19	0.162
18.64	75	47.31	3.57	0.148
18.57	90	28.66	3.47	0.103
30.86	78	74.98	3.90	0.157
30.55	81	67.26	4.00	0.153
31.60	100	45.21	3.67	0.113
31.89	116	25.36	3.90	0.077
31.29	120	19.84	3.77	0.062
37.96	104	69.45	3.93	0.162
38.80	118	42.99	3.90	0.111
40.47	126	27.56	3.83	0.071

say, that the effect was not over the eighth part of the force employed to produce it.

On another wheel of the same mill, they had 0.15.

Finally, upon another pit wheel, but better disposed, that of the *hospital*, they found this ratio as high as 0.27, and at a mean as high as 0.20.

In these different experiments, as well as in those which were made at the trough mills, the effect was diminished, and that considerably, with the increase of the velocity; most probably, that which gives the *maximum* of dynamic effect will be below the velocities adopted by the miller.

385. Notwithstanding the small effect produced by the pit wheels, compared to the water which they consume, their simplicity and solidity of construction cause them to be in frequent use, especially in places where there is an abundance of water.

These wheels move under water.

They have, moreover, a remarkable advantage, that of being able to work while submerged, and consequently in the freshets of rivers, so long as there is a marked difference of level between the upper and lower reach. We sometimes find them in certain localities, where there is but a slight fall, and where it is important that none of it be lost, established below the level of this latter reach; thus, upon the Aude, where the falls are only from 4.26 ft. to 5.25 ft., they are placed at from 1.64 ft. to 2.29 ft. below the surface of the common stages of this river.

3. *Turbine of M. Fourneyron.*

386. The great expense of water for horizontal wheels with curved floats, when every thing indicated a requisition for but little, denoted a great fault in their disposition. It was noticed by some authors, who observed that the evil might be remedied by causing the water to arrive through many mouths or inclined tubes, distrib-

Principle.

uted upon the periphery of the wheel.* Some trials were made; but the proposed machines, though well contrived (404), were unwieldy or complicated; and the problem remained to be solved, as far as concerns its application to practice. It has lately been solved, in a manner almost as successful as one could hope, by M. Fourneyron.

This young mechanist, instead of putting the wheel in a cylinder, as was done in pit mills, placed it outside. Like a ring, it surrounds the lower part, leaving a small play for the motion; this part, pierced with orifices throughout its circuit, delivers the water, in a direction which it is constrained to follow, upon the floats, which thus are properly struck, and all at once. From this simple disposition results one of the best hydraulic machines in existence; a glance at the Figs. 101 and 102 will give a first idea of it. Figs. 101 and 102.

Historical notice.

387. M. Fourneyron, after having conceived the idea, made a profound study of the principles on which it was based, and of the dispositions which should insure its success; and in 1827, he built one of six horse-power, in Franche-Comté. Its success surpassed all expectations; the effect was seen to be raised above 0.80 of the force of the current; and what increased the astonishment, in a country where pit mills were not known, was that the effect was not anywise diminished, even when the wheel was entirely submerged in water.

Four years passed away, however, before the author had occasion to make a second. It was made to move the bellows of the high furnace at Dampierre; it is represented in Fig. 101; it is remarkable for the elegance of its form, and for its small size; it has only a diameter of 2.034 ft., with a force of from seven to eight horse-powers. Its substitution in place of an old wooden wheel was so advantageous, that the proprietor of the establishment ordered another of fifty horse-power for his forges at Fraisans, five leagues from Dôle. It was established in 1832, upon the Doubs, and below the level of the water of the river. We will describe it hereafter.

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* Architecture hydraulique de Bélidor et Navier. Tome I., p. 454.

A short time before this period, the Society for the encouragement of national industry offered a prize of 6000 francs for "*the best application, on a great scale, of the hydraulic turbines, or wheels with curved floats, of Bélidor, to mills and manufactories.*" After the constructions which we have just mentioned, the prize was justly given to M. Fourneyron. Agreeably to the programme, he published in the Bulletin of the Society the description of his machine, with some practical directions, to serve as a guide to those who might wish to make similar constructions.* I should, however, observe, on this subject, that the author, having taken out a patent for his turbines, has the sole right to construct them during the period for which it is granted.

In 1834, he built one for a spinning mill at Inval, near Gisors, sixteen leagues northwest of Paris; it has been the object of many trials, made in some respects officially, and they accordingly serve better than others to fix our opinions as to the effects of which the machine is capable; we shall report upon them hereafter. Since then, the author has not ceased to multiply his turbines, in Germany as well as in France. In the first of these countries, at St. Blasien, in the Black Forest, he built one of forty horse power, though it was but 1.8 ft. in diameter; it worked under the enormous fall of 354 ft. Finally, this very year (1838), he has put up four near Paris, at the mill of St. Maur, where they drive forty mill-stones, (ten each).

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* Bulletin de la société d'encouragement pour l'industrie nationale. 1834. Cahiers de Janvier, Février et Mars.

Description of these turbines.

388. Turbines are divided into two classes, quite different in their construction: those designed to work continually under water (the *submerged turbines*), and those which are not. We will give an idea of the first by a short description of that at the smelting works of Fraisans. It is one of the greatest that we have; it is of fifty horse power; and can discharge 141.2 cub. ft. of water per second, under a fall of 4.59 ft.

Like all turbines, it is composed of three principal parts: the turbine properly so called AB, with its shaft C; the cylinder DEFG, with its bottom HH and the bottom supporter LL; and the gate MM. The whole is of cast iron.

Fig. 102.

The turbine, or rotating part, consists of two annular plates or crowns of cast iron, between which are placed the floats. The exterior diameter is 9.514 ft., and the interior diameter is 7.874 ft.; the width of the crowns *ab* is accordingly 0.82 ft. The space between them, or the height *bc* of the floats which they enclose, is 1.148 ft.: their number is 36. They are vertical, made of strong iron plates, with a simple curve: their first element, *d*, is very nearly perpendicular to the interior circumference, and the last element, *e*, makes an angle of about 15° with the exterior circumference. The turbine is supported by a spherical disc *ff*, cast in the same mould with the lower crown: its centre is pierced with a hole *g*, through which the revolving axle *c* passes, and to which it is fastened. This axle is of wrought iron; it is 0.574 ft. square and 17.55 ft. long. It is terminated at its lower end by a steel pivot *h*, which turns in a socket *ii*, contained in a strong cast iron shoe *ll*. By an ingenious contrivance, the socket, and consequently the shaft, may be raised, according to the wear of the pivot, so that the system remains always at the same height.

This situation of the pivot, as well as of the socket, had a great disadvantage. The rubbing surfaces, working continually in water sometimes salt, and sometimes charged with sand, &c., were destroyed in a short time, and it was necessary to change them. In his later constructions, M. Fourneyron has remedied the evil by a method as simple as it is skilful and efficacious. He fastens the socket upon the revolving shaft; below and within this is found a pivot with a steel head, contained in a cast iron shoe, which can be raised by the above-mentioned contrivance: between its exterior surface and the interior surface of the socket a small space is left. A small tube, whose lower end passes through the body of the pivot where it is fixed, empties into this space; the other part passes under the machine, and goes up into the edifice above; there the oil is poured in; it descends through the tube, and arrives at the space between the pivot and the socket; by virtue of its less specific weight, it rises into the upper part, and forces the water which it finds there out of the way; and thus these surfaces revolve as it were in an oil bath, and show but little depreciation in the course of a year.

Let us return to the machine at Fraisans. The cylinder has a diameter DG of 7.87 ft., and a height DE of 1.64 ft. only. On its upper part there is a rim or collar 0.59 ft. in width, by which it is fastened to the flooring NN, forming the top of the enclosure of the turbine, which has no other opening than that occupied by the cylinder. This cylinder does not descend as low as the wheel; its lower part is, as it were, replaced by the circular gate MM. When it is lowered, the water of the upper level, which fills the cylinder, cannot issue forth: but as soon as this gate is raised, it is precipitated against the floats, forcing them to yield, and to be put in motion; then, passing beyond, it enters and is lost in the lower reach. The bottom HH of the cylinder, which is established on a level with the lower crown of the wheel, is a strong cast iron disc; it has a tube II in its middle, 1.96 ft. in diameter, and as much in height. Against it, and upon the disc, are fastened the *guide curves* O, O, O, twelve in number, with a height of 1.97 ft., and a form represented in the figure: their extreme part *mn*, for a length of nearly 0.820 ft., is directed in a right line, and makes an angle of about 30° with the outer circumference. This disc rests upon a projection which the *disc supporting* pipe LL presents in its lower part; this pipe is 1.31 ft. in diameter and 11.48 ft. in height. By its upper part, which is disposed for this effect, it is as if suspended on a *chair* or platform let into a framing of carpentry. See the Memoir of M. Fourneyron for all the details of its construction and establishment.

The gate is also a cylinder, placed beneath and within the first, as the figure shows. Its height is also 1.64 ft.; but its diameter, in place of being 7.87 ft., is only 7.382 ft.: the space between them is closed by a leather packing *q*, which prevents the passage of the water. The interior of this moveable cylinder carries a lining formed of wooden blocks 0.59 ft. thick, which is placed between the guide curves. By this disposition, when the gate is raised a certain quantity, the lower part of the cylinder or reservoir presents, throughout its circuit, as it were a series of prismatic orifices, whose lower part is the bottom of the cylinder, whose sides are the guide curves, and whose upper part is formed by the bottom of the gate: we have thus a series of additional tubes, which deliver the water upon the turbine in the direction of the guides. Without these wooden blocks, the fluid would deviate from this direction, and would approach a perpendicular to the circumference. Naturally, the lower edges of the blocks will be rounded, so as to reduce the contraction of the vein. We shall also see, in the memoir of M. Fourneyron, the skilful mechanism with which he raised and lowered at will these circular gates.

389. We will give a short notice of the beautiful little wheel represented in elevation in figure 101, a kind of machine principally designed for great falls, and for wheels not immersed, though they also are able to work under water.

The cylinder B is entirely closed at the top, and narrowed at its lower end; it is 2.95 ft. in diameter and 4.36 ft. in height. At C is a tube connecting with the water leader D.

Below is the turbine AA, which is 2.95 ft. in the outer diameter, and 2.03 ft. in the interior. It has twenty-seven floats, having a height of 0.295 ft. only; they are of cast iron, and cast in the same mould with the rest of the turbine. Against the lower contraction of the cylinder, and within it, is the circular gate *aa'*, disposed like that at Fraisans, which is raised and lowered by a system of toothed wheels, and by three iron rods, whose ends are seen at *b*, *b*, *b*: upon the top of the shaft is a bevel wheel, by which motion is transmitted to a blast engine.

Such a turbine, however great the fall, may be established in any part of the mill thought best, for, as M. Fourneyron remarks, it takes up no more room than a stove or furnace.

390. The expression for dynamic effect deduced (383), from the theory of Borda, for other horizontal wheels with curved floats, will not answer for turbines. In these wheels, the water, in descending along the floats composed of elements more and more inclined to the horizon, will impress them, at every instant of its descent, with a new quantity of motive action, imparted by gravity. It is not so with the floats of turbines, which are formed of a series of vertical elements; the water does not act by its weight upon them; but while it advances upon them from its entrance to its discharge, another force, the centrifugal force, presses against this series of elements, and so produces the motion of rotation. So that, in a well arranged turbine, the water acts neither by its weight nor its impulse, nor even by its reäction, but only in virtue of its centrifugal force; it is perhaps the only kind of hydraulic machine in which this condition is fulfilled. This consideration induces us to dwell upon some effects of this force; they are produced, it is true, in a manner more or less striking, by most rotating machines; but much the most forcibly in turbines.

Theoretic effect.

391. Let ABCD be a cylindrical vessel, containing water up to the level IK. If we impress with it a uniform motion of rotation around its vertical axis EF, the fluid surface, by reason of the centrifugal force, will quit the plane and horizontal form; it will be lowered in the middle O, and raised towards the sides, taking in its vertical section the curved form GOH, a curve which we proceed to determine.

Fig. 67.

Form taken by the surface of water contained in a vessel to which a rotatory motion is given.

Since the movement of rotation is uniform, the fluid surface will have a permanent figure; its particles will then be in equilibrium, and will consequently be equally pressed in all directions, so that if, upon the horizontal OR, we take any particle, at P, for example, it will be as much pressed from above downwards by the vertical filament MP, as from the left to the right by the horizontal filament OP: these two pressures will be equal. Agreeably to the method adopted in questions of hydrostatics, we will consider only the two filaments, without regard to the rest of the fluid mass, and we will suppose them enclosed in the small tube OPM, open at both ends. For the filament MP, the action of the centrifugal force upon its particles, being directed perpendicularly to the sides of the small tube, will be destroyed by their resistance: the particles will experience no other action but that imparted by gravity, and consequently, the pressure at P will be equal to the sum of their weights: the weight of each is mg; their height MP, which we will designate by x, or the number of its points, represents the sum of the particles of the filament: so that their total weight will be mgx. For the filament OP, its particles resting on a horizontal plane, the action of gravity on them will be destroyed: they will only be animated with a centrifugal force; the force of that which at O, upon the axle of rotation, will be zero; and the force of that which at P, making $OP = y$, will be mw^2y, w being the angular velocity: from the point O to the point P, the forces, as well as the distances to which they are proportional, will increase in an arithmetical progression, and their sum will be $mw^2y \cdot \frac{1}{2}y$, y representing here the number of terms of the progression: this sum will be that of the efforts made by the particles of the filament OP, in passing from O to P, or in pressing upon this last point. We shall have, then, $\frac{1}{2}mw^2y^2 = mgx$: whence we deduce $y^2 = \frac{2g}{w^2}x$, the equation of a common parabola of which $\frac{2g}{w^2}$ is the parameter.

Action of the centrifugal force upon the velocity of issue, supposing the water in the vessel near the orifice has acquired all the angular velocity of the vessel.

392. Suppose now that at the point R, on the prolongation of OP, we make an orifice, through which the water issues from the vessel, while it turns around its own axis; suppose, moreover, that it constantly receives as much water as it loses: calling X and Y the coördinates HR and RO of the point R, and v its velocity of rotation, we have $v = wY$: moreover, the equation of the curve OMH gives $X = \frac{w^2Y^2}{2g}$: then $X = \frac{v^2}{2g}$; that is to say, that the height to which the centrifugal force will raise the water above the orifice R, open on a level with O, is equal to the height due the velocity of rotation of this orifice. X is also the head at R, and consequently, the velocity of discharge there will be that due

to $\frac{v^2}{2g}$, that is to say, that it will be equal to the velocity of rotation of the orifice.

If the water were brought to the vase by a tube, having the same axis, with a horizontal section considerably greater than that of the orifice of issue, and in which the fluid is maintained at L during the period of rotation, the water will issue at R, in virtue of its height X and of the new head LO ($=$ H'); thus the height due the velocity of issue will be $H' + \frac{v^2}{2g}$, and the velocity $= \sqrt{2gH' + v^2}$.

Even should a physical obstacle, such as a horizontal plate placed in the vase a little above the point O, obstruct the rising of the fluid above the orifice R, the effort X resulting from its tendency to rise, or from the centrifugal force, will none the less produce its effect upon the velocity of issue, which will always be $\sqrt{2gH' + v^2}$.

If, in the horizontal plane passing through OP, we make, at R', for example, a second orifice, placed at the distance Y_1 from the axis of rotation, the height due to the velocity of water issuing from it will be $H' + \frac{w^2 Y_1^2}{2g}$, just as it was at the first orifice, $H' + \frac{w^2 Y^2}{2g}$. Admit, then, that through one, as through the other, there issues the same quantity of water, $P^{lbs.}$ in 1", its dynamic force at the first orifice will equal (280) $P\left(H' + \frac{w^2 Y^2}{2g}\right)$, and at the second it will be $P\left(H' + \frac{w^2 Y_1^2}{2g}\right)$. Subtracting the former from the latter, we shall have for the increase of force of the same quantity of water, from one point to another, (an increase solely due to the centrifugal force,) $\frac{P}{2g} w^2 (Y_1^2 - Y^2)$; a value identical with that which we have already given in Sec. 298, observing that $\frac{P}{g}$ is equal to m, the mass of the running water.

393. Let us see now what will be the physical consequences of the two theorems we have just demonstrated, in the case of turbines in motion.

Since the water contained in a vase endowed with a movement of rotation around its vertical axis is depressed near this axis, the water of the basin in which a submerged turbine turns, will tend to a depression around the cylinder which delivers the motive fluid. From this tendency will arise, against the orifices of issue of this cylinder, a less pressure, or a *non-pressure*, analogous in its nature and effect to that described in Secs. 244 and 245; the interior pressure, by virtue of which the discharges take place, and which, in a state of repose, is H, or the difference of the two reaches, will be increased, the velocity of exit and the discharge of water will be considerably greater, and the force of the machine will thus be increased; it will be so much the greater as the wheel turns more swiftly. It may even happen that this increase of force will more than compensate the increased resistance experienced by a turbine moving in a fluid eight hundred times more dense than the air; and we may see, what seems paradoxical, but what experience nevertheless shows us to be true, a turbine produce an effect sensibly greater when it is immersed, (the difference of the two levels being taken for the fall).

By virtue of the second proposition, that, in a rotating machine, the velocity of the fluid issuing from it increases with its distance from the axis, the water will tend to be discharged from the turbine with a velocity greater than that with which it entered. Here, also, by reason of this tendency, notwithstanding the interposition of fluid found between the cylinder and wheel, and although the ducts of these two parts of the machine are discontinuous, the water, on quitting the turbine, may draw with it that issuing from the cylinder, and so augment its velocity; its action is nearly similar in character to what takes place in the lateral communication of the motion of fluids (106).

The increase of velocity and consequently of the discharge of water, according as the motion of rotation is more rapid, an increase which I suggested four years since, (p. 394 of the first edition of this Treatise), has been proved, by some experiments which M. Morin made upon a turbine established at Mühlback, in Alsace; it had a diameter of 6.56 ft. and a height of 1.08 ft.; the difference of level between the two reaches was 10.56 ft. I cite one of these experiments, made with 0.295 ft. raising of the gate; with the small weight of 77.18 lbs. put at the extremity of the arm of the brake, the turbine made 75 turns in 1′, and consumed 41.32 cubic ft. of water in 1″; the weight being increased to 396.98 lbs., the velocity was only 27½ turns, and the discharge 34.61 cubic ft.; thus, the velocities being diminished in the ratio of 273 to 100, the discharges of water were as 120 to 100. In another experiment, with a raising of the gate 0.492 ft., the first of these two ratios being as 100 to 289, we had for the second 100 to 128.[*]

Theory of M. Poncelet.

394. It remains now to bring into action the different elements of which we have just spoken, and to deduce from them an analytic expression for the effect of turbines in general. This labor has been performed by M. Poncelet, a *savant* well qualified to do it effectually; as one of the great propagators of the principle of *vis viva*, he would naturally make frequent use of it in arriving at the solution of the different parts of the proposed problem, and he has done it with rare ability. I limit myself to giving the expression of dynamic effect, indicating the course adopted by the author, and for the details, I refer to his memoir.[†]

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[*] Expériences sur les roues hydrauliques appelées turbines, par M. Arthur Morin, capitaine d'artillerie. 1838.

[†] Théorie des effets mécaniques de la turbine-Fourneyron. Dans les comptes rendus das séances de l'Académie des sciences. Séance du 30 Juillet, 1838.

Let

A	be the horizontal section of the interior of the cylinder.
O	the sum of the contracted sections of the orifices through which the motive water issues from the cylinder; each section being made by a vertical plane passing through the extremity of the guide curve, and directed perpendicularly upon the convexity of the following curve;
μ	the coefficient of the contraction which the fluid experiences on its entrance into the cylinder;
U	the velocity with which it issues from it. We have $Q=OU$;
O'	the sum of the contracted sections of the orifices through which the water is discharged from the turbine;
R' & R''	the radii of the exterior and interior circumferences of the wheel;
v' & v''	the respective velocities of these two circumferences; velocities which are as wR' and wR'', w being the angular velocity;
u	the relative velocity with which the motive fluid enters into the turbine;
u'	the relative velocity with which it issues from it;
i	the angle which, on its entrance, it makes with the interior circumference;
φ	the angle which, at its issuing, it makes with the exterior circumference;

We remark:

1st. That u' being the velocity of the water issuing from the wheel after deducting the motion of the latter, we have also $Q=Q'u'$, and consequently $U=\frac{O'}{O}u'$.

2d. That u is the resultant of the two velocities U and $-v''$, and that in consequence

$$u=\sqrt{\frac{O'^2}{O^2}u'^2+v''^2-2\,\frac{O'}{O}\,u'\,v''\cos.\,i.}$$

3d. That the absolute velocity of issue, being the resultant of the relative velocity u' and of $-v'$, is

$$\sqrt{u'^2+v'^2-2\,u'\,v'\cos.\,\varphi}.$$

These being the data, M. Poncelet determines all the losses of *vis viva* experienced by the fluid, from its entrance into the cylinder to its entrance in the turbine, inclusive, admitting (what is very nearly the reality) that in the wheels of M. Fourneyron, the first element of each float is perpendicular to the interior circumference; he finds, for the sum of these losses, $\frac{P}{g}(u^2+b^2u'^2-2bcu'^2)$; an expression in which $b=k'\frac{R'}{R''}\sin.\varphi$, and $c=\frac{O'}{O}\sin.i$, k being the coefficient of the perturbations which the fluid experiences between the floats.

Equating then the *vis viva* of the water on issuing from the turbine with the *vis viva* at its entrance, augmented by twice the quantities of action impressed, and diminished by the *vis viva* lost, he obtains an equation, which, all reductions being made, and supposing $\gamma=\left[1+\frac{O^2}{A^2}\left(\frac{1}{\mu}-1\right)^2\right]\frac{O'^2}{O^2}+b^2-2bc$, is

$$u'^2(1+\gamma)=2gH+w^2(R^2-R''^2).$$

It gives immediately the value of u', and since $Q=O'u'$ we have

$$Q=\frac{O'}{\sqrt{1+\gamma}}\sqrt{2gH+w^2(R^2-R''^2)}.$$

The first of these two factors of the discharge depends solely upon the dimensions of the machine; the other expresses, by its first term, the action of gravity in producing the velocity with which the water issues from the cylinder; and by the second, it expresses the action of the centrifugal force. This equation shows, that by reason of this last force, the discharge of water exceeds that which would be due simply to the difference of levels in the two reaches, and that the excess is in proportion to the angular velocity, as we have already observed (393).

As to the expression of effect, M. Poncelet established it by means of the principle mentioned in Sec. 297; the effect is equal to the force of the motor, minus the half both of the active forces lost and of the active force maintained by the water immediately after its exit; so that, with the values already given, we have

$$pv=PH-\frac{P}{2g}(v^2+b^2u'^2-2bcu'^2)-\frac{P}{2g}(u'^2+v'^2-2u'v'\cos.\varphi).$$

The author, passing then to the investigation of *maximum* effect, avoids a part of the difficulties which it presents, by taking the *maximum* ratio of pv to PH, and by causing only the velocity v'

to vary, or rather, the ratio of v' to $\sqrt{2gH}$. He gives the general expression of the first of these ratios, then that of the second for the case of *maximum* of effect, and finally, that of the *maximum* ratio. From these values, and admitting a rule of construction adopted by M. Fourneyron, he concludes, that in turbines, pv can never be equal to PH; but that it will approximate more nearly towards it, as the raising of the gate approaches more nearly the height of the floats, and as the angles i and φ are diminished. If they were zero, we should have $pv = PH$ and $v'' = 0.71\sqrt{2gH}$.

M. Poncelet concludes then from his calculations, 1st, that H not entering in the expression of the two ratios $\frac{pv}{PH}$ and $\frac{v'}{\sqrt{2gH}}$, the greatness of the effect, compared to the force of the motor, is independent of the fall; 2d, that the variations from the *maximum* effect are inconsiderable, though those of the velocities of the wheel corresponding to them may be quite considerable.

Having made various applications of his formulæ to the experiments of M. Morin, upon the turbine at Mühlbach, he found a satisfactory accordance. He remarks, however, that in great velocities, the real effect decreases much more rapidly than calculation indicates; he attributes the cause to the great resistance experienced by the turbine while moving with great velocity through the water in which it is submerged, a resistance whose action has not been introduced in the formulæ.*

Finally, M. Poncelet examined successively and succinctly what this resistance should be; what should be the influence of the annular play between the cylinder and turbine, as well as that of the plates which divide the height of certain turbines. See the memoir of the author on all these subjects.

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* Among the experiments made at Mühlbach, there are two series which enable us to appreciate with exactness two important circumstances in the motion of turbines. I cite a part of them.

1st. In the first of the annexed tables, we see that the effect pv, compared to the force PH, has been greater, as the raising of the gate approached more nearly the height of the floats, which was 1.062 ft. We had $H = 10.564$ ft. The ratio given in the last column is that which corresponds to the *maximum* obtained with the lift set against it.

2d. Beginning with the velocity of the wheel when it has no load, according as the load is increased and consequently its velocity diminished, the effect at first increases rapidly, then it gradually attains its *maximum*, and then it decreases

RAISING of the gate.	$\frac{pv}{PH}$
ft.	
0.164	0.37
0.295	0.52
.492	0.69
.656	0.74
.886	0.79

395. We pass to the real effect of turbines. There are few machines respecting which we possess, for this purpose, more full and more precise documents. Real effect.

We consider, first, those which have been furnished by the turbine of Gisors, already mentioned (387). It is in form and nearly in size the same as that of Fraisans, represented in Fig. 102; its exterior diameter is 9.51 ft., and its interior 7.874 ft.; the floats, in number 36, have a height of .984 ft.; their first element makes an angle of nearly 80° with the interior circumference, and the last, an angle from 10° to 12° with the exterior circumference. The cylinder has sixteen guide curves, meeting its surface at an angle of nearly 27°.

Shortly after its construction, in 1835, M. Fourneyron wished to measure its effect, by means of the dynamometric brake; but he could not fasten it immediately to the vertical shaft, and so he fitted it to a horizontal shaft geared with it; the brake therefore gave him but the useful effect $p'v$ measured upon the horizontal shaft. Still, he had from observation all the passive resistances, and consequently the total effect pv, or the force impressed by the current upon the turbine. His experiments, twenty-six in number, were divided into four series; I give, in the following

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gradually, the velocity diminishing considerably; as we see from the results in the annexed table, obtained with the same discharge of water. The velocities there varying from 34 to 73, the effects have not differed over ⅛ of the *maximum* effect.

N. B. The quantities of water P have been determined, at Mühlbach, by the common formula for weirs, with 0.41 for the coefficient; the experiments of M. Castel would indicate 0.432; thus the above ratios would be too great by about 5 in 100; but, on the other hand, pv has been taken on the shaft of the turbine, and, considering the friction of the pivot, it would be too small; they will thus nearly compensate each other.

Turns of wheel in 1'.	$\frac{pv}{PH}$
99.5	0.105
90	0.305
73	0.621
63.2	0.624
58.2	0.696
48.4	0.685
34.4	0.626

table, the mean result for each; in all of them, the turbine was entirely submerged.

Water in 1″.	Fall.	PH	$\frac{p'v}{PH}$	$\frac{pv}{PH}$
cub. ft.	ft.	horse powers.		
64.62	6.85	50	0.57	0.66
75.93	6.39	57	0.69	0.77
127.84	6.43	95	0.68	0.76
145.15	6.36	107	0.71	0.78

Such advantageous results attracted the attention of savans, and of the officers of government; M. Arago, member of the municipal council of Paris, thought they might be established in the heart of the city, upon the Seine, to raise its waters. At his suggestion, the prefect of the department appointed a commission of engineers, to revise the effects of the machine at Gisors, and to test them under small falls; for, at Paris, others could not be had: M. Fourneyron was made a member of the commission. Sixteen experiments were made with extreme care, on the 23d of January, 1837; and a report was made to the Academy, the 27th of the following month. There were three series, distinguished by the height of the fall; the turbine had from 2.526 ft. to 3.674 ft. of water upon its upper part. The water discharged was gauged at a weir, and by the formula $3.2603lh\sqrt{h}$ (the coefficient 3.26 was very likely too small by from four to five in 100; so that the ratios of $p'v$ to PH would be four or five hundredths too great). The effect was measured by means of a dynamic brake, placed upon the above-named horizontal shaft, and having a leverage of 13.46 ft.

I cite, in the following table, three experiments of each series.

FORCE.			EFFECT.			$\frac{p'v}{PH}$
			BRAKE.			
Fall.	Water in 1''.	PH	Load.	Turns in 1'.	$p'v$	
ft.	cub. ft.	horse powers.	lbs.		horse powers.	
3.838	98.427	43.93	242.5	44.25	27.88	0.641
3.746	95.460	41.16	330.8	35	30.07	0.731
3.743	94.365	40.64	463.1	26	31.27	0.769
. . . . .	. . . . .	. . . . .	. . . . .	. . . . .	. . . . .	. . . . .
1.962	66.28	14.96	286.7	12.33	9.18	0.614
2.044	66.91	15.74	264.6	15	10.31	0.655
2.044	67.56	15.90	242.5	18	11.34	0.713
. . . . .	. . . . .	. . . . .	. . . . .	. . . . .	. . . . .	. . . . .
0.991	45.55	5.20	110.27	10	2.86	0.552
1.007	46.37	5.38	99.24	13	3.35	0.622
1.040	47.60	5.70	88.21	14.50	3.32	0.582

What machine, other than the turbine, under the small fall of 3.77 ft., could acquire more than three quarters of the motive force, and a force of thirty horse-powers? or, under the slight fall of 0.984 ft., could take more than three fifths, and that, too, when entirely submerged in the water? Truly, the wheel of M. Fourneyron has an undoubted superiority in certain respects over all others; it is an admirable machine.*

Finally, it is not the turbine at Gisors only which has given such good results; let us remember, that in the first of those which M. Fourneyron has established, that built in 1827, the effect was .80 of the motive force (387). Among those last constructed, in 1837, if, at that of Moussai, M. Morin could not obtain so high a ratio as 0.70, at that of Mühlbach, he saw it raised as high as 0.793.

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* In a suit at law, now pending between Uriah A. Boyden, C. E., and the Atlantic Mills Company in Lawrence, Mass., in their answer to his writ, they admit that his turbines, which he built for them, have yielded an effect of 90 per cent. of the motive force. So great a result as 90 per cent. indicates a complete knowledge of the principles of these machines, with the details of their construction, and warrants us in the belief, that should he incline to publish his methods of construction, we may be possessed of information certainly equal, if not far superior, to any thing that can be derived from Europe. TRANSLATOR.

Recapitulating, and with the admission that in many cases, turbines acquire three quarters and more of the motive force, we will allow generally, with M. Fourneyron, for ordinary turbines, if well constructed and well run,

$$E = 0.70PH.$$

Peculiar advantages of turbines.

396. Thus, in respect to the amount of effect produced, turbines cannot be surpassed, except by some high bucket-wheels.

But they have over these wheels, as over all others, some important advantages. We have already remarked, that none, under very small falls, of .984 ft. for example, can produce such good effects. We will add, that none can work under such great falls; I doubt whether other wheels have ever been used with a fall of 49.21 ft.; and at St. Blasien, we have a turbine working under a fall of 354 ft.; and the effect, it is said, exceeds 0.75PH.

The space required for this kind of machine is inconsiderable; we have seen one of eight horse power, which was not unlike a piece of furniture, and could be put in a small room.

The velocity of turbines, as well as that of other horizontal wheels, (for there are many resemblances in their motions and in their properties,) will be quite often over a hundred turns per minute. But turbines being able to work under much greater falls, will often move incomparably faster; that of St. Blasien would make even 2300 turns per minute, (*Expériences sur les turbines,* par M. Morin, page 52); and turbines producing good effects will seldom have a velocity less than a half or third of that due the fall. If, in some cases, a great velocity admits of dispensing with gearing for the transmission of motion; in others, where the operating parts of the machine are to work slowly, we are obliged to have recourse to it. Generally, and as much as possible, its use should be avoided; not so much from the fact of its absorbing, without effect, a portion of the moving force, as that it multiplies, in mills, the chances of accident and of stoppages.

Precepts relating to the construction of turbines.

397. It would be desirable to give here the rules to be followed in the construction of turbines, so as to obtain the effects and advantages which we have just considered; but those which M. Fourneyron published on their introduction are very limited in number, and probably the experience he has since acquired may induce him to make some important modifications of them; however, as they were followed in the earliest constructions, and good machines have resulted from them, rendering, according to the author's statement, as much as .80 of the motive force, I think it proper to publish them.

The size of a turbine should be proportioned to the effect it is designed to produce, and, consequently, to the quantities P or Q and H. We give the principal of these dimensions, the interior diameter d, in its relation to these quantities. The turbine should afford, for the volume of water Q, which arrives with a velocity V, orifices of sufficient size; and for this purpose, we must have $Q = SV$ (108), S being the sum of the orifices of admission. Now, the water arriving at the same time upon the whole interior periphery of the turbine, upon the lateral surface of the cylinder forming this periphery, S will be equal to this surface (after deducting the thickness of the floats), and consequently to $\pi d h_2$, designating by h_2 the height of the floats. M. Fourneyron usually makes it equal to $\frac{1}{7}d$; thus $S = 0.4487 d^2$, and consequently, $Q = .4487 d^2 V = 3.60 d^2 \sqrt{H}$; whence $d = .527 \sqrt{\frac{Q}{\sqrt{H}}}$. This value should be affected by a coefficient expressing the effect of contractions and obstructions which the fluid meets in the cylinder, and at its entrance into the turbine, the effect of the obliquity with which the guide curves of the cylinder deliver the water upon the circuit of the wheel, etc.: according to the computations and practice of M. Fourneyron, I find that this coefficient, multiplied by 0.527, is 1.212, and consequently we have

$$d = 1.218\sqrt{\frac{Q}{\sqrt{H}}}.$$

The value Q to be admitted in this expression will be the greatest volume of water which the machine will have to consume, for a turbine can work with very different quantities of water, without a marked variation of effect, compared to the force employed.

The diameter *d* may also be expressed as a function of the force of the machine, that is to say, of the effect E which it should produce: we have (395) $E = 0.70PH = 43.624QH^{\text{lbs. ft.}} = 0.08041QH$ horse-powers: the value of Q, drawn from this equation, and put into the above expression of the diameter, changes it to

$$d = 4.297\sqrt{\frac{E}{H\sqrt{H}}};$$

E being expressed in horse-powers.*

As to the exterior diameter, M. Fourneyron makes it from 1.20*d* to 1.44*d*, according as *d* is greater or less. In the turbines known to me, *d* has varied from 7.87 to 1.47 ft.

The number of floats varies also with the diameter, but not proportionally; in the wheels just mentioned, there were from thirty-six to eighteen, and the guide curves were from sixteen to nine.

In the preceding numbers, we have given to the floats a height equal to a seventh part of the interior diameter of the wheel. But when the gate is only raised a little compared to this height, which will be necessary in case of a scarcity of water, the effect is very small, as I have already observed; the motive

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* This expression answers to the French "*cheval*," or 75 kilogrammes raised one metre in height every second = 542.5 lbs. ft. The equation for the English horse-power, or 550 lbs. raised one foot in height every second, would be

$$d = 4.3474\sqrt{\frac{E}{H\sqrt{H}}}.$$

TRANSLATOR.

action of the water is lost, as it were, in too great a space. It was probably to prevent this loss that M. Fourneyron, in some of his later constructions, has divided the turbines, in their height, into two or three stages, by means of one or two horizontal diaphragms, made of iron plates.

The theory of Borda (382) was a direct guide to this mechanist in the disposition of his floats. In order that the water launched by the cylinder should arrive upon them without shock, he established their first element in the direction of the resultant of the velocities of arrival and of the wheel; but as, in a turbine, the latter velocity may vary considerably, may be even doubled, without any marked change in effect, it became necessary to take a mean term; and very generally, M. Fourneyron has placed the first element nearly perpendicular to the interior circumference, and he has given the guide curves an angle of 30° with this same circumference. In order that the water may issue without velocity, it would be requisite that the fluid filaments, on leaving the wheel, should issue tangentially to its exterior circumference, and that, consequently, the angle made by them with it should be zero; but then they would quit it with difficulty, and this consideration has led to placing the last element of the float, which has a great influence upon the direction of the water at its issue, so as to make an angle of from 10° to 14° with the circumference.

Such are the principal rules to be followed in making turbines; but they are not to be adopted without some reservation, and some respect to local circumstances; it is thus that M. Fourneyron himself has done. The experience of more than fifty turbines, which he has probably built since the publication of his Memoir, must have suggested some new rules and numerous improvements. But he has published nothing upon this subject; it is a secret which he keeps to himself, wishing probably to manage his patent of invention to the best advantage. We hope, however, that when the term shall have expired, he will favor the public with his precious observations; and that then, competition lessening the cost of turbines, we may avail ourselves of them fully and freely.

4. *Duct-Wheels* (*Roues à couloirs*).

Turbine of M. Burdin.

398. M. Burdin has also resolved the problem of laying the water properly upon a horizontal wheel with curved floats. His machine is also composed of two parts, the one fixed and the other moveable; but, instead of making them concentric with each other, he has put the second below the first.

To get an idea of his turbine, imagine a basin in the form of a circular trough, the bottom of which, being quite thick, is pierced with holes or *injecting orifices*, widened at the top to prevent contraction, and directed so as to deliver the fluid at an angle indicated by theory.

Immediately below this feeding basin is the wheel. Its upper part presents also a circular trough, but of very small depth, upon the bottom of which are a series of short tunnels adjoining each other; at the bottom of each of them is a pipe, or "*couloir*" (a small *duct* of sheet iron,) bent so as to have its upper part vertical and its lower nearly horizontal. The water, on issuing from the injectors, is received in the trough, or rather, by the tunnels which compose its bottom; it descends along these pipes, and presses against the bottom of them; and, acting thus by its weight and by its centrifugal force, it causes the machine to turn. The vertical planes, which we may imagine as passing through the pipes, are not all perpendicular to the radii of the wheel adjoining their origins; alternately, one plane deviates a little to the right, the following one is perpendicular to the radius, and the third deviates a little to the left; so that the extremities of the pipes or ducts are found, alternately by threes, upon three circumferences of a different radius, but having a common centre at the same point of the axis of rotation. In this manner, the water is delivered upon three distinct circumferences; the fluid issuing from one pipe, and nearly without progressive motion, incurs no risk of being struck by that issuing from the following pipe. This disposition induced M. Burdin to give his machine the name of *turbine of alternate discharge.*

He established one at Pontigibaud, in Auvergne. But, simplifying the construction, instead of the annular basin established above the wheel, he made use of a water-lead, closed at its extremity, and with a block of wood fixed upon its bottom, in which were placed several injectors; so that the water was delivered, at one time, only upon the part of the periphery of the wheel lying immediately beneath the course. The effect obtained, measured by a brake, was as high as 0.67PH, and with a consumption of only 3.284 cubic ft. of water, instead of 9.89 cubic ft., which the percussion wheel (for which this was substituted) would have required.*

Wheel with a conical core.

399. I shall here mention a duct-wheel upon a conical core, designated sometimes under the name of *pear-shaped wheel* (*roue en poire*), and which Bélidor has described in these terms: "We see in some places, on the Garonne, mills of a very singular construction. The wheel is a species of drum, having the figure of a reversed cone, and which turns in a well of masonry made expressly for it. The floats are applied obliquely upon the surface of the drum, where they form portions of a spiral. These floats, thus disposed, compel the wheel to turn with great velocity, and also the millstone upon the same axle; and for this there is needed but a mere thread of water." (*Architect. Hydr.* § 668).

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* See a description of this machine in "Annales des Mines," 3d series, tome III., 1833. The wheel was 4.59 ft. in diameter by 1.31 ft. in height; and had thirty-six pipes or conduits.

If, instead of enclosing this wheel in a curb of nearly its own form, which compels us to leave a space through which the water, urged by its centrifugal force, escapes without effect, we should surround its floats with a conical envelope, concentric with the surface of the core, we should have an excellent duct-wheel, and, says M. Navier, the best of the *danaïds.*

Danaïds.

400. The name of danaïd was first given, by Carnot, to a machine of M. Manouri d'Ectot, the principal piece of which was a cask or small tub made of tin, and pierced at the bottom with a hole, through which issued the water, which entered at its upper part. The axis of rotation passed through it also. In this tub there was a drum, closed at its ends, with a diameter so much smaller than that of the tub as to leave a space of from $1\frac{1}{2}$ to $1\frac{9}{10}$ in. between them. There was a like distance between the lower base of the drum and the bottom of the tub. This last space was divided into compartments, by vertical partitions, terminating at the edge of the circular opening in the middle of the bottom.

The motive water was delivered, through spouts, tangentially to the interior surface of the tub. It advanced upon this surface, rubbing against it, and imparting thus a movement of rotation to the machine. While whirling round, it descended; on arriving at the bottom, it entered the compartments, and was directed towards the orifice of issue; but as it was retarded by the centrifugal force, it issued nearly without velocity, having expended nearly all its force upon the machine. Carnot, wishing to test its effect, caused it to raise different weights, and he found that it exceeded 0.70PH, and sometimes even 0.75PH.*

I have made mention of this machine simply because it is a type of a new kind, often alluded to by authors; for it has not been built upon a large scale.

It is not so with the danaïd which M. Burdin established at a saw-mill near the Bourg-Lastic (*Puy-de-Dôme*).† This also was a tub, with its bottom pierced with a circular orifice of about

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* Rapport de M. Carnot à l'Institute, in the "Journal des Mines." Vol. XXXIV. page 213.

† Annales des Mines. 1836. p. 504.

0.984 ft.; the diameter of the tub was 3.93 ft., and its height 7.54 ft. At 0.328 ft. above the orifice is a vertical tube of the same diameter, which rises to the top of the cask, and through which passes the axis of rotation. Between its convex surface and the concave surface of the cask are eight vertical partitions, descending to its bottom.

The water issuing from a reservoir, whose height, as in most of the turbines of M. Burdin, is equal to that of the moveable part, so as to arrive with a velocity due to half of the fall, the water, I say, let on with a slight inclination, and tangentially to the interior surface of the cask, impinges against these partitions; it presses against them, urges them forward, and so puts the machine in motion; arriving at the bottom, the horizontal velocity which it tends there to take, to escape through the orifice at the middle, is in a great measure destroyed by the centrifugal force, and there remains scarcely any at its exit.

5. *Reaction Wheels.*

Reaction wheels.

401. We designate by this name, machines in which the water contained in them, and which issues from them with a certain effort, reäcts upon the parts of the machine opposite the orifices of issue with an equal effort, in consequence of which it constrains these parts to recoil, and so occasions the motion of rotation. The following example will enable us to appreciate this mode of action; but before giving it, I revert to a principle.

The equality between action and reäction, which is regarded nearly as an axiom in mechanics, has been directly demonstrated by Daniel Bernouilli, in the case of a jet issuing from a vase (*Hydrodynamica*, pp. 279 and 303). He found, by calculation and experiment, that the effort exerted upon the vase by the reäction of the jet was equal to the weight of a prism which had for its base the orifice, and for its height twice the height due the velocity of issue; and we know that such is the measure of the effort of which the jet is capable (234).

Let there be a vase or great vertical tube, of which A is the base, which is moveable around its axis C, at the foot of which is fixed a horizontal tube BD, open at B, and closed through its remaining extent. If this apparatus be filled with water, the fluid will exert an equal pressure on all parts of the tube; that which takes place at any point will be destroyed by the pressure upon the point diametrically opposite, and there will be an equilibrium. But if we make an orifice at *a*, for example, there will no longer be a pressure upon this point; that exerted upon the opposite side will be no longer counterbalanced, and it will drive the tube in the direction from *a* to *e;* the jet issuing at *a*, acting by its *reäction*, will cause the machine to turn around its axis C, and in a direction opposite to its own; in the same manner as the elastic fluid arising from igniting the powder contained in the charge of a squib or rocket, issuing downwards, drives it rapidly upwards. Fig. 66.

Segner's machine.

402. If, at the lower part of the great vertical tube A, we have radiating from it many tubes similar to BD, and similarly pierced, we shall have the machine of reäction designed, towards the middle of the last century, by Segner, professor of mathematics at Göttingen, which the Germans consequently name *Segner's wheel* (*Segnersche Wasserrad*).

Euler, having made this an object of his studies, (*Académie de Berlin*, 1750,) proposed, 1st, to give a curved form to the horizontal tubes, so as to obtain a pressure resulting from the centrifugal force; 2d, to cause the water to issue through the extremities of the tubes, which extremities he curved so as to make them perpendicular to the radius of the wheel drawn to them.

403. Lately, M. Manouri d'Ectot, profiting by the indication of these improvements, planned a machine such as we see in Fig. 68. Its tubes, swelling in the middle, and curved like an ῶ, were united and held by iron bars. The motive water is conveyed to them by means of a great vertical tube, which is bent horizontally at B, and, passing under the wings or revolving arms, rises vertically, and terminates at the common centre C.

Manouri's machine.

Fig. 68.

These wheels have been successfully established in the mills of Brittany, of Normandy, and of the environs of Paris; "from authentic experiments, they produced an effect superior to that of the best executed 'pot wheels,'" says Carnot, in the name of the commission of the Institute appointed to the examination of this machine (*Journal des mines*, 1813, tom. XXXIII). I believe, however, that in common practice, we cannot, without difficulty, keep tight the junction of the stationary part, the tube conducting the water, with the moveable part, the wings or arms of the wheel. Otherwise, this wheel seems better fitted than any other to transmit the action of a current of water directed from below upwards, such as issues from certain Artesian wells.

404. Euler, whose ideas upon these reäction machines were derived from Segner's, designed one which seemed to him better fitted to reap the full advantage of this mode of the action of water. It had the form of a great bell, or rather, it was a truncated cone, hollow in the middle; consisting of two concentric surfaces, made of sheet iron plates, with a space between them, open at the top and closed at the bottom; small bent pipes were fitted vertically all around, and at the bottom, their extremities being horizontal and in the direction of the

Euler's machines.

motion, or rather, in a direction opposite to it. The motive water entering at the top of the machine, filled the space between the two conical envelopes, and issued through the small tubes. Though unwieldy, this machine has been used advantageously in France.

Three years after, Euler gave a more complete theory of reäction wheels; and on this occasion, he projected a second, which is described in the *Memoirs de l'Académie de Berlin*, 1754. It consisted of two parts, placed one above the other. The upper was immovable, and formed a cylindrical and annular reservoir, with small tubes fixed to the bottom, rectilinear, but inclined at an angle determined by calculation, and delivering the water upon the lower part. The latter, moveable around its axis, presented at the top an annular trough, from the bottom of which projected twenty tubes, diverging in their descent, the ends of which, bent horizontally, delivered the water in the air. All of these pipes were covered, as far as the bending, by a smooth sheet iron surface, designed to lessen the resistance of the air. Fig. 68 *bis*.

Such a machine, with tubes uniformly curved, not being obstructed at their extremity, and not being entirely full of water, has a close resemblance to the duct wheels of M. Burdin, Sec. 398; and the theory of Borda would be equally applicable to it.

Machines of M. Burdin. Reäction turbines.

405. The learned engineer whom we have just named, and to whom the works of Euler were unknown, also made a *reäction turbine*, which bears a great resemblance to that of the illustrious geometer. We give a short description of one which he established at the mill of Ardes, in the department of *Puy-de-Dôme*.

Fig. 69.

The fall is 6.56 ft. Under a wooden basin, where the water is maintained at a constant height of 3.28 ft., is placed the machine of rotation represented in Fig. 69. Three injecting orifices, fitted to the bottom of the basin, deliver the water horizontally in the crown, or small annular basin, which forms its upper part. It then enters into three pyramidal enclosures, with vertical axes, whose extremities are bent horizontally, having an orifice of issue. The height of the machine is 3.28 ft.; and generally, it is one half the fall.

It is contrived so that the turbine, under the injecting orifices, may have a velocity of 14.53 ft., that due the height of 3.28 ft. The water arriving upon the machine with a velocity equal to that of the points which receive it, there is no shock. Moreover, the head upon the orifices of the conduits being 3.28 ft., the water will issue from them also with the relative velocity of 14.53 ft.; and as that of the orifices in an opposite direction is the same in value, the absolute velocity of the fluid will be zero. The two conditions necessary for the *maximum* of effect are thus fulfilled, and the dynamic effect of the turbine will be PH.

But in practice, many circumstances always occur to change the conditions of this greatest effect. Still, M. Burdin has never seen the useful effect of his reäction turbines below 0.65PH, and sometimes it has been as high as 0.75PH (*Annales des mines*, tom. III. 1828).

Note upon the theory of reäction wheels.

406. Nearly a century has elapsed since the theory of reäction machines was the object of Euler's researches (402, 404): his memoirs upon this subject, which, however, I am not in a situation to properly appreciate, bear, according to competent judges, the impress of his analytical genius. But since their publication, and partly in consequence of the works of this great man, the theory of machines in motion, especially in all pertaining to their dynamic effect, has reached a much greater degree of generality and simplicity.

For a summary application to reäction wheels of this theory, the principal points of which I have already mentioned in Sec. 297, I will suppose, with M. Navier, that the water enters them without shock, and runs through them without a sudden change of velocity; I shall only, then, have to consider its absolute velocity immediately after its exit from the machine. We have demonstrated (392) that when water issues through orifices made in the circumference of a wheel in motion around its vertical axis, its velocity, relatively to that of the machine, is, upon the last element of the orifices, $\sqrt{2gh+v^2}$, h being the height of the reservoir above these orifices, and v their velocity of rotation. We suppose their extremity to be horizontal, and perpendicular to the radius of the circumference described; then, their velocity v is found directly opposed to that which the fluid possesses upon this extremity, and its absolute velocity, immediately after quitting it, is then $\sqrt{2gh+v^2}-v$. But the dynamic effect is equal to the force of the motor, minus the half of the *vis viva* which the water possesses after issuing from the machine (297), and we shall thus have

$$E = Ph - \frac{P}{2g}\left(\sqrt{2gh+v^2}-v\right).$$

This equation shows that the effect is greater, as the complex factor of the second term in the second member is smaller, and that it will be at its *maximum* and equal to Ph, when this factor is zero; now, we cannot have $\sqrt{2gh+v^2}-v=0$, except v is infinite. Whence we conclude, that in reäction machines, the effect can never be, even in theory, equal to the force of the motor, and that it is greater, in proportion as the velocity of rotation is the more considerable.

Finally, this very year (1838), M. Combes, mining engineer, took up the theory of reäction machines, and extended it to all the circumstances of motion: after having studied carefully that of Euler, he established a more general one, which he presented to the Academy of Sciences; but as yet, it has not been published. From the short notice upon this subject, inserted in the reports of the sessions of the Academy of Sciences (session of 6th August), the formulæ of M. Combes indicate in reäction machines, what those of M. Poncelet have shown for turbines, that the velocity of the wheel may experience great variations, either increasing or decreasing, from that giving the *maximum* of effect, without a marked diminution in this effect. "It is necessary," observes the author, "that the gates of the reäction wheel should be fixed upon the wheel itself; and in order that the useful effect may remain always the same, notwithstanding the variations in the volume of water, it is requisite that the gates should act at once upon the whole of the orifices of entry and issue of the moveable pipes, which should have between them a constant ratio, determined by the equation of motion."

Appendix, containing some observations upon the effect of grist-mills.

The horizontal wheels of which we have been speaking, especially the wheels properly so called (377, 380), are usually attached to grist-mills; these also present the most frequent examples of vertical wheels; their product is of the most general use, and is most intimately connected with our first necessities; these considerations induce me to state the little that is precisely known as to their useful effect.

Useful effect.

407. What is the resistance opposed by grain to the millstone? this is the first question to be resolved. Its solution will differ for each kind of grain; we restrict ourselves to the most important of all, that of corn or wheat.

Fabre, from some observations made upon the mills of Provence, estimates the resistance or effort opposed by corn to grinding, supposing that this effort acts at two thirds of the radius of the runner-stone, as the twenty-second part of the weight of this stone, inclusive of its fixtures.

Calling δ the diameter of the millstone, ε its thickness, ϖ the weight of a cubic foot of the material composing it, and v the number of turns made by it in one minute; its weight is $\frac{\pi}{4}\delta^2\varepsilon\varpi$, and its velocity at the extremity of the radius $\frac{\pi\delta v}{60}$. The dynamic effect, being the effort of resistance multiplied by the velocity of its point of application, will be

$$\frac{1}{22}\cdot\frac{\pi}{4}\delta^2\varepsilon\varpi\times\frac{2}{3}\cdot\frac{\pi\delta v}{60}=0.00125\delta^3\varepsilon\varpi v.$$

The specific gravity of siliceous or calcareous stones, of which millstones are made, never varies more than from 150 to 170 lbs. per cub. ft. On account of their fixtures, we raise it to 190 lbs., which will then be the value of ϖ; and we shall have for the expression of the useful effect of the millstone,

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* Essai sur les machines hydrauliques, et en particulier sur les moulins à blé, p. 234.

$$.238\delta^3 \varepsilon v^{\text{lbs. ft.}}$$

This value should only be regarded as approximate.

Force to grind a given quantity of corn.

408. The question of the useful effect of mills may be solved by a method of more direct interest to us, in determining, by experiment, the force necessary to grind a given quantity of corn.

M. Navier, combining and investigating the various published documents upon this subject, concludes that to grind 2.205 lbs. of corn, would require us to impress the millstone with a dynamic force or quantity of action equal to $40202^{\text{lbs. ft.}}$:* there would then be $3015200^{\text{lbs. ft.}}$ for a hectolitre or 2.838 bush. of corn, the weight of the hectolitre being 165.4 lbs. as a mean term. We usually estimate the work of a millstone by the number of hectolitres ground in one hour; so that the quantity of action which must be developed during this time will be 3.015200 lbs. ft. per hectolitre, or $839.36^{\text{lbs. ft.}}$ in one second; a force equivalent to that of 1.54 horse-powers. This value is much too small. M. Hachette, measuring the force by means of the dynamometric brake applied to the shaft of the motive wheel of a mill near Paris, which worked only on a large scale, found it 2.26 horse-powers. At the mills in the environs of Toulouse, MM. Tardy and Piobert, with a brake fitted to the shaft carrying the grinding-stone, giving immediately the force of this stone, found it from 2.80 to 2.87 horse-powers. M. Egen, among his numerous dynamometric observations, found it 3.56 at one of the good mills of Westphalia,—mills whose yield is, in truth, very small.

From these facts, and some others, I shall infer, that the force of a millstone, to grind 2.84 bushels of corn per hour, exceeds that of two horses; most frequently, it will be nearer three. To prevent all misreckoning, we will adopt the last estimate; especially if we refer it to the moving wheel of the mill, a wheel which usually transmits its action to the runner-stone through the intervention of gearing, which absorbs a part of this action. We shall consequently admit, *that generally, the force which a mill-wheel should possess is a three horse power per hectolitre* (2.84 bush.) *of corn ground in an hour.*

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* Architecture hydraulique, par Bélidor et Navier, tome I. p. 464.

409. As a wheel only takes the m^{th} part of the force of the motive current, the force of this current, on the basis we have laid down, should be $\frac{3}{m}$; the value of m, for the different wheels we may employ, has been given in this chapter. It is about 0.70 for good vertical wheels and turbines; thus, in employing such machines, we should have to count upon a force of water of four horse-powers, at least, for each hectolitre to be ground in an hour. For any wheel, this will be the force of the current, or .11507QH (282), divided by $\frac{3}{m}$; or .03835mQH.

Effect of mills according to experience.

410. That we may be enabled to judge of the actual amount of work of different mills, and of their mechanical as well as economical effect, I give, in the following table, the results of some authentic observations. I there indicate the kind of wheel used, as well as the value of m corresponding to it, according to the basis above established (408). In a note concerning each observation, I shall furnish some data in regard to the mill where it was made.

But first, I remark, that the same grinding-stone, with the same discharge of water, and with the same fall, may grind, in the same time, quantities of grain which may vary as one to three, and even more, according as the grain is coarse or fine, hard or soft, or according as it is to be made into the fine flour for the bakeries or the coarse for military stores. So that we must regard only as mean terms the ratios indicated in this table, as well as in the works of different authors, between the quantity of grain ground and the force employed to grind it.

KIND OF WHEEL.		WATER expend'd in 1.″	HEIGHT of fall.	GRAIN ground in 1 hr.	FORCE per hec'litre	m
		cub. ft.	ft.	cu. ft.	horse.	cu. ft.
	(*a*)	31.537	1.000		3.0	1.00
Bucket-wheels (good)	(*b*)	22.991	3.645	8.75	3.9	0.77
" " (medium)	(*c*)			6.21	7.3	0.41
" " (ordinary)	(*d*)	12.184	5.653	1.87	14.8	0.20
Wheel of trough-mills . .	(*e*)	7.204	10.105	3.53	8.4	0.35
	(*f*)	8.864	13.287	7.06	6.9	0.43
	(*g*)	8.829	13.123	4.69	10.0	0.30
	(*h*)	10.842	14.304	7.62	8.2	0.36
Wheel of pit-mills . . .	(*i*)	25.851	7.093	6.39	11.7	0.25
	(*k*)	26.028	7.513	4.41	18.0	0.17
	(*l*)	30.089	7.710	4.41	21.4	0.14

(*a*) "I learn," says Evans, (p. 131 of his *Millwright and Miller's Guide*, translated by M. Benoit,) "that, from exact experiments, made at the expense of the English government, it is ascertained that a power of forty thousand cubic ft. of water falling one foot, can grind and bolt one bushel of corn." Does the power act directly upon the millstone? If there was an intermediate machine, what was it? Evans does not tell us. The fact which he reports indicates a force of 3.6 horse-powers, for grinding and bolting the hectolitre, equal to 2.84 bushels; I take three for the grinding only.

(*b*) Observation made by M. Mallet, engineer, upon a mill of the English pattern, in the neighborhood of Paris. The grinding-stone was 4.265 ft. in diameter, and made from 100 to 120 turns per minute.

(*c*) General result of very numerous observations of Evans upon the mills of the United States of America. The bucket-wheels employed are badly constructed and badly disposed, and present too great a height of water above the summit. The millstones are generally five feet in diameter, and make 100 turns in a minute. (*Miller's Guide*, pp. 118—124).

(*d*) Egen made this observation upon a mill in Westphalia. The wheel, which was 12.66 ft. in diameter, drove a millstone having only 4.65 ft. diameter, and making sixty-two turns per minute. It made per hour only 82.22 lbs. of fine flour; the other mills of the country do not yield more, according to the report of the author.

(*e*) This fact relates to a mill established upon a small stream, near Montauban, and working only at intervals; according to the supply of water, it yields 198, 165 and 132 lbs. of flour.

(*f*) I made this observation upon one of the best mills in the neighborhood of Toulouse, the *Bayard Mill*, established on the canal of Languedoc. It was a merchant-mill, and yielded an unusual product; one pair of stones ground 5⅔ bushels of corn per hour; and the other, newly sharpened, went as high as 11⅓ bushels.

(*g*) The ordinary product of the good mills on this canal, which I indicate in this line, is in no case above 220 lbs. of flour, when they work for the bakeries.

(*h*) A mile below the mill of Bayard, is that *des Minimes*, upon which MM. Piobert and Tardy, after having executed the dynamometric experiments mentioned in Sec. 379, also made various observations upon the grinding; that noted in the table was done by a millstone newly picked, and working for traffic; it yielded about six bushels. But at its side was another millstone, which had been picked a month and a half, and which, with a nearly equal force, only made three bushels (of fine flour, it is true); it expended thus per hectolitre a power of more than sixteen horses, though the mechanism was properly disposed.

(*i*, *k*, *l*) These three observations were also made by MM. Tardy and Piobert, upon three different stones of the mill of Bazacle. The first had been dressed an hour and a half only, and made flour for ammunition bread. The second had been dressed eight days, and worked for a bakery. Finally, in the last, the flour was ground very fine, and the millstone had been lightly picked some days previous. These grinding-stones, as well as all those of the country, are made of porous silex; they are generally 5.74 ft. in diameter, and make about eighty turns per minute. They do not accomplish, per hour, more than one to one and a half hectolitres, rarely two; and the proprietors of the mills are satisfied if they obtain regularly one hectolitre. Elsewhere, it is said, more is accomplished; and according to M. Taffe, the trough-mills of Provence yield more than six hectolitres per hour, and do not expend a force of six horses per hectolitre.*

CHAPTER III.

MACHINES WITH ALTERNATING MOTION.

Hydraulic machines, which, instead of a rotatory motion, work with a reciprocating motion, are but little used in the industrial arts; I know of but two that are extensively used, the *water-pressure engine* and the *hydraulic ram.*

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* "Application des principes de mécanique aux diverses machines." A current furnishing 14.444 cub. ft. per second, with a fall of 21.65 ft., yields per hour 1056 lbs. of flour; that is 6.40 hectolitres, and a force of 5.62 horse-powers per hectolitre.

ARTICLE FIRST.

The water-pressure engine.

411. This machine consists of a cylinder, or working-barrel, in which moves a piston impelled by the weight of a high column of water, contained in an upright pipe. To the piston rod is fitted a connecting rod or working beam, which transfers the motion to the common pumps or other *operators;* sometimes, though rarely, we fit to it a mechanism which transforms the reciprocating into a rotatory motion.

The first idea of such a machine is due to Bélidor, who, in the second volume of his *Architecture Hydraulique*, published in 1739, makes known the considerations which led him to this discovery, and enters into all the details of its construction. It was not, however, till ten years after, that a machine of this description was made; it was made by Hœll, at the mines of Schemnitz, in Hungary. Then some others were built at these same mines, as well as at those of different parts of Germany, where they were called *Hœll's machines.*

But their construction and establishment required artists of a superior order to those commonly employed; they required, especially for their maintenance, much care and expense; and the effect which they rendered was not proportioned to the expense. Thus, they were falling into disfavor and disuse, when a peculiar circumstance, thirty years ago, drew towards this machine the attention of a man of genius, Reichenbach, one of the most accomplished mechanists of our age. Being occupied, by order of his sovereign, at the salt-pits of Bavaria, in the extreme branches of the Tyrolean Alps, the working of which (becoming more and more expensive) was on the point of being abandoned, he conceived and executed the grand and bold design of taking the salt water immediately from its sources, and leading it across a mountainous country, a distance of 68 miles, to a district where there was abundance of wood necessary for the manufacture of the salt. Eleven water-pressure engines, some single acting and some double, and all on a new principle, were employed, with great success, for this purpose; one of them, that of Illsang, raised water, at one jet, to a vertical height of 1168 ft., and thus carried it across a deep valley.

Some years after these gigantic works were completed, which was in 1817, a quite vague report of it came to M. Juncker, engineer, director of the mines of Poullaouen and of Huelgoat, in Brittany, at a time when he himself was occupied with the establishment of a water-pressure engine at the last of these mines. He repaired to Bavaria; there saw Reichenbach and his wonderful constructions, submitted to him his plans, received his advice, and, after his return in 1831, executed the greatest and the most beautiful hydraulic machine which we have in France.

Machines of Huelgoat. Fig. 70.

412. This machine, or rather these two machines, for there are two precisely alike, side by side, are designed to drain the water from the mine, the quantity of which may be as high as 7000 cubic ft. per hour. M. Juncker established these machines at about 360 ft. below the surface of the ground, in the middle of the pits, to the bottom of which all the water was conducted, at a depth of 1080 ft. For this purpose, he threw over the chasm of the pits a cast iron bridge, resting upon

freestone abutments, and with all the appliances that art could furnish to insure its stability.

On this bridge he planted the two great cylinders A, the principal pieces of the engines.

They are of cast iron, and open at the top; each is 3.37 ft. in diameter by 9.02 ft. in height. The piston B is made of brass, and has only a simple leather packing (425); its stroke is 7.54 ft., and it makes 5½ per minute. At its centre is fitted an iron rod C, which passes through the base of the cylinder, and descends vertically to the bottom of the pit, where it is fitted immediately to the piston of a pump established there, and which, at one jet, raises the water 754 ft. in vertical height; there it is delivered into the discharge gallery.

At the foot of the cylinder is a tube D, through which enters the motive water designed to raise the piston, and through which it afterwards issues when it descends. Another piston, the *regulator* E, which moves to and fro in the cylindrical box F, puts alternately this tube in communication with the water-pressure tube ending at G, and with the discharge tube H, which, being bent in a vertical direction, ascends to a height of 45.93 ft. or to a level with the discharge gallery. The height of the pressure tube is 242.78 ft.; this is the head which impels upwards the piston B; in an opposite direction, as it were, we have a head of 45.93 ft., by reason of the ascent just mentioned; so that the head or effective fall is but 196.85 ft. If the machine had been established on a level with the discharge gallery, the pressure pipe would have had but this last height, and we should not have been compelled to raise all the motive water this height of 45.93 ft. But it was desirable that it should equipoise in part the enormous weight (about 35287 lbs.) of the rod C, which would have drawn too forcibly the piston in its descent, and would have increased too much the weight to be raised; an equilibrium is thus produced by the weight of a column of water having the piston for its base, and 45.93 ft. for its height. Such a *hydraulic balance* is worthy of note.

Notwithstanding all the interest which the machine of M. Juncker possesses, I shall not enter into the details of its construction, of its regulating mechanism, nor even speak of its accessories, such as the iron bridge, the pits, the aqueduct galleries, &c. All these objects are amply discussed in a complete and philosophical description which the author himself has published. (*Annales des mines*, tom. VIII. 1835.)

413. Still, I will endeavor to give an idea of his *system of regulation*, a system the basis of which is due to Reichenbach, and which is admitted to be that best fitted for water-pressure engines.

Regulator system.

The principal piece is the regulator piston E. It is a hollow brass cylinder, and is perfectly turned and polished: its height, which is triple that of the junction pipe D, is divided into three parts; that of the middle, being a little over a third of the height, is smooth on its exterior surface; the two others are fluted, each having eight grooves, whose depth, at first nothing, increases as they approach their respective bases; so that their vertical section is a right angled triangle. Suppose, now, that the great piston B is at the bottom of its stroke, and the regulator, being midway of its descending stroke, and entirely covering the communicating pipe, continues its descent; as is represented in the figure (made on a scale of $\frac{1}{80}$). The water, which is upon the head of the regulator, under the pressure of the entire fall, passing at first through the foot of the grooves, will begin its arrival under the piston in very small quantities, and will accordingly urge it upwards with an extremely small velocity: the flow of the water and its velocity gradually increases, and will be at its *maximum* when the upper base of the regulator, in its descent, shall be found at the level of the lower edge of the connecting tube; then the piston B will be in the middle of its ascending stroke. At this moment, the regulator, by means of a mechanism which we shall soon describe, will take an ascending direction, and will contract the orifices for the entrance of the water, in the same ratio as it had opened them in its descent; so that in the middle of its stroke, it will entirely cover the opening: no more water will arrive in the cylinder, and the piston B, having reached the limit of its stroke, will stop. The regulator continuing to ascend, its lower grooves will present themselves by degrees before the connecting tube; the water in the cylinder, pressed by the weight of the piston and its appurtenances, will issue through the grooves, and reach the emission tube H, at first in small quantities, and the piston B will begin gently to descend; then it will descend more and more rapidly, until the regulator reaches the end of its stroke; then it will again descend, diminishing the emission more and more, till it becomes nothing. It follows from this, that the velocity of the piston, whether ascending or descending, is at first extemely small; that it then increases gradually up to the middle of its stroke; and then it diminishes gradually to zero. In this manner, all sudden action and shocks are avoided, so that one standing by the machine does not hear the least noise, and is astonished at the ease and smoothness with which it performs its great movements. By simple grooves, suitably made in the upper as well as the lower part of the small piston, have been thus completely solved both the great theoretic problem of preventing every loss of *vis viva*, and the no less important practical problem of avoiding concussions, a principal cause of the destruction of machines.

To counterbalance the effort exerted by the column upon the head of the regulator or piston E, another piston, I, is placed immediately above, which moves in the box K, having a diameter a little greater than that of F, and this piston is connected with the first by an iron rod. In this manner, the water contained and pressed in the two cylinders will exert upon the piston I, from below upwards, an effort a little greater than that which it exerts, from above downwards, upon the piston E; consequently, the system will rise, and will naturally be held at the top of the common stroke. To make it descend, the piston I is surmounted by another hollow cylinder L reversed, and having an annular space between its exterior surface and the interior surface of the cylinder K: a leather packing, placed at the top of this cylinder, closes the upper portion of the empty space. We have further the small bent tube *abc*, and the straight tube *gf:* in this last move the two small pistons *m* and *n*, united and disposed between themselves similarly to E and I. The water which is in the cylinder K enters through the orifice *a*, follows the tube *abc*, then *cd*, traverses the small communicating tube *de*, empties in the annular space which encircles the cylinder L, and fills it: it acts there, under the entire head of the pressure column, upon the annular border of the upper surface of the piston I; this effort, united with that exerted upon the head of the regulator, surpasses that which takes place from below upwards upon the piston I; and the system descends. If, after the descent is effected, we raise and place the small piston *m* between the orifices *c* and *d*, the communication between the pressure column and the annular space is cut off, the effort exerted at the upper surface of the piston I no longer exists, and the regulator ascends. Thus, to make it ascend or descend, all that is necessary is to bring the small piston *m* above or below the orifice *d*. The force necessary for this purpose is inconsiderable, the effort which the fluid exerts upon this piston being in a great measure equipoised by that which takes place in the inverse direction upon the piston *n*. When the machine is put in motion, the machinist himself, taking in hand the small lever *lo*, brings successively the piston *m* to a suitable position. But after that, the great piston B continues the work of itself. For this purpose, near its edge is fixed the rod *pq*, having two cams, *s* and *t*, fixed upon its two opposite faces. They act upon two catches, placed also upon the two opposite faces of the sector fitted to the extremity *l* of the lever *lo:* when the piston ascends, one of the cams raises the lever, and consequently the small pistons; and lowers them in its descent.

These cams may be fixed upon different points of the rod *pq*, and as they are more or less distant, the stroke of the great piston is the more or less extensive. We may vary this stroke by opening more or less the cocks *b* and *f* through which the water enters into the annular space, or issues from it. There is also, in the pressure pipe, as well as in the discharge pipe, a circular valve or register, by means of which we contract at will the passage of the water running in, as well as the effluent water: the cut off of the first diminishes the ascending velocity of the piston, and that of the second its descending velocity.

Such are the means by which are governed at will, and with great ease, the two enormous engines of Huelgoat. Seeing them as it were suspended, midway of the pits, at more than 656 ft. above the bottom; seeing them raise a very great volume of water, at one jet, to a height of 754 ft., without the intervention of levers, gearing, &c.; seeing them accomplish their great movements, with a surprising smoothness and silence, I cannot withhold saying of these engines of M. Juncker, what he himself said, at Illsang, on seeing that of Reichenbach: "All is admirable for boldness, for simplicity and precision."

Effect of water-pressure engines.

414. In water-pressure engines, the piston receives immediately all the weight of the motive water, except the small quantity which is taken to put and keep the regulator in play; moreover, nearly the entire head of the water H is made useful; so that their dynamic effect should be very nearly expressed by PH. But then, the friction of the pistons in their respective cylinders, the resistances experienced by the water in the pipes and in passing numerous contractions, absorb an important part of the force of the motor; and the useful effect is never greater, even in good constructions, than two thirds of this force.

In the ancient machines, those of Hœll, we only find it from 0.33PH to 0.46PH; though in one it was raised to 0.52PH.* But it is more considerable at the establishments made in later times, at the mines of Hungary, of the Hartz, &c. At those of Freyberg, in Saxony, according to the report of the sub-director of the mining engines of this kingdom, the useful effect, according to very exact observations, was not below 0.70PH; and in some, when the pumps which they drive worked with all the water they could carry, it was raised to 0.75PH.[*]

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* Hachette, Traité des machines, pages 171 and 323.

Such will probably be the case with the engines at Huelgoat, when they shall have their entire load. According to a gauging of the infiltrating water of this mine, the quantity has not been over 1.06 cub. ft. per second; but when the subterranean works shall have attained their full depth, we presume that the volume will be doubled, and so each of the machines will have to raise 1.06 cubic ft. in 1″; to meet which its dimensions have been determined. So that the useful effect which they will have to produce will be 66.16 lbs. raised 754 ft., or 49887 $^{\text{lbs. ft.}}$ The head being 196.8 ft., we presume that it will require from 352.87 lbs. to 385.95 lbs. of motive water, which will give an effort of from 0.72 to 0.66PH. M. Juncker, for still greater certainty, reckons upon 392.57 lbs. of water, and upon a useful effect of 0.65PH. At this time, when the height of elevation is only 587.28 ft., with but little water to be raised, we have in reality but 0.45PH.

From what has been said upon the effect of water-pressure engines, upon their useful effect alone, we conclude that, in general, as regards dynamic effect, they do not yield to any other description of machines; and that it is fit that we should employ them in preference, in many circumstances, as when it is desired to make the best use of a great fall of water, especially if the work is to be accomplished by a reciprocating motion, like that of pumps. M. Juncker has shown, in his memoir, the very great economy that the water-pressure engines of Huelgoat have produced, in the expense of draining the water of that mine.

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[*] Page 418 of the translation in German of the first edition of this "Traité d'hydraulique," made, with some additions, by the sub-director whom I have just mentioned, M. Theodore Fischer. "Handbuch der Hydraulik. . . . . Leipzig, 1835."

ARTICLE SECOND.

The Hydraulic Ram.

Its parts. 415. This machine, of a very peculiar character, remarkable for its simplicity as well as for its mode of action, is the invention of M. Montgolfier, who took out a patent for it in 1797.

Fig. 71. It is composed, independently of the feeding reservoir or leading conduit M, of a pipe or *body of the ram* AB, which conveys the water to the operating part of the machine; this part, or *head of the ram*, consists of a short pipe CD, open on its upper side through an orifice *e*, against the edges of which is applied the plate or *stop-valve a*, designed to close it; the extremity of this head bears the *ascension clack-valve b;* it empties into a receiver, whose upper part is full of air, and is consequently called the *air reservoir;* this receives in its lower part, which is filled with water, the extremity E of the *ascension tube.*

The arrangement, as well as the form of the pieces which we have just named, may, however, be varied; thus, it is quite different in Fig. 72 from that in Fig. 71. Fig. 72. In the former, instead of the common valves and clack-valves, spheres or hollow balls are substituted, of a specific gravity double that of water; they are retained in an iron frame, which allows them the necessary play; the edge of the openings which it is their function to close is provided with cushions of tarred linen.

I cite, as an example, the largest among those which have been built, at least, in France: it was established by Montgolfier's son, at Mello, near *Clermont-sur-Oise*. The body is a cast iron tube, 0.354 ft. diam., 108.2 ft. long, and weighs 3198 lbs.: the head weighs 441 lbs.: the capacity of the air reservoir is but 0.21 cub. ft. The stop-valve consists of a horizontal plate, pierced with seven openings, covered by as many hollow balls, 0.13 ft. in diameter: it beats sixty blows per minute.

Action of the ram.

416. Let us give an idea of the action of this singular machine.

Fig. 71.

Let us first suppose it to be at rest; the water in the ascension pipe will be at the same level with that in the reservoir M; the valve at *e* will be closed by the pressure of the fluid against the sides of the ram; and that at *b* will be closed by its own weight. Let us depress the plate or stop-valve *a*, by pressing upon its end; the water will issue through the orifice *e*, by virtue of the head in the reservoir; it will establish, in the body of the ram, a current from A to C; on arriving at the head, it will take an ascending motion from *a* to *e*, in consequence of which, the plate *a* will be driven upwards, and strike against the edges of the opening *e*, which will thus be smartly closed. The efflux will cease, it is true; but the fluid column AB, in virtue of its acquired velocity, will act still with all its *vis viva;* it will butt like a ram against the clapper *b*, and will open it; the fluid will penetrate into the reservoir N; it will compress the air found there, and cause the water already in the ascension pipe to rise. It will continue to rise there, followed by the water of the reservoir, but progressively diminishing in velocity, until the movement impressed upon the column AD, gradually reduced by the continued action of resistance of the compressed air and the weight of the water to be raised, is entirely destroyed. Then these resistances, predominating and becoming active in their turn, will impress another motion, but in an opposite direction, upon the water which was in the reservoir and in the ram; a phenomenon analogous to that of a fluid, oscillating in a tube, which descends again after being raised to a certain height. At the first instant when the retrograde motion commences, the clapper *b* will be shut; but after its closing, the motion from D to A will continue; consequently, it will tend to create a vacuum under the stop-valve; the stop-valve *ae*, pressed by the weight of the atmosphere, will descend; the collar or enlargement at the end of the stem, designed to limit its descent, will strike forcibly against the band that retains it; and the orifice *e* will again be reöpened. As soon as the retrograde motion is exhausted, the fluid AD, urged anew by the head on the reservoir, will recoil; it will issue through the orifice *e*, re-shut the valve *a*, and will produce, a second time, an order of results similar to the first.

These operations will succeed each other without interruption, as long as the reservoir shall continue to furnish a fresh supply of water, or until its communication with the head of the ram is cut off by a gate or otherwise.

Real effect of the ram.

417. The oscillating motion of the water in the hydraulic ram, with the indication of the mechanism which produces and maintains it, well explains the physical cause of the action of the machine; but its circumstances are far from being well enough known to furnish a basis for a mathematical theory; experiment alone instructs us as to their useful effect. As to the total dynamic effect, the passive resistances, and especially those arising from the shock of the valves, will present difficulties

in estimating them, which render its determination nearly impossible.

Before reporting the results of experiment, I observe, that, in the estimate of the effect of the ram, we need not, as in the case of hydraulic wheels, take into consideration the velocity of motion, and consequently, its reference to a unit of time. The effect will be the weight of water raised a certain height, in a certain determinate time; calling p'' this weight, and H_1 this height, it will be $p'' H_1$. The corresponding force (P being the weight of the fluid furnished by the current in the same time, and H the height of the fall) will have PH for its value; consequently, the ratio will equal $\frac{p''H_1}{PH}$; it will also be $\frac{qH_1}{QH}$, designating by q the volume of water raised, and by Q the volume of water expended; since $Q : q :: P : p''$.

418. The following table shows the ratio and effect of our common rams. The first of the observations reported was made upon the ram which Montgolfier set up at his house in Paris; the second refers to a great ram constructed by his son, which we have already alluded to (415); the following relate to three rams, located in the environs of the capital, which he mentions in his *Traité des machines* (p. 161).

NUMBER of experiment.	HEIGHT		WATER		$\frac{qH_1}{QH}$
	of fall H	of elevation H_1	expended Q	raised q	
	ft.	ft.	cub. ft.	cub. ft.	
1	8.53	52.69	2.401	.22037	0.570
2	37.30	195.01	4.944	.61811	0.653
3	34.77	111.87	2.966	.60037	0.651
4	3.21	14.92	70.173	9.50011	0.629
5	22.96	196.85	0.459	.03425	0.671

The average of these experiments give 0.65 for the mean ratio of qH_1 to QH.

With a view to determining this ratio, Eytelwein, one of the most accomplished and expert of hydraulicians, made observations upon two rams, constructed for him in 1804, at Berlin. According to a well digested plan, he varied gradually and successively the dimensions of the different parts of these machines; by 1123 experiments, he determined the effect produced in each case, and deduced rules as to the dispositions, and dimensions of parts, adapted for the best effect. (Eytelwein's *Observations on the effects, etc., of the hydraulic ram.*)

I limit myself to giving, in the following table, some experiments made with the larger of the two rams, such as it was when admitted to be disposed in the most advantageous manner. Its dimensions were:

Length of body, ...	43.734 ft.
Diameter, ...	0.186 ft.
Capacity of air reservoir, ...	0.31078 cub. ft.
Area of opening of stop-valve, ...	0.0258 sq. ft.
This area, in the first experiment, was . .	0.04305 sq. ft.

The two valves were arranged as indicated in Fig. 71.

NUMBER of beats in 1'.	HEIGHT of fall H		WATER IN 1' expend'd Q		$\frac{qH_1}{QH}$ according to experiment.	
	of fall H	of elevation. H_1	expend'd Q	raised q	experiment.	formula.
	ft.	ft.	cub. ft.	cub. ft.		
66	10.059	26.30	1.709	0.543	0.900	0.97
54	10.167	32.35	2.242	0.615	0.873	0.92
50	9.931	38.64	1.928	0.421	0.850	0.87
52	7.995	32.35	1.310	0.271	0.847	0.85
45	8.730	38.64	1.758	0.336	0.845	0·84
42	7.425	38.64	1.592	0.241	0.787	0.78
36	6.046	38.64	1.426	0.169	0.754	0.71
26	4.447	32.35	.840	.079	0.672	0.67
31	5.062	38.58	1.292	.113	0.667	0.65
23	4.117	38.64	1.783	.104	0.548	0.56
17	3.003	32.18	1.734	.074	0.473	0.51
15	3.218	38.64	1.981	.058	0.352	0.45
14	2.486	38.64	1.935	.035	0.284	0.32
10	1.971	38.64	1.575	.014	0.181	0.18

419. The first of these experiments gave the greatest effect; its useful effect alone was 0.90 of the force employed to produce it; no machine presents so advantageous a result. But this advantage, possessed when the height of raising is small compared to the fall, diminishes as the height increases, and ends with being below that of other machines; a single glance at the last column but one of the table is sufficient to show this, the experiments there being ranged in the order of the magnitude of the elevations, compared to those of the falls. Thus, in the ram, the ratio of effect to the force diminishes as the height of the elevation increases.

I express, with sufficient simplicity and exactness, the results of the experiments at Berlin, by the following equation, by means of which the numbers of the last column of the table were calculated; they differ but very little from those given by observation:

$$\frac{qH_1}{QH}=1.42-0.28\sqrt{\frac{H_1}{H}}.$$

Expression of effect.

420. The above expression, being deduced from experiments which refer in some measure to the *maximum* effect of rams, will usually give too great products. We shall have them sufficiently exact, by reducing the numerical coefficient by about a sixth, and establishing, with our usual symbols,

$$pH_1=1.20P\,(H-0.2\sqrt{HH_1}).$$

Let us apply this formula to those of the above experiments which gave the greatest effect: this effect being reduced to the second of time, we have for the

2d experiment of 1st table in 418, 124.81$^{\text{lbs. ft.}}$ instead of 125.39$^{\text{lbs. ft.}}$
4th " of same table, 160.42$^{\text{lbs. ft.}}$ " 147.61$^{\text{lbs. ft.}}$
1st " of 2d table, 14.54$^{\text{lbs. ft.}}$ " 14.90$^{\text{lbs. ft.}}$

421. The hydraulic ram has not yet been used except to raise

Observations upon the use of the ram.

small quantities of water, and consequently but to produce small effects. The greatest which Eytelwein obtained, in his 1123 experiments, was not over $24.602^{\text{lbs. ft.}}$ in one second. The greatest for rams constructed in France has been, as we have seen, only from 123 to $144.7^{\text{lbs. ft.}}$, but half the effect of a horse harnessed to a gin.

Can the ram be equally well employed for raising great volumes of water? This is to be doubted. The violent shock of the valves, and the strong blows which the machine makes, shake its supports. Attempts have been made to reduce these jars, by increasing the weight of the machine, and thus diminishing the injurious effects proceeding from its vibrations; but the evil is only partially remedied. For great rams, the strong masonry and carpentry employed to hold them, are themselves shaken and impaired at the end of a certain period. So that there are grounds for believing that this machine, otherwise so remarkable, may be restricted in its use; that it is not adequate to furnish a supply of water sufficient for the wants of a large building or a manufactory.

SECTION FOURTH.

MACHINES FOR RAISING WATER.

422. We proceed here, also, with the discussion of hydraulic machines, but of a different kind from those treated of in the preceding section; in them, the water was the motor, the power; in these, it is the body moved, the resistance. We say, in this connection, that we by no means intend to dwell upon all the machines which have been used or devised for raising water, but simply upon those in most common use; such as *pumps*, the *Archimedean screw*, and bucket machines, such as *norias*, chain pumps, Persian wheels and *tympana*.

CHAPTER FIRST.

PUMPS.

Parts of pumps.

423. A pump consists of a cylinder, or working-barrel, in which moves, with a reciprocating motion, a *piston*, to which is fitted one or two cylindrical pipes; the one below is the *suction pipe;* the other, above or at the side, is the *lifting pipe.* The upper opening of the first is covered with a plate or *valve,* which rises and falls alternately, according to the circumstances of motion; there is still another, either upon the piston or at the lower opening of the lifting pipe.

I shall not enter into details relating to the making and arrangement of these different parts; they may be found in the *Architecture Hydraulique* of Bélidor, and in some special treatises; I limit myself to the consideration of their most important features.

Working-barrel. 424. The working-barrel of the pump is a cylinder, which was formerly made by boring and hollowing a piece of wood, but now is most generally made of cast iron or brass, the interior surface of which should be perfectly polished and bored true. Its diameter determines the force of the pump; if it is below 0.39 ft., this is small; and great, if it is above 1.082 ft.; it seldom exceeds 1.31 ft., and very rarely 1.64 ft. The length of the barrel is but little over that of the *stroke* or *lift* of the piston.

Piston. 425. The piston is the most delicate part of a pump, requiring the most care, and on it depends chiefly the good effect of the machine. Its form is various; I shall consider only those forms which seem to be justly preferred in common practice.

The most simple piston is made of elm, sometimes Fig. 73. boiled in oil; its form is indicated in Fig. 73; its lateral surface is convex; its upper part A, somewhat resembling a basket handle, is traversed by a rod which serves to raise and lower it; its body is pierced with a cylindrical opening, with a diameter nearly half that of the working-barrel of the pump. Most generally, pistons have the form represented in Fig. 74; this, also, is Fig. 74. a piece of perforated elm, traversed by two bolts, which form a part of the iron stirrup to which the rod is fastened. In some kinds of pumps (Fig. 76), the piston is solid.

The exterior surface of all pistons has a packing, designed to stop all communication between the water or air which is above it, and the water or air below it. It is necessary that it should lie quite close to the interior surface of the pump, so that the interruption may be complete; but without being too close, as it would then occasion friction, which would consume, without useful effect, a portion of the motive action. In the most common pumps, it consists of a band of thick leather, which surrounds, and reaches a little beyond, the upper part of the body of the piston (Fig. 73), being wider at the top; the upper edge of this kind of collar, being urged by the weight of the water or the atmosphere, presses against the sides of the working-barrel of the pumps, and interrupts the communication above and below it. In other pistons (Fig. 74), the leather collar is supported by a copper ring; at the bottom is another ring of the same material, and the space between them is packed with hempen wicks dipped in melted tallow; they surround the middle, projecting somewhat, and so rub against the body of the pump. The common packing for cast iron or brass pistons, especially when they have a high column of water to lift or force, consists of two circular plates of strong leather, turned up at their upper edge, a height of from .078 in. to 0.118 in., and presenting thus the form of a cup; one is fixed upon the upper base of the piston, with the fold upwards, and the other upon the lower base, with the fold downwards. In pistons pierced in the middle, wide leather rings, also turned up at their outer edge, are used. These leathers, thus bent by a peculiar process, bear the name of *crimped leathers*. Fig. 73.

In well managed establishments, where the pumps force up the water, instead of the above-mentioned pistons, they used for many years long brass cylinders, turned and well polished, solid or hollow, called by the

English *plungers;* their length somewhat exceeds the stroke, and their diameter is from 0.039 in. to 0.078 in. less than that of the working-barrel. They have no packing, but they pass through the middle of some, enclosed in a *stuffing-box* placed at the top of the *barrel,* and disposed as follows: Upon its bottom, which answers to the flange or collar of the working-barrel, is placed a crimped leather ring, bent downwards at its interior edge; when the pump forces, the water presses the bent portion against the cylinder; upon this leather ring is placed another of brass, the upper surface of which, instead of being horizontal, like its lower, is inclined towards the interior of the *stuffing-box;* above that, the cylinder being put into its place, is wound around it, one over the other, several hempen hards, soaked in melted tallow, to which is added a little oil; this then is covered by a second brass ring, with its lower face inclined towards the exterior; finally, the cover of the box is put on; it is traversed by screw bolts, which pass through the bottom or pump-collar; when they are tightened, the cover and the upper ring are lowered; they press the hempen hards, and urge them against the cylindric piston. The cover of the box is often made to take the place of the second bronze ring, which in this case is useless.

Valves. 426. The valves generally used are of two kinds.

The one, a truncated cone of small height, is simply a circular brass plate, its upper surface being a little larger than its lower. It enters and is completely embedded in the opening which it is designed to close. Below is a stem, which passes through a guide, and is terminated by a *stop-button;* the stem holds the valve in its position. These valves are called *stem-valves* (see Fig. 76 at *b*).

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The others, *clack-valves*, consist usually of a circular plate of thick oiled leather, supported, upon the plate or tube whose opening it is meant to close, by a small leather band, which serves as a hinge. In the most common pumps, a lead plate is simply nailed upon the circular leather valve, which keeps its form plane, and loads it with a sufficient weight. But usually, the leather is held between two iron or copper plates; the upper being a little larger than the opening to be closed, and the lower a little smaller, as we see at M (Fig. 75). Frequently, in large pumps, the clack-valves are simple brass plates, about 0.39 in. thick, which move around a common hinge; when the openings are large, they may be divided into two or three compartments, each of which is covered by its proper plate; Fig. 80 presents these double and triple clack-valves. Fig. 75.

In the construction of either, it is requisite, while preserving the necessary solidity, (a condition of the first importance,) that their upper surface, which is subjected to the pressure of the liquid from above, should exceed as little as possible in size the portion of the lower surface susceptible of being pressed from below upwards, a portion which is the same as the orifice covered.

These valves being subject to frequent repairs or re-packing, it becomes essential that these operations should be executed promptly; for example, in machines designed to drain the waters of a mine. To do this with ease, we swell or enlarge, for a height of about a foot, the parts of the pump immediately above the valves (see Fig. 79); these enlargements, or chambers, are closed by a door or cast iron plate, which is opened when we wish to repair or change a valve.

Pipes. 427. The suction pipe can never have a height above 26.25 ft., as we shall soon see. Its diameter is almost always smaller than that of the working-barrel; it is two thirds or a half of it; but it is not well to reduce it more, unless constrained by some special considerations.

The same condition applies to the diameter of the lift pipe. As for its length, it has no other limit but that of the disposable motive force; it was 728 ft. for the pumps of the mine at Huelgoat (412).

Kinds of pumps. 428. After these general observations upon pumps, we proceed to the characteristics which distinguish them from each other.

Their pistons raise the water, either by exhausting the air found at first beneath it, or by forcing the water in the lift pipe, or by these two modes conjoined; whence the ancient division of pumps into ***suction pumps***, ***force pumps***, and ***suction and force pumps***.

ARTICLE FIRST.

Suction Pumps.

Parts. Fig. 75. 429. The essential parts of a suction pump are, 1st, a working-barrel A; 2d, a suction pipe B, with its extremity plunged in the well containing the water to be raised; 3d, a piston C, pierced in the middle; 4th, a valve *a* covering the opening of the piston; 5th, a second valve *b*, placed at the top of the suction pipe, and called the *fixed valve.*

Usually, the lower end of this pipe is widened, and a strainer is affixed to it, to exclude bodies which the water might carry up; sometimes we enlarge this extremity, and pierce it with small holes (Fig. 76).

Without stopping to describe the action of the suction pump, which is universally known, and the circumstances of which, relating principally to our object, will be manifested from what follows, I pass to the considerations whence we deduce the rules for the proper establishment of this machine.

Height to which the water can be raised.

430. Let us take a pipe 40 ft. long, for example; and let it be placed vertically, so that its lower end, to which we have fitted a piston, may be plunged in a well. If we raise the piston, the water follows it, and it will ascend in the pipe to a height, such that the weight of its column shall be equal to that of a column of the atmosphere, resting upon the well (having the same base). There it will stop; and if the piston continues to ascend, it will cause a perfect vacuum between it and the surface of the raised water. Designate by b the height of a barometer put in this place; supposing that the mercury of this instrument is reduced to zero of thermometric temperature, 13.6 will be its specific gravity, and $13.6b$ will express the length of the column in the pipe; this will be the greatest height to which the water can be raised by suction. At the level of the sea, where the barometer stands, as a mean, at 30 in., this height will be, as a mean term, 33.99 ft.; it varies, in our latitudes, between 32.809 ft. and 35.10 ft.

431. Now, place above this same pipe the working-barrel of the pump, furnished with its fixed valve, and containing a common piston. Let us determine the height to which it can raise the water, by its alternate play, and by suction.

Designate by k the height ($13.6b$) of the column of water representing the atmospheric pressure, by E the space comprised between the fixed valve and the piston at the top of its stroke, and by e the space between this same valve and the foot of its stroke. We admit that, after some strokes of the piston, the water has reached in the pipe a height ψ, and that φ is the elastic force of the air comprised between the surface of this water and the valve; that is to say, that φ is the vertical height of a column of water whose weight measures this force: we shall have $\varphi = k - \psi$, since this force, plus the weight of the column ψ, is in equilibrium with the atmospheric pressure. The piston being supposed at the bottom of its stroke, the mass of air which is found between it and the same valve, in the space e, will have an elastic force equal to that of the atmosphere, and consequently equal to k. When we raise again the piston to the top of its stroke, this mass will dilate, and finish by filling the space E: its density will be diminished in the ratio of e to E, and the elastic force, which follows the same law, will only be $k\frac{e}{\mathrm{E}}$: if the force φ of the air which is below the valve is found to be greater, it will open it (deduction being made of the weight of the clack or stem valve); a portion of this air passes above it; φ will diminish and become φ', and the water will be raised a new quantity in the suction pipe. When the piston descends, this same portion of air, or a part equal to it, will escape by raising the valve of the piston; and there will only remain between it and the fixed valve, or in the space e, an aeriform mass similar to the first, having always a force k. When the piston reäscends, if we have $\varphi' > k\frac{e}{\mathrm{E}}$, the water will still rise in the pipe. Finally, when, after a number of strokes of the piston, starting from that where the height of the column raised was ψ, the equality between the two forces, above and below the fixed valve, is established, so that we have $\varphi^n = k\frac{e}{\mathrm{E}}$, this valve will no longer open, and the water will rise no more, although the piston continues its play. Then the relation $\varphi^n = k - \psi^n$ will become $k\frac{e}{\mathrm{E}} = k - \psi^n$: whence we deduce $\psi^n = k\left(1 - \frac{e}{\mathrm{E}}\right)$, an expression in which ψ^n indicates the greatest height which the water can attain in a long suction pipe.

It would consequently be superfluous to give this pipe a greater height: it would be necessary to make it sensibly shorter, both because it is requisite that the water, passing beyond it, should arrive in the working-barrel of the pump, and because of the weight of the valves; we make it about 0.65 ft. less.

When e is zero, we shall have $\psi^n = k = 13.6b$; the water will thus rise the whole height to which suction can carry it. But in every other case, ψ^n will be less; it will be but $\frac{2}{3}$ of $13.6b$, if $e = \frac{1}{3}E$. We see from this how prejudicial to the effect of suction is the space between the bottom of the stroke and the fixed valve; thus the Germans name it the prejudicial space (*schädlicher Raum*). It is necessary to make it as small as possible, and to dispose the machine in such a way, that the piston, in its descent, may arrive very near to the fixed valve; it is well, however, to leave a small interval, so that in the play, which the pieces of mechanism moving the piston always make, it may not strike upon this valve.

432. When, by the effect of the less height given to the suction pipe, at the n^{th} stroke of the piston, a certain volume of water shall have entered the working-barrel of the pump, another order of things will be presented. This volume, remaining there during the descent of the piston, will by so much diminish the space e: at the following stroke, the air will be still more rarified, φ^n will diminish in value, and the water will ascend further in the body of the pump; and at the end of a few strokes, it will fill entirely the prejudicial space. When, therefore, the piston which is now in contact with it shall ascend, it will tend to make a perfect vacuum beneath it, and the water will follow it, provided it is not raised above $13.6b$; it will no longer leave it, and the working of the pump will be definitely established. We may, however, demonstrate, in a manner analogous to that used in the preceding number, that there may be two points of stoppage, if the length of the stroke is less than $\frac{H^2}{4k}$, H being the height of the most elevated point of the stroke above the reservoir: but we need not fear these stoppages, when the height of the prejudicial space is small compared to the stroke.

433. Recapitulating, and observing that the height of the barometer, or the atmospheric pressure, varies from day to day, in the same place; and that consequently, for a pump to perform its functions at all times, we should admit the lowest of these pressures; we say, that in the establishment of suction pumps, it is

necessary, 1st, that the piston, when it is at the top of its stroke, should not be more than $12b'$ above the well, b' being the mean height of the barometer in the place where the pump is; it will be from 29.5 ft. to 26.25 ft., according as the elevation of the place above the sea is 300 ft. or 3000 ft.; 2d, that the space between the bottom of the stroke and the fixed valve should be only a few hundredths of a foot; say 0.16 ft., when the length of the stroke exceeds 1.64 ft.

In default of direct observations giving the value of b', if we should wish to know approximately the elevation, above the sea, of the place where the barometer is, we shall have its value by the equation

$$\log. b' = -\left\{ .6021070 + \frac{\varepsilon}{60286\,(.928. + 184 \cos\lambda - .000003807\varepsilon)} \right\}$$

ε being this elevation and λ the latitude: for France, and elevations below 1640 ft., we shall have simply, and with all the exactness necessary in such cases, $b' = 2.5005^{\text{ft.}} - 0.000089\varepsilon$.

If, instead of supposing, as we have done, that the mercury is at 0° centigrade (=32° Fahrenheit), we take the mean temperature at 12° (=54° Fahrenheit), the mean height of the barometer will be $2.5052^{\text{ft.}} - 0.00009\varepsilon$. I remark, in passing, that this height gives the point marked *variable*, in the barometer regarded as prognosticating a change of weather.

Lift pumps. 434. The upper limit which we have assigned to the stroke of the piston, concerns suction pumps properly so called, where the water is discharged through a delivery pipe, fixed upon the working-barrel of the pump at the level of the highest point of the stroke. But, usually, this point is not established more than 16 ft., 20 ft., or 23 ft. above the well, according to local circumstances; and in order to lose none of the height at which the water is to be discharged, the working-barrel of the pump is prolonged by an upright pipe, at the extremity of which is placed the discharge pipe. The piston, in its ascent, supports and raises the fluid column contained in the pipe. When the height of these machines exceeds 33 ft. by a few feet only, they are the high suction pumps (the *hohe Sätze* of the Germans). But if they exceed 66 ft., they are called lift pumps; and their height has no other limit but that of the power which puts it in action.

These pumps are now frequently used in the drainage of the water which collects at the bottom of mining shafts; a single one performs the work which was hitherto accomplished by ten and fifteen suction pumps, placed in succession, one above the others. The two pumps of the mine of Huelgoat (412), though far from having the half of their destined load, perform the work which, a few years since, would have required fifty-nine common pumps.

I describe succinctly one of these pumps, the strongest we have in France, the principal parts of which are seen in Fig. 78. Fig. 78. The suction pipes and lift pipes have the same axis and diameter, 0.9022 ft.; at their junction is an enlargement, which acts the part of chamber, and encloses the two valves; one at the foot of the lift pipe, and the other at the top of the suction pipe: the latter is 23 ft. in height, and the other is 728 ft. The working-barrel of the pump is at the side, and communicates at its upper end with the chamber: it is of brass, and perfectly bored; it is open at the bottom, which admits of greasing the inner surface easily. The piston, also of brass, is 1.384 ft. in diameter, and has a stroke of 7.55 ft.: its packing consists simply of two bent leathers, one bent upwards and the other downwards, completely retaining the water, notwithstanding the enormous load of twenty-three atmospheres: its rod, cast in the same mould with it, traverses the cover of the working-barrel, through a leather stuffing box, and joins the rod which descends from the water-pressure engine placed 690 ft. above. When the piston descends, it creates a vacuum above it, and the water from the well, passing through the suction pipe, rises to fill it; when the piston ascends, it raises and bears upon it a column of 75 ft. of water: so that, in the first half of its oscillation, the pump is a sucking pump, and in the second, a lifting.

There are lifting pumps in which the two pipes and the working-barrel have all the same axis, and where, consequently, the long rod of the piston is enclosed in the lifting pipe. These pumps, occupying but little space in width, are best adapted for narrow wells: they serve exclusively for extracting the water from wells bored with the augur; it is by means of such wells and such pumps, having only a diameter of 0.262 ft., that the salt springs of certain countries are worked. The diameter of the working-barrel of the pump is a little smaller than that of the ascension pipe, so that when the piston has been raised, for the frequent repairs which it needs, it can be easily introduced from the top.

Load of water upon the piston.

435. *Whatever may be the height at which the pump discharges its water, whatever may be the diameter and inclination of the suction and ascension pipes, the piston always bears a load of water equal to the weight of a column of this fluid, having for its base that of the piston itself, and for its height, the difference of level between the surface of the well and the point of delivery.* Let H be this difference of level, and D the diameter of the piston; let us observe the piston at any point of its motion, and designate by h the vertical distance between this point and that of the delivery, and by h' the elevation of this same point above the well; we have always $h+h'=\mathrm{H}$. The piston will be pressed from above downwards by the weight of the atmosphere, and by that of the column of water which is above it; it is $62.45\pi'\mathrm{D}^2\,(k+h)$; it will also be pressed from below upwards, by the weight of the atmospheric column minus the weight of the column of water which is below its base, that is to say, by $62.45\pi'\mathrm{D}^2(k-h')$. These two pressures being opposite, their resultant, or the effective load of the

piston, will be $62.45\pi' D^2(k+h) - 62.45\pi' D^2(k-h') = 62.45\pi' D^2(h+h') = 62.45\pi' D^2 H$, agreeably to the enunciation of the theorem.

The diameters of the suction and ascension pipes do not enter into this expression, and the load is independent of them, by reason of this hydrostatic principle: when a vessel encloses a liquid, the pressure which takes place upon the bottom depends only upon the magnitude of the bottom and the vertical height of the liquid above it, whether the vessel, at its upper part, is reduced to a long and narrow tube, or whether it presents a great widening.

Passive resistances.

436. Independent of the load just considered, and which corresponds to the useful effect of the machine, the force applied to raise the piston will also have to overcome the passive resistances arising,

1st. From the friction of the piston against the sides of the working-barrel;

2d. From the friction of the water against these same sides, and against those of the pipes;

3d. From the contraction of the fluid vein at its entrance into the suction pipe, and at its passage through the opening of the fixed valve;

4th. From the weight of this valve;

5th. Finally, from the inertia of the mass of water to be moved.

A rigorous determination of these resistances is impossible, and the values which we may assign them should only be regarded as simple approximations, in which we have especially avoided any error in defect.

Friction of the piston.

437. This friction depends:

1st. On the number of points of the periphery of the piston, in contact with the sides of the working-barrel; a number which is proportional to the diameter or D. (We disregard the height of the periphery.)

2d. On the pressure of each of these points against the sides. When the packing of the piston consists of a simple bent or crimped leather, its upper edge being pressed against the working-barrel of the pump by the column of water raised, the pressure is proportional to H. In other cases, and generally, the packing should clasp the more lightly, according as the water makes a greater effort to pass between it and the body upon which it presses; and this effort is also proportional to H.

3d. Upon the smoothness of the friction surfaces. Consequently, the friction of the piston will be expressed by μDH, μ being a number to be determined by experiment, depending principally upon the polish of the surfaces of the working-barrel. Langsdorff, though I do not know on what grounds, admits for an approximate value of μ for the working-barrel, when made of

	per sq. ft. lbs.
Well polished brass, ...	1.434
Cast iron, merely bored, ...	3.0733
Quite smooth wood, ...	5.1221
Wood worn by use, ...	10.244

Friction of the water.

438. Water moving in the pipes of pumps meets there a resistance of the same nature as in conduit pipes; with this difference, however; that in pumps, all the particles move with a very nearly equal velocity, which is that of the piston; while in conduits, the velocity of the particles adjoining the sides, and on which the friction depends, is less than the mean velocity, or that which is introduced into the formula. So that, if we would use the same formula for pumps (186), it will be necessary to admit, for them, a velocity greater than that of the piston, in the ratio of the velocity of conduits near the sides to their mean velocity. Dubuat, after having made this remark (*Principes d'hydraulique*, § 305), proposes to take for this ratio that which he found between the velocity of the bottom, and the mean velocity, of water running in a canal: and according to what has been said in Sec. 109, this mean velocity will be $v' + .29886\sqrt{v' + .04462}$; or simply, $v' + .30792\sqrt{v'}$, v' being the velocity of the bottom. Consequently, for the quantity v of the formulæ of the motion of water in conduits, we shall substitute $v + .30792\sqrt{v}$, where v represents the velocity of the piston.

According to this, if D is the diameter of the working-barrel, and L its length, we shall have for the expression of the friction which the water experiences, that is to say, for the height of the column of water, whose weight expresses the resistance due to this friction,

$$0.0004175\frac{\mathrm{L}}{\mathrm{D}}\left[(v+.30792\sqrt{v})^2+.18044\,(v+.30792\sqrt{v})\right]$$

or, more simply, but less exactly,

$$0.0004358\,(v+.30792\sqrt{v})^2\frac{\mathrm{L}}{\mathrm{D}}.$$

So also, if D′ represent the diameter of the suction pipe, and L′ its length, observing that the velocity of the water there is greater than in the working-barrel of the pump, in the ratio of D^2 to D'^2, we shall have for this pipe

$$.000436\,(v+.3079\sqrt{v})^2\left(\frac{\mathrm{D}}{\mathrm{D}'}\right)^4\frac{\mathrm{L}'}{\mathrm{D}'}.$$

In a lift pump, where D″ is the diameter of the ascension pipe, and L″ its length, we shall again have

$$.000436\,(v+.3079\sqrt{v})^2\left(\frac{\mathrm{D}}{\mathrm{D}''}\right)^4\frac{\mathrm{L}''}{\mathrm{D}''}.$$

The piston must overcome these resistances; upon its base press the columns of water whose height we have given; thus the absolute value of the resistances proceeding from the friction of water against the sides of the pump will be

$$62.45\pi\mathrm{D}^2.000436\,(v+.307\sqrt{v})^2\left[\frac{\mathrm{L}}{\mathrm{D}}+\frac{\mathrm{L}'}{\mathrm{D}'}\left(\frac{\mathrm{D}}{\mathrm{D}'}\right)^4+\frac{\mathrm{L}''}{\mathrm{D}''}\left(\frac{\mathrm{D}}{\mathrm{D}''}\right)^4\right]$$

Resistance due to contractions.

439. For greater simplicity, we will determine the resistance at each of the contractions which the fluid column experiences in the pumps, according to the principle, that such a resistance is represented by the height due to the velocity of the water in its passage through the contraction, minus the height due the velocity which the fluid had immediately before.

For the contraction on entering the suction pipe, calling m the coefficient of contraction, which will vary from 0.82 to 0.95 (50), according to the form of the widening, and observing that the ascensional velocity of the water in the well is zero, we shall have $\frac{v^2}{2gm^2}\left(\frac{\mathrm{D}}{\mathrm{D}'}\right)^4$.

For that which occurs at the opening of the fixed valve, if we designate by s the section or area of the opening, by m' the coefficient of contraction relating to it, and by γv the velocity of the water immediately before, and remembering that $\pi' D^2$ is the section of the piston, there results $\frac{v^2}{2g}\left(\frac{\pi' D^2}{m's}\right)^2 - \frac{\gamma^2 v^2}{2g}$.

Thus the absolute resistance proceeding from the two contractions will be

$$62.45\pi' D^2 \frac{v^2}{2g}\left[\frac{1}{m^2}\left(\frac{D}{D'}\right)^4 + \left(\frac{\pi' D^2}{m's}\right)^2 - \gamma^2\right].$$

Resistance due to weight of valve.

440. At the first instant of the raising of the piston, when the water operates in opening the fixed valve, it experiences a resistance arising from the weight of the plate to be raised. To overcome it, it must exert upon the lower part of this plate an effort whose action must be at least equal to this weight. Let us determine the height of a column of water which represents it, and for greater generality, let us take the case of a clack-valve.

Let P be its weight, λ the distance of its centre of gravity from the axis of rotation, σ the area of the opening, λ' the distance of its centre from the same axis, and x the height sought: $P\lambda$ will be the moment of the resistance due to the weight of the clapper, and $62.45\sigma x\lambda'$ will be that of the force opposed to it; and since the two actions should be equal, we shall have $P\lambda = 62.45\sigma x\lambda'$. Deducing from this equation the value of x, and multiplying by $62.45\pi' D^2$ for the effort to be exerted by the piston, it will be $\frac{P\pi' D^2\lambda}{\sigma\lambda'}$.

If the clapper, instead of being horizontal when it is closed, should make an angle ω with the horizon, we should multiply the above expression by cos. ω.

For a stem-valve (*à coquille*) covering a circular orifice, whose diameter is d, we shall have simply $P\frac{D^2}{d^2}$.

When the valve is opened by the effort, whose expression we have just given, there is required still another to hold the clapper up during the whole ascent of the piston. In default of positive ideas upon the extent of this last effort, and though it should be inferior to the first, we will admit that the first, though it acts but for an instant, is exerted during the whole ascent.

Resistance due to inertia.

441. The effort or statical force employed to overcome the inertia of the water depends upon the nature of the motion which the piston is constrained to take.

If it were entirely free, and this force, independently of those which equilibrate the other resistances, acted constantly upon it, there would result a uniformly accelerated motion. Calling l the length of the stroke, and t the time of the piston in passing through it, $\frac{2l}{t^2}$ will represent the accelerating force, and $\frac{\Pi}{g} \cdot \frac{2l}{t^2}$ will be the motive force sought, Π being the weight of the water to be moved: after having reduced all its parts to the velocity of the piston, and according to the notations already employed, $\Pi = 62.45\pi D^2 \left(L + L' \frac{D^2}{D'^2}\right)$.

But, nearly always, the piston is connected with a machine, which, in moving it, regulates the circumstances of its motion. For example, if it is connected, directly or indirectly, with the crank of a wheel endowed with a uniform motion, it will start from its rest with the water which follows in its train; it will rise at first with an accelerated motion: the acceleration will diminish by degrees, and it will be nothing at the middle of its stroke: then its velocity will be retarded, more and more; and finishes by being nothing at its highest point. During the first half of the stroke, the motion will have required an accelerating force, diminishing progressively; and during the second half, a retarding force, increasing by the same progression, and which will have destroyed the effect of the first. Thus, whatever was required to be taken, above the entire force employed to move the machine, to surmount the inertia of the mass during the first part of its stroke, will be rendered back, by the same inertia of this mass, to the same force, during the second part, and, in short, the inertia will not have occasioned any expenditure or loss of force.

If the piston is required to move with a given uniform velocity v: as it starts from repose, there will be a certain time, however small, required to attain this velocity. Let τ be this time; the force necessary to impress it with v, or to overcome the inertia, will be $\frac{\Pi}{g} \cdot \frac{v}{\tau}$.

Calculation of the resistances of a pump.

442. Let us make an application of all these formulæ to an experiment which I had occasion to make.

The pump had the following dimensions:

Diameter of the working-barrel of pump, $D = 1.0656$ ft.
Length of the working-barrel, ... $L = 5.9056$

Diameter of suction pipe, ... $D' = 0.44407$ ft.
Length of this pipe, ... $L' = 25.105$
$L + L' =$ . . . . $H = 31.011$
Length of the stroke, ... $l = 4.7671$
Mean velocity of piston ($4\frac{1}{2}$ strokes in 1′), $v = 0.71523$
Weight of the clapper nearly . . . . $P = 2.2054$ lbs.
Coefficient of contraction at entrance of suction pipe, ... $m = 0.85$
Coefficient at the fixed valve, ... $m' = 0.62$
Effective section of opening of valve, . . $s = \frac{3}{4}\pi' D'^2$
(For approximation, we have taken $\frac{3}{4}$ of section of suction pipe.)
The water arriving at the valve with the velocity which it had in this pipe, and which was $v\left(\frac{D}{D'}\right)^2$, we have . . . . $\gamma = \left(\frac{D}{D'}\right)^2$

We pass to the calculation of the different resistances, and remark, that the quantity $62.45\pi' D^2$, which is found in nearly all of them, is equal to 55.696 lbs.

1st. Weight of the column of water to be raised (435)
55.696×31.011 . . . . 1727.20 lbs.

2d. Friction of the piston (437)
$3.0733 \times 31.011 \times 1.0656$ . . . . 101.56 lbs.

3d. Friction of the water (438) $55.696 \times .00043738\,(0.71523 + 0.30792\sqrt{.71523})^2 \times \left[\frac{5.9056}{1.0656} + \frac{25.105}{0.44407}\left(\frac{1.0656}{0.44407}\right)^4\right]$ . . . . . 43.60 lbs.

4th. Contractions of the fluid column (439).
Observing that $\left(\frac{\pi' D^2}{m's}\right)^2 = 4.6248\left(\frac{D}{D'}\right)^4$, and that $\left(\frac{D}{D'}\right)^4$ is found in all the terms of the complex factor, we have $55.696\,(.71523)^2 \times .0155366\left(\frac{1.0656}{0.44407}\right)^4\left(\frac{1}{(0.85)^2} + 4.6248 - 1\right) =$ 74.39 lbs.

5th. Resistance due to weight of valve (440)
$2.2054 \times \left(\frac{1.0656}{0.44407}\right)^2 =$ 12.70

6th. For inertia, the pump being moved by a hydraulic wheel (441), ... 0.00

Total of resistances, active and passive, . . . 1959.45

Amount brought up, ...	1959.45
Deducting weight of water displaced by piston, ...	30.88
There remains	1928.57 lbs.

Experiment has given 1896.69 lbs.

These two results may be regarded as identical.

In this example, the suction pipe was narrower than usual, and occasioned a resistance, from the friction of the water, much greater than we commonly have.

The experiment, the result of which has been just reported, is one which M. Duchène and myself made upon one of the draining machines at the mines of Poullaouen, of which mention has already been made (364). In making them, we also observed the effects of inertia. A dynamometer, bearing a weight of 5403 lbs., was suspended from one end of the working beam which raised the pistons. At the first moment of the raising, the effort necessary to overcome the inertia occasioned a jerk which bore the index of the instrument to a point far above the division 5403, but which could not be observed, the movement being made in the twinkling of an eye; the index immediately returned to 5403, where it remained, trembling the while, during the five or six seconds of the time of ascent. If the velocity was increased, the elevation of the needle, at the first instant, was still greater; but it soon fell below 5403: having once increased the velocity in the ratio of three to four, and consequently the action of inertia in that of nine to sixteen, the needle, after its fall, marked only 5293: it might have been said, that the impulse of the force employed at the first moment, to overcome the inertia of the body raised, an impulse whose direction was opposed to that of gravity, had diminished the weight of this body.

Effort to raise the piston.

443. The effort to raise the piston should be equal to the weight of the column of water, plus the passive resistances.

These resistances are of two kinds; one, such as the friction of the piston, is independent of the velocity; the others are dependent upon it. These last will always be very small compared to the total resistance to motion; when we caused the velocity of the piston to vary in the ratio of four to five, the load remaining the same, we did not observe a sensible difference in the resistance indicated by the dynamometer. Accordingly, and excepting extraordinary cases, the passive resistances may be estimated at a certain part of the weight of the column of water raised. The determination of this portion was one of the objects of our experiments at Poullaouen; they are reported in the *Journal des mines* (vol. XXI., pp. 169—178); I confine myself to giving the results of them.

The first column of the following table indicates the nature of the load; thus, for the fifth experiment, it was a long vertical connecting rod, (to which were attached the pistons of the pump, placed one under the other,) plus six pistons, plus the sum of the resistances of the first pump, plus that of the second, plus that of the third. The second column presents the weight of this load, as indicated by the dynamometer. The third and fourth contain the principal dimensions of the pump of the number marked against it in the first column. The fifth shows the sum of the resistances of this same pump: it is the difference between two consecutive numbers of the second column. In the sixth, we have noted the weight of the column of water borne by this same pump: it is 49.046D^{2}H. Finally, the last indicates the ratio between the two numbers of the two preceding columns, taken upon the same horizontal line.

LOAD OF THE MACHINE.		PUMP.		RESIST. total by pump.	WEIGHT of water by pump.	RATIO of resistance to weight.
NATURE.	WEIGHT.	DIAM.	HEIGHT.			
	lbs.	ft.	ft.	lbs.	lbs.	
Rod	5403	.....	.....	.....	.....	.....
Rod + 6 pistons	5932	.....	.....	.....	.....	.....
Do. + 1st pump	7763	1.066	31.972	1830	1779	1.03
Do. + 2d pump	9659	1.066	31.010	1896	1725	1.10
Do. + 3d pump	11600	1.073	31.972	1940	1806	1.07
Do. + 4th pump	13497	1.058	31.598	1896	1735	1.09
Do. + 5th pump	15394	1.066	32.034	1896	1784	1.06
Do. + 6th pump	19672	1.073	34.977	2139	1976	1.08

The machines upon which these experiments were made had cast iron working-barrels, but their polish had been much impaired, the packings of the piston had been freshly placed, the suction pipes were narrow, and without widenings; so that the resistances were much greater in them, than those commonly experienced; consequently, and without any inconvenience in practice, the mean term 1.08 of the last column may be generally admitted.

The effort to raise the piston will then be $52.97D^2H^{\text{lbs.}}$ $(=62.45 \times .7854 \times 1.08)$; avery simple expression, which will dispense, in most cases, with long calculations, relative to each kind of resistance, and which will give results sufficiently accurate. We may raise it to $53.08D^2H$; to this, we then add the weight of the piston and its rod. The *dynamic* load of a pump would thus be one twelfth greater than the *static* load.

Effort to lower the piston.

444. When the piston descends, we must exert upon it an effort to surmount the resistances arising, 1st, from the contraction which the fluid mass experiences in passing through the piston; 2d, from the friction of its packing against the working-barrel of the pump. Both will be calculated in the mode already given (437 and 439). Concerning the last, I remark, that it will be nothing in the case where it depends only upon the pressure of the fluid column; as when the packing consists simply of a flexible leather. The effort exerted upon the piston in its descent, favored otherwise by the weight of this piece and its rod, will always be small compared to that required in raising it.

Coupled pumps.

445. Thus, during half the time of the working of the pump, the force which moves it remains nearly unemployed. The better to utilize it, we usually couple two pumps, by means of a balance-beam or

other contrivance, so that one piston may ascend while the other descends. The force acts then continually with the same intensity, and should be equal to that required to raise and lower one only of the two pistons.

Most frequently, we place two working-barrels of a pump upon the same suction pipe.

The two working-barrels, or the two pumps, deliver their water in the same trough, which thus furnishes a nearly continuous jet.

We obtain this continuity of jet with but one working-barrel, by means of a reservoir of air, similar to that which we shall mention when on the subject of fire-engine pumps (455).

Quantity of water raised by a pump.

446. When a pump is in perfect order, that is to say, when the valves fit very exactly, and the packing of the piston does not suffer any part of the fluid to repass which has already passed above it, it raises, at each stroke of the piston, a volume of water equal to the volume of space generated by the base of the piston during its upward stroke, that is to say, equal to $\pi' D^2 l$, or differing only by the minute quantity which the suction valve, in closing, forces beneath it.

While the piston ascends, it is true, the volume of water discharged is diminished by the volume of the space occupied by the rod; but in its descent, when the water which was under the piston passes above it, the rod displaces the same volume, and causes its discharge; so that, by the entire oscillation of the piston, the quantity of water delivered is always $\pi' D^2 l$.

But, in reality, we do not obtain such a product: the valves and the packing allow a portion of the water already passed to escape; and all that has been sucked up does not arrive at the delivery pipe. When the pumps are well made and kept in repair, the loss is inconsiderable: thus, in the beautiful pumps of Huelgoat (434), M. Juncker found it but 3⅓ in 100. M. Castel, at my request, has made some careful gaugings of the water delivered by the pumps of the water-works at Toulouse (454): I give below the results obtained. There were two sets, each with four pumps, (plunger pumps,) whose pistons were .889 ft. in diameter, and stroke 3.77 ft.: it was known that in the set No. I, one of the fixed valves, being broken, did not close exactly: as to the set No. II, it seemed to be without fault. These experiments show, that even in very good machines, the loss increases when the velocity of the piston is diminished. In common pumps, it is more considerable, and generally reaches from one to two tenths, according to the condition of the pump; so that the volume of water discharged, in place of being $0.785D^2l$, would be given by an expression varying from $0.7D^2l$ to $0.6D^2l$. It is more especially in such pumps, that the loss of water is so much the greater, as the piston is more slowly raised.

No.	STROKE in 1′.	PRODUCT IN 1′.		LOSS in 100.
		Theoret.	Real.	
	ft.	cub. ft.	cub. ft.	
I	16.66	47.35	45.19	4.55
	12.50	35.52	32.98	7.16
II	19.06	54.06	53.36	1.50
	11.41	32.45	31.50	2.94

Velocity to give the piston.

447. It should not, however, be moved with such a velocity, that the working-barrel of the pump, in which the water mounts by virtue of the atmospheric pressure k, overcoming at the same time different resistances, may not have time to be filled before the piston commences its descent. Deducting the slight resistance experienced by the water in the suction pipe, if we suppose that the piston, raised suddenly, has left a perfect

vacuum behind it, and that the water has already arrived at the entrance of the working-barrel of the pump, at the fixed valve, the time of filling will be determined by the rules given in Secs. 97 and 98. Designating it by t, and by L' the elevation of the valve above the well, we shall have

$$t = \frac{2\pi D^2}{ms\sqrt{2g}}\left(\sqrt{k-L'} - \sqrt{k-L'-l}\right).$$

Representing by u the mean velocity with which the water rises in the working-barrel of the pump while filling it, we shall have $u = \frac{l}{t}$. The resistance of the suction pipe will diminish a little this value of u; we shall obtain this diminution by reducing a little the value of m.

If the piston has a velocity v greater than u, the water cannot follow it; it will quit it, and will be rejoined by it before having arrived at the top of the working-barrel of the pump, which will not be entirely filled at each lift. It is necessary, then, that v should be less than $\frac{l}{t}$; prudence dictates, that we should not allow it to be over two thirds of it.

In the above example (442), where we have $D = 1.0656$ ft., $l = 4.7671$, $L' = 25.105$ ft., $s = .15489$ sq. ft., and $m = 0.667$, making $k = 32.809$ ft., we find $t = 2.2852''$, and $u = 2.086$ ft., a velocity more than double that of .71523 ft., which is that of the piston. Even when we make $m = 0.50$, we shall then obtain $u = 1.5673$ ft. Thus, we should have no fear that the water might not follow the piston.

The expressions u and t indicate that the velocity with which the water ascends in the working-barrel of the pump, and consequently, that which we give to the pistons, is so much the greater as the suction pipe is shorter, and as its diameter, as well as that of the opening of the valve, is more considerable.

In great pumps, working with a continuous motion, and the stroke of whose pistons may be 3.937 ft., for example, we have usually from four to six strokes per minute, which corresponds to a velocity of from .5249 ft. to .787 ft. This limit is never exceeded, even in fire-engine pumps: notwithstanding the quick movements of the pumpers, they do not make over sixty strokes of 0.3936 ft.; which gives only a velocity of 0.7874 ft. There are few cases where it goes as high as 0.984 ft.; though in the pumps at Huelgoat (434), it has reached as high as 1.377 ft.

I will observe, that with equal velocity, it is advantageous to increase the length of the stroke, in diminishing the number of those which are made in the same time; we have to surmount less frequently the inertia of the masses to be again set in motion; the quantity of water which, at each shutting of the fixed valve, returns below it, is less; and the changes of direction, which produce shakings in the joints of the mechanism, and end in wearing them out, are less frequent.

ARTICLE SECOND.

Force Pumps.

Their character. Fig. 76.

448. In these machines, though not in frequent use at present, the working-barrel of the pump is plunged into the well; it is joined to an ascension pipe, at the lower extremity of which is the *stop-valve*.

If the water which is in the working-barrel of the pump is removed, that of the reservoir penetrates there, and it rises to the same height as the exterior surface, by reason of the law in virtue of which all parts of the surface of a fluid mass tend to take the same level.

Their kinds.

449. On entering there, it raises the fixed valve *b*, which is in its lower part, and which closes when the fluid has attained the level MN. Then, the piston, descending, presses and forces the water between its base and the valve; forces open the stop-clapper *e*, and rises in the ascension pipe. When the piston has reached the bottom of its stroke, and ascends again, the

fixed valve is opened anew, and the working-barrel of the pump is filled a second time; and so in succession. Such is the *force pump*, properly so called.

Fig. 77. In others, the piston is pierced in the middle, and surmounted with a valve; when it descends, the water which was below it, opening the valve, passes above it; in reäscending, it raises this water, as well as the whole column which is in the ascension pipe. This is the *lifting pump;* it only differs from that described in Sec. 434, in that the latter has a suction pipe below the working-barrel of the pump.

In some force pumps, the piston, which is also provided with a valve, is introduced through the lower opening of the working-barrel of the pump, and is supported by an iron frame, attached to a rod.

Load. 450. It is evident that such pumps can carry the water to any desired height, provided the disposable force is sufficient.

It is also evident, that the load of the piston, whether forcing or lifting, is always equal to the weight of a column of water which has for its base that of the piston itself, and for its height, the difference of level between the well and the delivery pipe.

What we have said, in the article on suction pumps, upon the resistances arising from the friction of the piston and of the water, from contractions at the valves, &c., applies equally to force pumps.

Resistance at the stop-valve. 451. There is, however, a resistance which is more considerable in these last, and of which no mention has yet been made; it is that experienced when we attempt to open the stop-valve, and in general, every valve bearing a mass of water upon it, having its upper surface greater than that of the opening, (and it cannot be otherwise).

Let Σ be this upper surface, H′ the height of the fluid mass upon it; $62.45\Sigma H'$ will be the pressure exerted by this mass. To surmount it, we must oppose to it an effort whose momentum must be at least equal to it: preserving the denominations of Sec. 440, we shall have then $62.45\Sigma H'\lambda = 62.45sx\lambda'$, whence $x = \frac{H'\Sigma\lambda}{s\lambda'}$: thus this effort, acting upon the piston, or being exerted by it, will be $62.45\pi D^2 H' \frac{\Sigma\lambda}{s\lambda'}$. If the two surfaces of the valve had been equal, that is to say, if the upper surface had been equal to the orifice, there would always have been requisite, to raise this mass, an effort equal to $62.45\pi D^2 H'$; then that arising from the excess of the upper surface will be

$$62.45\pi D^2 H' \left(\frac{\Sigma\lambda}{s\lambda'} - 1\right).$$

This effort should act but a single instant, at the commencement of the opening of the valve.

In a pump whose piston is moved by a hydraulic wheel, or by any mechanism carrying a fly-wheel, if the physical duration of this instant could be appreciated, and should be represented by θ, Θ being the time of the entire lift of the piston, we might convert the effort of an instant into an effort acting continually upon the machine, in multiplying it by $\frac{\theta}{\Theta}$.

ARTICLE THIRD.

Suction and Force Pumps.

452. Most commonly, the two kinds of pumps are united into one, and it is consequently called the *suction and force pump.*

It is composed of a working-barrel, of a short suction pipe, of an ascension pipe, of a solid piston, or of a long cylindric piston (plunger), and of two valves, the suction and the stop-valve.

Commonly, the suction pipe, which is never over a few metres in length, is placed immediately below the working-barrel, in the same straight line, and the ascension tube is placed at the side. Sometimes, however, these two pipes are in the same line, making, as it were, only one, and the working-barrel is at the side, as we see in Figs. 78 and 79.

We also couple suction and force pumps. Often the two working-barrels have but one suction tube, and sometimes also but one ascension tube. Pumps are also made with two pistons, moving in the same body. Finally, Lahire,* MM. Arnollet,† Cordier,‡ and Carcel (in his lamps) have employed but one working-barrel, with only one piston, which exhausts and forces at the same time in its reciprocating motion.

Dynamic effect. 453. In whatever manner the two coupled pumps are arranged, the dynamic force which the motor must employ to keep them in action, will be $52.956D^2H\times v$ (443); or rather, $56.203D^2Hv$, the force destined to raise the piston having to be increased by that necessary to lower it (444). The velocity v is estimated usually by the number N of strokes of each of the pistons in one minute; thus, l being the length of the stroke, we shall have $v=\frac{2Nl}{60''}$; and for the force impressed, or dynamic effect produced in 1″,

$$1.8734ND^2Hl^{\text{lbs. ft.}}.$$

454. I will give, as an example of good distribution of the parts of a suction and force pump, designed to accomplish a considerable and continuous work, one of those which M. Abadie has established, with complete success, at the water-works of Toulouse. They are eight in number, divided into two entirely distinct sets: each is moved by a great hydraulic wheel, whose turning axle carries, at each of its extremities, a crank, which

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* Mémoires de l'Académie des sciences. 1716.

† Bulletin de la société d'encouragement pour l'industrie nationale.

‡ Annales des ponts et chaussées. 1831. Machines de Beziers.

moves, through the intervention of a beam and connecting rod, two coupled pumps.

Figure 80 presents one of them, with its essential parts. Fig. 80.

The working-barrel is of cast iron: it is 0.98 ft. in diameter, and 4.92 ft. long. The piston consists of a beautiful brass cylinder, perfectly polished; still, after twelve years' service, its surface has all the lustre of a metallic mirror: its interior is hollow, and filled with lead small-shot: the exterior diameter is 0.889 ft., and its length is 5.57 ft. The stuffing box, besides the usual packing, contains at the bottom a crimped leather, bent downwards.

Below this box, the working-barrel is pierced with a small hole, furnished with a cock, through which issues the air that may have entered there. The suction pipe is 4.527 ft. long and 0.525 ft. in diameter: it is covered by a brass plate, carrying two semi-circular clapper-valves.

At the foot of the working-barrel, and upon one of its sides, is fitted a square cast iron box, 0.984 ft. in height and width in the clear. It contains a species of bronze box, open at one end, and its upper surface, being inclined 45° to the horizon, is pierced with three rectangular openings, 0.787 ft. long and 0.328 ft. wide; upon each is a clack-valve of the same metal. Above this box, the square box has an opening, which is closed by a cast iron plate, retained by iron straps, which are taken off when there is occasion to repair the valves, (which has not yet happened since their construction.)

This box is prolonged to the other pump of the same couple, whose water it also receives. In the middle of its upper surface rises an upright pipe, 0.886 ft. in diameter: at a height of 21.325 ft., it reünites with that which proceeds from the second couple of the same set. After this reünion, being then 0.984 ft. in diameter, it continues vertically, and discharges its water, 78.74 ft. above the well, in a basin placed at the top of the water-works.

The stroke of the piston is at will 2.62, 3.28 and 3.93 ft.

When the pumps are in full work, with the great stroke, we have 6½ strokes, and consequently a velocity of 0.853 ft.

455. One of the most useful combinations of suction and force pumps is found in the fire-engine pump.

Fire-Engine pump. Fig. 81. The two working-barrels, made of brass, have generally a diameter of 0.393 ft. and a length of 1.97 ft. The pistons are surrounded with leather rings; above and below are crimped leathers, disposed according to the description of Sec. 425; all are contained and pressed between two iron plates. The suction-valve is a stem-valve, and the stop-valve is a clapper. Between the two working-barrels is the reservoir or air receiver, made of copper plates about 0.118 in. thick; its diameter is 0.82 ft., and its height 1.804 ft.; in its lower part, it is pierced with a circular hole, to which is soldered a brass pipe, from the top of which issues a leather or strong impermeable canvass pipe, bearing at its extremity a long ajutage or spout, which is about 0.052 ft. in diameter at the orifice, and is directed towards the fire to be put out.

This pump is placed in a wooden box, mounted on four wheels, and drawn to the place where the fire breaks out. The firemen then continually supply the water with buckets made for the purpose, while the pumpers, placed at the two ends of the beam, working the rods of the two pistons, keep the engine in play.

The water passes from the pumps into the air reservoir; and as it arrives there in much greater quantity than can be vented, under a small pressure, through the lower aperture, it rises, condenses the air more and more, and gives it an elastic force, very often greater than that of three atmospheres. The reäction being equal to the action, the air presses the water with this same force; it causes it to issue with velocity through the spout, with a continuous jet.

Eight pumpers, working well, give sixty strokes to the beam per minute; the stroke of the pistons is 0.393 ft., and they impel the water a height of 65.62 ft.

Deducting all losses, this is $195.36^{\text{lbs. ft.}}$ of useful effect in 1″ per man.

Hydraulic press.

456. Towards the end of the last century, an application of the suction and force pump was made, too important to be passed in silence; it has given rise to the *hydraulic press.*

Fig. 82.

This machine consists of a piston A, which rises in the working-barrel of the pump B, communicating with the small pump C, by the pipe D. The great piston is covered with a plate, upon which we place the objects to be pressed; these are forced against an immovable plane, fixed a little above it.

The pressure which the base of the small piston exerts upon the water, when it descends, is transmitted, by the intervening fluid contained in the pipe, to the base of the great piston; and as it is equal upon each of the points of the two bases, its total effort upon each will be in the ratio of their surfaces; so that, if the ratio of the two diameters is as one to five, the effort exerted upon the great will be twenty-five times greater than that upon the small piston. Let us suppose a man, capable of exerting a pressure of 66.16 lbs. upon a weighing machine, acts at the end of a lever 3.2809 ft. long; and that the point of this same lever, to which is attached the rod of the small piston, is but .164 ft. from the other extremity, where the fulcrum is. The arm of the lever, where the power is, is twenty times longer than that of the resistance, and the effort at the great piston will evidently be $25 \times 20 \times 66.16 = 33080$ lbs.; an effort equal to that which 500 men, acting at the same time, would be capable of exerting.

I shall not enter into any details as to the very simple mechanism used to feed the pump with water, and to direct it suitably under the great piston. I merely remark, that it is very essential that the packing of the leather box through which the piston passes should allow no water to drop through it; this packing consists of a single crimped leather ring, so rounded upwards, that the cover of the box, pressing upon its convex surface, extends it in breadth, and brings it to bear forcibly, with one edge against the piston, and the other against the lateral surface of the box.

Rotatory pumps.

457. A continuous rotatory motion produces generally a greater effect than alternating motion; two distinguished mechanists, Bramah, of England, and M. Dietz, of France, have attempted to procure for pumps the advantages of the former. Having had no occasion to use their ingenious machines, I confine myself to giving a simple idea of their structure and mode of action; I will take for example, the *pump of Dietz.*

Fig. 83.

The body of the pump is composed of a drum or cylindrical copper box, A, having, in the clear, a diameter of from 0.656 ft. to 1.312 ft., and a thickness of from 0.131 to 0.393 ft., according to the power of the machine. It contains, between its two ends, a second box BB′, also of copper and cylindrical, but of less diameter, and without a cover: it is moveable about the turning axle C, furnished with a crank. In the interior of the box or wheel BB′, and adjoining its concave surface, there is an eccentric D fastened by screws upon the drum. The latter encloses also, at the sides of the pipes E and F, a large iron plate G*b*H, which is pressed at *b* against the convex part of the wheel, and is pierced with two openings: through one, *c*, the water passes from the suction tube E into the space *aaaa* between the two boxes; and through the other, *d*, it enters the ascension tube F. Finally, the box BB′ has, throughout its thickness, and as far as the axle, four cross formed cuts, in which slide four iron tongues, I, I′, I″ and I‴: their width (parallel to the axle), as well as that of the band G*b*H, is equal to the distance between the two ends of the drum: one of their extremities is constantly bearing against the exterior edge of the eccentric D, and the other is against the concave side of the space *aaa;* so that, like partitions, they divide this space into separate parts.

When the machine is put in motion, and the wheel BB′ goes from *b* towards B′, the tongue I, after passing the point *b*, leaves behind it a vacuum, and as soon as it gets beyond the opening *c*, the water enters in to fill it. The tongue I′, which follows, pushes before it this water, causes it to run through the interval *aaa*, forces it to pass through the orifice *d*, and to rise in the pipe F. So on successively, and we have a continuous motion and jet.

From what has been said, in order that the machine may raise all the water possible, it is necessary that the fluid be completely retained in the spaces, so as not to pass from one to the other, and consequently, that the moveable box and the tongues join perfectly the two ends of the drum, without, however, occasioning any considerable friction ; and for this purpose, we must have great perfection in the adjustment of the pieces of the machine. Even should this perfection exist on coming from the hands of the artist, we have to fear lest it may be damaged by much work, and by the raising of saline waters, etc., and that, at the end of a certain period, the useful effect may become far inferior to what it was at first : this latter, in an experiment made by MM. Molard and Mallet, has been $\frac{44}{100}$ of the force employed to produce it.

CHAPTER II.

ARCHIMEDEAN SCREW.

Parts and dimensions.

458. If, upon the surface of a wooden cylinder, we trace a helix of several *spirals*, so that in a groove cut according to this curve are set small plates, all of the same height, and joining well upon each other, the combination will present, as it were, the thread of a screw, very salient and of a uniform thickness; and if we then cover them with a cylindrical envelope of staves, the whole will constitute the *Archimedean Screw.* Its envelope will be the *barrel*, the plates forming the

thread of the screw will be the *steps*, and the solid cylinder the *newel* or *core;* the space comprised between the newel, the barrel, and the thread, will form a *helicoidal canal.*

In the common screws, we have upon the same newel three equidistant threads, and consequently three canals. The diameter of the screw, which is the interior diameter of the barrel, varies from 1.066 ft. to 2.13 ft.; that of the newel is a third of it; and the length of the screw is from twelve to eighteen times the diameter, according as it is more or less strong. The angle made by the helix with the axis, or rather with a right line traced upon the newel, and consequently parallel to the axis, has undergone great variations; the ancient Romans made it but 45°; at Toulouse, according to prescriptions derived from Holland, they make it about 54°; the Paris constructors make it generally at 60°; and Eytelwein, in a small screw, carefully made, went as high as 78°. At the upper extremity of the axis is a crank, and at the lower is a pivot, which is received in a socket, embedded in one of the small sides of a frame supporting the machine.

Use. 459. If we place it in a mass of water, giving it an inclination less than that of the helix upon the axis, which is usually from 30° to 45°, and impress upon it a motion of rotation, in an opposite direction to that of the helices, the inferior orifice of the canals passing in the water, will draw up a certain quantity, which will rise from spiral to spiral, and will issue at the upper orifice.

The screw is peculiarly adapted to the draining of water from places where we wish to lay, unobstructed by water, the foundations of any hydraulic structure, such as the pier of a bridge, a lock, &c. Its simplicity, the small space it occupies, the facility of transporting and setting it up, as well as that of setting up many at the same point, cause its use to be very general in such drainings, and give it a preference even over other machines, which have some advantages in other respects. It was well known to the ancients, and the illustrious name which it bears, shows that it has been known for more than twenty centuries. Vitruvius, who lived in an early age of the Christian era, made mention of it, and what he said shows that at that epoch, its construction was as well understood as now.

Method of working.

460. I attempt to give a precise idea of the mode in which the water rises in the screw.

Fig. 84.

For greater simplicity, let us take a screw formed by a tube, bent and wound round a cylinder. We first place it horizontally; if, through the orifice at the base, we introduce a bullet, in rolling, as upon an inclined plane, it will advance towards the other extremity of the tube, and it will stop upon the lowest point of the first spiral; by turning the machine, the point on which it rests will be raised; it will leave it, and, as if descending, it will pass to the following point; and in succession to the others, remaining always at the same level, but advancing towards the outlet of the tube, which it will finally attain, and so pass through it. Now, incline the machine a little, and again introduce the bullet through the lower end; it will still settle itself upon the lowest point of the first spiral; when it will be raised by means of the motion of rotation, and will pass upon the following one, which will also be raised, but in a less quantity; in this manner, by a movement at once progressive and ascensional, it will gain the upper outlet; it will have risen by descending, the plane on which it rested rising more than itself. If the inclination of the screw had been such, that the helix should present no point lower than that upon which the bullet is first placed, it would have continued to remain there. Finally, if the inclination had been still increased, the bullet could not have entered it; and if it had been introduced through the upper orifice of the tube, it would have descended in following all the windings, and have issued through the lower orifice.

What we have said of the bullet applies equally to the water which enters through the base into the spiral tube. It will flow to the lowest point of the spiral; it will then rise on both sides, in the two branches, to the level of the most elevated point of the branch of entry. The arc of the spiral, containing all the water it can then admit, is the *hydrophoric arc* of the screw. If, after the first spiral is filled, we make a revolution of the machine, the water it contains will advance, like the bullet, with a double motion, progressive and ascensional, and it will be found in the hydrophoric arc of the second spiral; it will be replaced in the first by a new and equal quantity of water. In the following revolutions, these two bodies of water, as well as those which follow after them, will ascend from spiral to spiral, even to the orifice of exit. Thus, at each revolution, the screw will evidently discharge a quantity of water equal to that contained by the hydrophoric arc.

The depth to which the screw should be plunged in the well.

461. But for this purpose, the base of the screw should be plunged in the well a certain quantity.

Fig. 84.

It should be at least so much submerged, that the mouth of the helicoidal tube, after having traversed in its rotation the water of the well, on its arrival at the surface, shall be found at the summit of the hydrophoric arc of the first spiral; then this arc will be entirely filled; and it is evident that it could not be so, if the level of the reservoir was below this point, whose position we shall soon determine. When the mouth, in pursuing its rotation, shall have passed this level, the atmospheric air will enter in the tube, will take the place vacated by the water, and at the end of the first revolution, it will fill the upper part of the first spiral, that which is above the hydrophoric arc. It will be the same with the following spirals; the water and the air will be then disposed as indicated by the figure; each of the columns of the former fluid will be entirely supported by its spiral; it will not exert any pressure upon the inferior columns, and throughout, the air will have the same density as that of the atmosphere.

It will not be so, if the level of the well should be raised above the summit of the hydrophoric arc, even though the orifice of the tube may be found, in some portion of its revolution, outside the water. The air, it is true, will be introduced among the spirals, but the water will occupy more than the hydrophoric arc; it will rise, in the ascending branch, above the summit of this arc, that is to say, above the summit of the descending branch; it will bear upon the inferior column with all this excess, and will compress the air comprised between that and itself. Often this air, striving to regain its density, traverses the column which is above it. On the other hand, and by reason of the movements which take place, and of the irregularity with which the water and the air are reciprocally disposed, the last of these fluids may be found rarified in certain parts; and we may see the atmospheric air introducing itself in the tube, passing briskly through the water of some spirals, and going to establish the equilibrium; these shocks and irregular movements diminish considerably the product of the machine.

Finally, when the base is plunged entirely in the well, the air cannot enter the screw; nothing but water can enter there. If the velocity of rotation be very great, the centrifugal force resulting from it may raise this water, and cause it to be discharged through the upper outlet, as in the case mentioned in Sec. 392. But with a less velocity, the water will only reach a certain height in the tube; forming a continuous whole, it will press, with all the weight due to its vertical height, upon the orifice of entry, and will thus counteract the centrifugal force. In great machines, the air which is already in the helicoidal ducts, and that which arrives there through the upper opening, also produce irregularity in the motions, and the diminution of the product already alluded to. When, however, the canals are very large, and the machine is properly disposed and inclined, the exterior air arriving without commotion in all the spirals, these inconveniences no longer occur, and we obtain nearly the usual product..

Eytelwein, who made a particular study of the movements of water in different kinds of screws, published a series of experiments which show the bad effect of a too great or too little submersion of the base in the water to be drained; at least, for screws with small ducts. I give here some of the results obtained. He was provided with a model of a screw made with great care: it was 0.512 ft. in diameter and 3.608 ft. long: it had two helicoidal ducts, intersecting the axis at an angle of 78° 21″, and having, in the direction of the radius, a height of 0.138 ft. This screw was placed in a reservoir, in an angle of 50° to the horizon, and when it yielded the greatest product, the level was 0.042 ft. above the centre of the base. I indicate in the first column of the annexed table, the height of the water above or below the centre of the base; and in the second, the volume of water raised at each revolution.

HEIGHT of level.	PRODUCT per revoln.
ft.	cub. ft.
.400	0.008
.180	0.009
.082	0.009
.049	0.010
.041	0.012
.032	0.011
.016	0.011
.019	0.010

Theory of the screw, the canal being narrow.

Though the Archimedean screw is very ancient, and simple in its character, still, there is no theory to be found for the machine as it is now used. The essays of some learned mathematicians are far from enabling us to determine its effects exactly. That which Bernoulli and most authors have given, applies only to the case (now out of use) of a tube, with a very small diameter, rolled spirally round a cylinder: I make an elementary exposition of the principal features of it, both to guide our first impressions upon this subject, and to avoid leaving a gap in this work.

Fig. 85.

Let AMCND be a vertical projection of the axis of the helicoidal tube, wound round the cylinder ABED, and the circle *anòma* a projection of the base of the cylinder, upon a plane perpendicular to its axis. Through the point F of the arc AMM′C draw the tangent GH; it will make with the edge OI an angle IFH, which we designate by a; and through the extremity B of AB draw the horizontal BK, the angle EBK, or b, will measure the inclination of the screw.

462. Let us determine the length of the hydrophoric arc MCN.

And first, the height LP of any point L of the helix, above the horizontal plane BK. Project L at l upon the circumference of the circle of the base, and draw the horizontal lr, we shall have $\mathrm{LP} = \mathrm{L}r + r\mathrm{P}$. For greater simplicity, make the radius $oa = 1$; designate by α the length of the arc $\mathrm{A}l$ $(= al)$; the angle which the helix makes at A with the plane of the base, being the complement of a, we shall find $\mathrm{L}r = \mathrm{L}l \sin. b = \mathrm{A}l \cot. a \sin. b = \alpha \cot. a \sin. b$. We shall also have $r\mathrm{P} = lq = l\mathrm{B} \cos. b = sb \cos. b = (1 + \cos. \alpha) \cos. b$. Then $\mathrm{LP} = \alpha \cot. a \sin. b + (1 + \cos. \alpha) \cos. b$.

The summit or commencement of the hydrophoric arc of the spiral ACD will be at M, the most elevated point above BK. It corresponds consequently to the *maximum* value of LP. Differentiating the above expression, equaling the differential to zero, we have $\sin. \alpha = \cot. a \tan g. b$; which gives the value of the arc α, or dm, for the case of the *maximum*. Calling m this particular value at the point M, we have for the height of this point above BK, $m \cot. a \sin. b + (1 + \cos. m) \cos. b$.

If through M we imagine a horizontal plane, the point N, where it intersects the ascending branch of the spiral, will be the end of the hydrophoric arc; since the commencement and the end should have the same level. Project N upon the circumference of the base; it will fall upon the point n; call n the arc bn; the arc of the circle $ambn$, corresponding to the arc of the helix AMCN, will be $\pi + n$; and for the elevation of N above the horizontal plane passing through B, we shall have $(\pi + n) \cot. a \sin. b + [1 + \cos. (\pi + n)] \cos. b$.

This elevation should be equal to that of M. Making the two expressions equal and reducing, we have $(\pi + n) \sin. m + \cos. (\pi + n) = m \sin. m + \cos. m$: an equation from which we may deduce the value of n, by means of successive substitutions.

This value being found, we shall know the arc mbn corresponding to the hydrophoric arc MCN. But an arc of the helix is equal to an arc of the corresponding circle, increased in the ratio of the radius of the tables to the cosine of the angle comprised between the two arcs, that is to say, divided by this cosine. Here the arc of the circle is $\pi - m + n$, the angle comprised between the two arcs is

$90° - a$: the length of the hydrophoric arc will then be $\frac{\pi+n-m}{\sin. a}$; and, for a cylinder whose radius is r,

$$r\frac{\pi+n-m}{\sin. a}.$$

463. If s is the section of the helicoidal tube, the volume of water raised at each turn of the screw will be the above expression multiplied by s.

Calling N the number of turns made by the screw in a given time, L its length outside of the water, and observing that the height of the elevation is L sin. b, we shall have for the value of the useful effect, during this time,

$$\mathrm{NL}sr(\pi+n-m)\frac{\sin. b}{\sin. a}.$$

464. The expression sin. $m=$cot. a tang. b, obtained by differentiating, and making equal to zero the general value of the elevation of any point of the first spiral, answers equally to the case of *maximum* and *minimum;* it gives the smallest as well as the greatest elevation. Moreover, the sin. m applies as well to the arc am' as to the arc am, by taking $bm'=am$. Consequently, if we project the point m', upon the hydrophoric arc, M′, which is its projection, will be the lowest part of the arc, as M is the highest point.

The expression cot. a tang. b, representing a sine, cannot exceed 1. When it is equal to it, the arcs am and am' will become ad'; the points M and M′ will be merged in the point F; there will no longer be a hydrophoric arc, and no more water raised. But cot. a tang. $b=1$ gives tang. $b=\frac{1}{\cot. a}=$ tang. a or $b=a$; that is to say, that when the angle of inclination shall be equal to the angle made by the helix with the edge of the cylinder, the discharge will cease; it is necessary, then, in order that it may take place, that the first of these angles should be smaller than the second, as we have already remarked (459).

That of the values of b giving the greatest effect is impliedly embraced in the above expression of effect. For the same screw, moved with the same velocity, there will be no variable in this expression but sin. b $(\pi+n-m)$, and it will be necessary to determine the value of b which will render this quantity a *maximum.*

465. From what was said at the commencement of Sec. 461, in order that the hydrophoric arc should take all the water it can contain, the level of the fluid in the well should be as high as the point *m*, or as the point *p*, which is on the same horizontal; and consequently should be raised above the centre of the base by the quantity $op=r \cos. m=r\sqrt{1-(\cot. a \text{ tang. } b)^2}$. For the vertical elevation, we shall have

$$r \cos. b \sqrt{1-(\cot. a \text{ tang. } b)^2}.$$

Influence of velocity upon the product.

466. In what has been said, we have supposed the hydrophoric arc had time to be filled with water, without any mention of the velocity of the water. It has, however, a great influence upon the amount of the product, especially when the bottom of the screw is entirely submerged. This influence is shown by the experiments of Eytelwein. They were made with the small screw already mentioned, with an inclination of 50°. In the first series, the end of the screw was entirely submerged; an unfavorable circumstance, the disadvantages of which are not sufficiently appreciated by workmen. The second was made under more favorable circumstances, with the base submerged only a suitable quantity (465). In practice, it will suffice to establish the screw in such a manner as that the end of the vertical diameter of the core may project a little above the surface.

NUMBER of revolut. in 1′.	WATER raised per revolut.
	cub. ft.
22	0.0099
41	0.0094
51	0.0088
74	0.0081
121	0.0068
56	0.0113
60	0.0118
73	0.0121
85	0.0123
98	0.0123
120	0.0118

Comparing the terms of the two series, when the velocity of the machine has been nearly the same, we see that when the inferior extremity was entirely submerged, the product was about one third less.

Real effect of the screw.

467. We pass to the effect of which great screws are capable.

I make known what this product would be, by giving, in the following table, the results of experiments made with three pumps, of $1^{\text{ft.}}$, $1\frac{1}{2}^{\text{ft.}}$ and $2^{\text{ft.}}$ (French measure) in diameter, the latter limit never being exceeded. I give the length and velocity of each, as well as the angle of inclination at which it stopped delivering water; an angle which, according to theory, is equal to that made by the helix with the axis (464). The greatest effect was produced at an angle of 30°; I have taken it for the unit, and have compared with it those obtained under different angles; this comparison shows the great influence of the inclination.

Angle of inclination.	Diameter = 0.066ft. Length = 19.182ft. Revolut. in 1′ = 90 Limit of incli. = 60°			Diameter = 1.597ft. Length = 27.69ft. Revolut. in 1′ = 60 Lim. of incli. = 62°			Diameter = 2.10ft. Length = 25.57ft. Revolut. in 1′ = 41⅔ Lim. of incli. = 65°		
	WATER raised in 1 hour.	HEIGHT of elevat.	SERIES of effects.	WATER raised in 1 hr.	HEIGHT of elevat.	SERIES of effects.	WATER raised in 1 hr.	HEIGHT of elevat.	SERIES of effects.
	cub. ft.	ft.		cub. ft.	ft.		cub. ft.	ft.	
30°	1486.8	8.98	1.00	4576	12.36	1.00	9149	10.66	1.00
35°	1236.	10.10	0.93	3630	14.62	0.94	7164	13.12	0.97
40°	872.3	11.25	0.74	2397	16.85	0.71	4841	14.90	0.74
45°	443.8	12.36	0.50	1306	19.12	0.44	2613	16.49	0.44
50°	307.2	13.48	0.31	508	20.23	0.18	893		
55°	91.8	14.62	0.10	180	21.35	0.07	367	17.84	0.07

Though the volumes of water indicated in the table have been admitted, as the results of experiment, by a commission of engineers, still, as they are presented by a constructor of the Archimedean Screw, we may fear that there is some exaggeration; and in application, we should not reckon upon more than two thirds of the product indicated.

It seems that the quantities of water raised by these machines, they having been reduced to the same number of turns in the same time, should be proportional to the capacity of the hydrophoric arc, and consequently to the cube of the diameters, if the screws were similar solids; yet I find that these quantities are very sensibly proportional to the 3⅔ power of the diameter, or to $D^{\frac{11}{3}}$. Consequently, by reducing one third the quantities given in the preceding table, the volumes of water raised in one hour, under different angles of inclination, by a screw of a given diameter D, would be such as are indicated in the adjoining table.

ANGLE of inclination.	WATER raised in 1′ by 40 revolutions.
	cub. ft.
30°	$364D^{\frac{11}{3}}$
35°	288 .
40°	191 .
45°	104 .

Number of workmen employed.

468. These screws are usually put in motion by men, who act indirectly upon the crank, through the intervention of beams or connecting rods, upon which they impress a reciprocal motion, which converts that of the crank into a rotatory. What is the number of men to be employed to produce a given effect?

A screw 1.607 ft. in diameter, and 19.19 ft. long, used for draining by M. Lamandé, engineer, moved by nine men, (working in spells of two hours, and then relieved by a similar number of fresh hands,) inclined about 35°, making forty turns per minute, raised in one hour $1589.2^{\text{cub. ft.}}$ of water 10.82 ft. For each of the nine workmen, this was $176.58^{\text{cub. ft.}}$ raised 10.82 ft., or $1910^{\text{cub. ft.}}$ raised $1^{\text{ft.}}$; he did not work over five hours in the day; thus, the day's labor of each was only $9550^{\text{cub. ft.}}$. In another experiment, six workmen, working six hours, raised each $10660^{\text{cub. ft.}}$, and consequently, $1776^{\text{cub. ft.}}$ per hour.

According to these positive and authentic facts, we may admit that a workman, employed upon a well arranged screw, can raise in one hour $1738^{\text{cub. ft.}}$ one foot in height, and that he may labor in this manner six hours per day. He might even work eight hours in the twenty-four, in a continuous draining, if the relays were properly established; so that the number of workmen to accomplish such a draining would be $\frac{Q'H'}{579}$, or, to prevent any mistake, $\frac{Q'H'}{463}$, Q′ being the volume of water to be raised in one hour, and H′ the height of the elevation.

Hydraulic screw.

469. We also employ for draining, screws without the envelope or barrel, consisting simply of a newel, upon which are placed the helicoidal threads. We place them in a canal or semi-cylindrical box enclosure, made of carpentry or masonry, and having a

suitable slope: it is as it were a half-barrel, but immovable. But a very small interval is left between its sides and the edges of the threads. These machines, called *hydraulic screws*, (*Wasser-Schraube*,) by the Germans, are much used in Holland, where they are frequently set in motion by windmills.

They have a great velocity imparted to them, lest a great quantity of water, raised at first, should fall back into the well, following the sides of the trough, before it has reached the point of discharge. They have the advantage of being independent, in their product, of the height of the water of the reservoir compared to their extremity, and, without shifting their place, they may drain a reservoir whose level is gradually reduced. But this advantage is more than counteracted by an inconvenience: very often, the core or newel, at least if it is not large, bends, and the edges of the threads rub against the sides of the canal; which wears out the machine, and occasions a resistance, absorbing a portion of the motive force.

Spiral pump. Fig. 84. 470. I will make brief mention of a machine, which has some resemblance to the Archimedean screw, and which may be used for raising water to great heights: this is the *spiral pump*. It consists of a conical or cylindrical turning shaft, upon which is wound, screw fashion, a tube of lead or other material: one of its extremities takes up the water, and the other is enclosed exactly in the curved end of an upright tube, which conveys this water to the desired point.

This machine, invented and made, in 1746, by a tinman of Zurich, has been made the subject of a work by Daniel Bernoulli, who has given its theory, and proposed some improvements, which have been adopted in a construction made at Florence. Since then, Nicander and Eytelwein have devoted their attention to it: the latter reported that, in 1784, he had established such a pump, near Moscow, with complete success; it conveys 4.09 cub. ft. in 1′ a distance of 761 ft., and 75.46 ft. in vertical height. This author extols all the advantages of this machine, and recommends its use.

Notwithstanding this recommendation and these facts, as it is but little used, and is unknown to me, I shall not enter into any details, but simply refer to the principle upon which it is based. When the mouth takes up alternately water and air, these two fluids advance, from spiral to spiral, up to the upright pipe: they enter it; the air is disengaged and escapes into the atmosphere, the water ascends gradually, and is discharged' through the spout placed at the top of the pipe. During the motion, the two fluids are disposed in the spirals as shown in the figure: the water on one side, the air on the other; the latter occupying less and less space. In the first spiral from the entrance mouth, the air is loaded, not only with the atmospheric weight, but that of the column of water of the second spiral: the air of the latter sustains also the weight of the third column; and so on, so that in the last spiral, that which is near the upright tube, it is as it were loaded with the weight of a column of water, whose height is the sum of the heights of this fluid in all the spirals. This same air supports, by the elastic force due to such a pressure, the column of water in the upright tube; it can therefore support one whose height is equal to the sum of the heights of the water in the spirals. Thus the height to which we can raise water, by means of a spiral pump, depends upon the length and the number of spirals of the helicoidal tube.

Blast or blowing screw.

471. If the compressed air, on issuing from this machine, were properly received and directed, it would produce a blast, which might easily be made nearly continuous. An Archimedean screw, containing also in its spirals alternate masses of air and water, might yield an analogous effect, if it were disposed and moved in an order in some sort the inverse of that followed in draining.

In this manner, M. Cagniard-Latour, well known for his many inventions, has made a new blast machine, which has been used successfully for various purposes. It is an Archimedean screw of great diameter compared to the core, placed in a basin filled with water, with a certain inclination, so that the upper end of the axis shall be very near the liquid surface. When the screw turns, the upper mouth of the helicoidal canal passing in the atmosphere during one half of its revolution, there takes a certain quantity of air, which at first has its place above the first hydrophoric axis, and which then descends from spiral to spiral, issues through the lower mouth of the canal, and tends to rise in the water of the basin, with an elastic force measured by the height of the liquid surface above this mouth.

CHAPTER III.

BUCKET MACHINES.

(*Buckets, Norias, Chain Pumps, Persian Wheels.*)

472. In the machines we are about to describe, the water is drawn by a bucket, or machine of that kind, which conveys it and delivers it at the desired height. We have, then, the case of a weight immediately raised a certain height, for which there are no special theories; we have only to give a clear idea of the machine by which it is accomplished, and to estimate its effects in practice, as well as the ratio between this effect and the force employed to produce it.

This force is usually that of a man, working upon a winch, or a horse harnessed to a gin. The useful effect of the first is $39.797^{\text{lbs. ft.}}$ in 1″ (475), and that of the second is $289.43^{\text{lbs. ft.}}$ (291); a man may thus raise $2317.4^{\text{cub. ft.}}$ of water one foot in one hour; and a horse $16839.4^{\text{cub. ft.}}$.

ARTICLE FIRST.

Elevation of Water with Buckets.

Baling. 473. Buckets alone are seldom used to raise water by continuous labor. Sometimes, however, we have recourse to this method; for example, for draining required to be done at once, and of but short duration. Many workmen, each provided with a bucket or scoop, placed in the foundation or trench, may thus be employed in bailing out the water. But as, at each discharge, they have to raise not only the weight of the water, but that of the bucket, as high if not higher than their heads, they must necessarily work in uneasy positions, and so accomplish little. According to Perronet, when they lift the water a height of 5.9 ft., they can only bale 1.2 cub. ft. per minute; and twice this amount when the elevation is 3.28 ft.; this would be, as a mean, but 463.33 cub. ft., raised one foot per hour; and consequently, the fifth part of what a man can do, when he employs the force of his arms in the most advantageous manner.

Swipe with buckets.

474. When we have to raise only a small quantity of water from a depth of 16 to 20 ft., for one or two hours of the day only, the object is conveniently accomplished by suspending a bucket from the end of a swipe, (supported by a post,) having at its other end a counterpoise; so that the effort of working it is exerted solely in drawing down the empty bucket. In this manner, a workman raises from 1390 to 1740, and even to 2320 cub. ft., one foot an hour, according to his skill in such labor.

Buckets upon a wheel. Work at the winch.

475. For greater depths, the best mode of using buckets is to suspend two upon a wheel, by means of a rope, so that one ascends while the other descends. Laborers upon the winches, fixed at the ends of the revolving axis, put and keep the machine in motion.

Coulomb, in his important memoir upon *the quantity of action that can be produced by man in his daily labor, according to the different modes of exerting his force*, examines also the case where a man raises water or a weight by means of a winch, a mode of action which this author has found to be the most advantageous. In default of direct experiments, he concludes, from experiments made upon draining machines, that, in a continuous labor of six hours (216000″) out of the eight or ten of the common day's work, a workman exerts an effort of 15.437

lbs. upon a winch, which moves with a velocity of from 2.526 to 2.756 ft. So that the quantity of a day's work would be, as a mean, 15.438 lbs. × 2.6247 ft. × 216000″=8752300 lbs. ft. (281). In an hour, this is 2337 cub. ft. of water raised one foot.

I shall admit this last result, not for the hour of continuous labor, but of ordinary labor, that is to say, intermixed with resting spells, which may occupy a fifth or even a fourth of the time appointed for the work. A man can labor, by the day, eight hours in this manner, and consequently can raise about 18700 cub. ft. a height of one foot, or produce, in a day's work, a useful effect of 1157740 lbs. ft.

The experience, not of a day, nor of a year, but of several centuries, (which, too, I have often verified,) leads me to this conclusion. I particularize the fact, as appearing to me the best calculated to give a positive measure of the effect produced in the day's work, by a common workman, at a winch.

At the mines of Freyberg, in Saxony, one of the most important among the mines of Europe, and probably the best regulated, a great part of the mineral worked is raised from lower stages to upper stages, by means of axles .72 ft. in diameter, with winches whose radius or arm is 1.443 ft. The daily task of two miners employed at the winch of each of these drums, is to raise 120 buckets of mineral products from a depth of twenty "*lachter*," the "*lachter*" being equal to 6.5027 ft., and the bucket equal to 1.1654 cub. ft.; its load, that is to say, the weight of the fragments of rock or of mineral with which it is filled, varies from 115 lbs. to 132 lbs.

Thus, in the day's work, each man produced a useful effect of from 894246 to 1032434 lbs. ft. But he only worked six hours and a half at the winch; if he had worked his allotted eight hours, he would have produced, as a mean, 1185672 lbs. ft. In any case, this is, per hour, 2320 cub. ft. of water raised one foot.

Setting the mean load at 123½ lbs., and observing that the diameter of the cord which bears the bucket is .03 ft., we find the effort exerted upon the winch by each of the two workmen, to equipoise this load, is 16.077 lbs. The velocity of the point upon which they act, admitting an entirely continuous motion during the $6\frac{1}{2}$ hours of work, is 2.561 ft. (It would be about three feet at the time of effective motion; and then, our common winches, with an arm of 1.31 ft., would make twenty-four turns in one minute.) Thus, the useful effect produced by each of the two miners will be $16.074^{\text{lbs.}} \times 2.562^{\text{ft.}} = 41.18^{\text{lbs. ft.}}$ in $1''$.

For the dynamic effect, or measure of the force impressed by the motor, the passive resistances of the machine should be added to the load; they will increase it about a tenth, and will thus cause the effort exerted by each of the workmen to be 17.64 lbs.; so that the quantity of action developed and impressed by them will be $17.6432^{\text{lbs.}} \times 2.561^{\text{ft.}} = 45.18^{\text{lbs. ft.}}$; and in a working day of eight hours, or $288000''$, it will be $1302462.^{\text{lbs. ft.}}$.

However advantageous may be the raising of a weight by means of a winch, it is not used, in connection with buckets, in great drainings; as these buckets may be but imperfectly filled, may lose their water in rising, may swing, come into collision, &c.

Buckets on a fixed pulley.

476. Coulomb, in examining the quantity of daily action produced by a man raising water by means of two buckets, hung at the two ends of a cord passing over a fixed pulley, found but half of that impressed upon a winch; it was but 513685 lbs. ft. It is a little less than that furnished by workmen employed on a pile-driver, and which Coulomb estimates at 542625 lbs. ft.

ARTICLE SECOND.

Norias.

Idea of a Noria. Advantages and inconveniences.

477. When we raise water by means of buckets borne by a wheel, besides the two men placed at the winches, there is needed a third in the well, to see that the bucket, at its descent, shall be quickly and completely filled; and so the cost is increased. Sometimes a fourth

is placed at the top of the pits, to empty the full buckets on their arrival.

To avoid this increased expense, as well as to increase the volume of water raised, by preventing the interruption caused by the filling and emptying of the buckets, we attach a series of them to an endless chain, passing over a drum or great axle, established above the reservoir whence we draw the water. Their opening is turned upwards on the ascending branch, and downwards on the other. This machine, called *noria*, is put in motion by winches, or by a gearing at the end of the axis of the drum. The buckets, passing into the well, are there filled with water, which they bear all along the ascending branch; arrived at the top, they incline along the upper convexity of the drum, and deliver their water in a trough or basin appointed to receive it.

478. In this manner, the buckets fill and empty themselves, and a continuous motion is perfectly established. But by the side of these advantages, there are some inconveniences; the water is necessarily raised to a greater height than that of the point of its reception; and the great weight of the apparatus, as well as its numerous joints, increase greatly the passive resistances and the repairs to be made.

Notwithstanding these defects, the noria is a good machine. It is much used in the south of Europe; for many centuries, it has served for watering all the great gardens in the environs of Toulouse, where it is worked by horse-gins.

Description of a Noria.

479. It is not long since these buckets were simple cylindrical earthern pots; the chains consisting of twists of straw, and the wheels were bits of joist, joined in the form of a double cross. Now, the buckets are made of choice woods, or, more frequently, of copper plates; the

chains are of wrought, the gearing of cast iron; and the machine is generally arranged like that built in 1781 at *Vitry-sur-Seine*, near Paris, a description of which was published by the Agricultural Society of that capital, in 1817, recommending its use.

I can give no better idea of a good noria, with its principal dimensions, than by a short description of one established by M. Abadie near Toulouse.

The drum, in its vertical section, is a regular hexagon, each side of which is 1.47 ft.: it is a trundle, with six spindles. It is formed by two cast iron plates, .065 ft. thick, 1.41 ft. apart, and connected by spindles or iron bolts .098 ft. in diameter. One of the plates is pierced with a simple opening for the passage of the axis of rotation, which is composed of a piece of iron 0.177 ft. square. The other presents at its centre as it were a nave, formed of two concentric rings, projecting 0.262 ft., or of that width; the small one, 0.196 ft. in diameter, embraces the axle; between it and the great one, which has a diameter of 0.426 ft., are six small partitions, placed in the direction of radii: the whole is of cast iron, and run in the same mould as the plate. Between the two plates, like a newel in the middle of the drum, is placed horizontally a truncated hexagonal hollow pyramid; its height is 1.41 ft.; the side of the great base is 0.656 ft., and that of the small is 0.164 ft.: this small base is fastened against the small ring of the nave, and the great base against the inner side of the opposite plate. Its six edges correspond with the six small partitions of the nave, and with the six spindles. Between each edge and its corresponding spindle is a cast iron plate or great partition, and the drum is thus divided into six compartments.

The chain is 45.01 ft. long, and is composed of twenty-eight great links. Each one carries a bucket made of copper plates: Fig. 87 presents a section of one, made perpendicular to the axis of rotation: AC = 0.889 ft., AB = 0.688 ft., CD = 0.427 ft., and their width, parallel to the axis, is 1.099 ft.: their capacity is thus 0.529 cub. ft., double that of the common norias. In the middle of the bottom CD is a circular hole, 0.088 ft. in diameter, covered by a small wooden valve.

Upon the two opposite sides of each bucket are fixed two small strips of iron M, 0.016 ft. thick, 0.105 ft. wide, and 1.74 ft. long. Their extremities are traversed by a bolt 0.065 ft. in diameter, so that the one traversing the upper end of the strip of a bucket shall traverse also the lower ends of the strips of the bucket just above it. It is this which composes the links, and great care should be taken that their length, and the distance of the bolts apart, should be such that, in the part of the chain which bends upon the upper part of the drum, the bolts should correspond perfectly with the spindles of the trundle, that is to say, to the summits of the angles of the hexagon.

One of the ends of the axis of rotation carries a vertical wheel, with twenty-three teeth, geared into those, thirty-eight in number, of a horizontal wheel. The latter is traversed by a vertical iron shaft, 0.177 ft. square and 3.608 ft. long: its lower end rests in a socket, and its upper end, fixed in a ring, receives the arm of a horse-gin 13.12 ft. long.

Upon the horizontal axle we have also a ratchet-wheel, to prevent a retrograde motion.

When the machine is in motion, and the upper end of the link arrives at the trundle, it is taken by a spindle, and carried along with it. As soon as, in rising, the bucket of this link begins to incline, its water also begins to pour into the corresponding compartment; it ceases flowing before it has attained a horizontal position, and consequently before it has begun its descent. This water descends into the compartment; arriving at the bottom, which is one of the inclined faces of the truncated pyramid, it follows it, and issues through the corresponding opening of the nave, without losing a drop during the discharge.

Effect of the Noria.

480. The noria which we have just described is established upon a well whose level is 17.06 ft. below the axis of rotation. It is worked by an ordinary horse, and raises 812.28 cub. ft. of water in one hour, and delivers it in a receiving basin, whose surface is 0.229 ft. below the axle, and consequently 16.831 ft. above the well. Thus, the useful effect in one hour is equivalent to 13670 cub. ft. raised 1 ft. (=812.28 × 16.83). We have seen (472), that a horse working in

a gin may raise 16685 cub. ft. 1 ft. We have, then, a loss of 18 per cent.

M. Navier reports, that a noria used at the drainages near Paris, worked by two horses, raised in one hour 2476.39 cub. ft. of water a height of 11.81 ft., or 29249 cub. ft. raised 1 ft.; this would be, per horse, 14624 cub. ft. raised 1 ft., and the loss would be only 12 per cent. It is usually much greater; it ranges between 20 and 30 per cent.

It arises from two causes; 1st, from the buckets in rising suffering a portion of the water which they had previously drawn to fall back; though this portion never arrives at the receiving basin, it has, during a certain time, borne upon and resisted the action of the motor; 2d, from the fact that the machine always raises the water higher than the surface of the basin, a surface which is of necessity somewhat below the axis of rotation.

We may allow for the first of these, and for some other sources of loss, by reducing the volume of water which a horse can raise one foot in one hour, from 16685 cub. ft. to 13904 cub. ft. raised 1 ft. We may allow for the second, in diminishing these 13904 cub. ft. in the ratio of H to $H + r'$, H being the height of the surface of the basin above that of the well, and r' being the vertical distance between the first of these surfaces and the highest point to which the water is borne; r' will usually be the radius of the drum, increased from four to eight inches.

Consequently, the useful effect that a horse can produce in an hour, in working a noria, is expressed in a cub. ft. of water raised 1 ft. by $13904 \frac{H}{H+r'}$. Thus, the volume of water which he can raise to a height H will be $\frac{13904}{H+r'}$.

Whence it follows, that the number of horses to be employed, on one or more norias, to raise a volume of water of Q′ cub. ft. in an hour, to a height H, is $Q'\frac{(H+r)}{13904}$.

Dynamic effect.

481. M. Emmery, engineer, has made some experiments to determine the ratio between the useful effect of the noria and the quantity of action developed by men employed to produce it. In one of them, five strong workmen, working all together, and exerting upon the windlass an effort of 102.26 lbs., with a velocity of 2.749 ft., raised in one hour 900.53 cub. ft. to a height of 11.8 ft.: which gives 0.657 for the ratio sought.

In good pumps, this ratio is greater: thus, provided we can have such pumps and the means of maintaining them, they will be preferred. Otherwise, and if the machines are only to work at intervals, we would construct norias, any blacksmith being able to make the requisite repairs.

ARTICLE THIRD.

Chain Pumps.

Chain pumps, which are also a series of buckets, but of a particular kind, were formerly almost exclusively employed for drainage on a large scale; they are now used only in such localities as do not admit of a convenient use of the Archimedean screw. There are two kinds, the *vertical* and *inclined*.

Vertical chain pump.

482. The vertical chain pump consists, 1st, of a wooden cylindrical tube, or *trunk*, from 13 to 19.6 ft. long, and with a diameter of from 0.42 to 0.52 ft.; its lower end is plunged into the water to be drawn: 2d, of a spur-wheel, placed above the tube, armed with iron clutches; it is traversed by a turning shaft, furnished with winches at its ends: 3d, of an endless chain, bearing, from space to space, beads or *paternosters*, formed each of a greased leather washer, held between two iron plates; 4th, finally, of a trundle

placed at the foot of the chain, to keep it extended and properly directed. See, in the *Architecture hydraulique* of Bélidor, the details of the construction and establishment of these machines.

When the chain pump is in motion, the claws of the spur-wheel seize successively the links, and the chain ascends. The paternoster arriving at the lower end of the tube, takes the water which is beneath the preceding one, intercepts its communication with the reservoir, and raises it up to the discharging pipe.

The vertical chain pump is well adapted for drainage, when we have to deliver the water at a height of over 13 feet; its apparatus is less complicated and less heavy than that of the noria, and it offers less resistance. It allows, it is true, a large quantity of water to fall back, which passes between the leathers and the sides of the tube, especially when the velocity is small. This loss is diminished by a good care of the machine, and by putting at the lower end of the tube a pipe of metal, well bored, of a diameter a little smaller, and of a length somewhat exceeding the distance of the paternosters apart.

From four to eight men may be employed at once upon a chain pump; its winches are 1.31 ft. at the elbow, and make from twenty to thirty turns per minute. The workmen are relieved every two hours; each works eight hours in the day, and there will be needed from twelve to twenty-four, divided into three relays, to pump night and day.

483. Perronet, who had twenty-two chain pumps in one pit of the foundations of the bridge of Orleans, has determined their useful effect. In an experiment made on one of them, moved by four men, who made thirty turns of the winch, they raised to a height of 15.98

cub. ft., 18.1598 cub. ft. in 108″; which would make 605.26 cub. ft. in one hour; and for each man, 2419 cub. ft. raised one foot in this time. Perronet admits twenty-five turns of the winch for ordinary work, and consequently, 2016 cub. ft. per hour. Perhaps it would be better, in common practice, to admit only twenty turns, and we shall not have over 1613 cub. ft. for the quantity raised, (deducting the greatest loss due to a less velocity). This result would accord with that deduced from the observations of the engineer Boistard upon three chain pumps, of about 0.49 ft. in diameter and 11.48 ft. in height; admitting that a fifth of the time (1 in 5.14) is taken for a resting spell, and that the loss is but a sixth of the water at first drawn, we find that the volume of water raised one foot high in an hour, by each of the six or eight workmen employed at the same time, is for the three chain pumps respectively, 1815 cub. ft., 1479 cub. ft., and 1441 cub. ft.; as a mean term, 1579 cub. ft.

Adopting this, the number of men, working eight hours per day, to be employed upon a continuous draining, to raise, by means of vertical chain pumps, in one hour, Q' cub. ft. of water to a height H, will be $0.00189Q'H$.

Inclined chain-pumps.

484. In the inclined chain pump, the tube is only a rectangular trough, and the paternosters are simple wooden plates. The descending branch of the chain rests either upon the upper part of the trough, if it is covered, or upon a platform placed above it, if it is not. Between its jaws and the sides of the plates we leave a space of only ⅛ to ¼ in.

In the *Architecture hydraulique* and in the *Traité des machines* will be found a detailed description of one of these machines.

Let ABHI be a section of a portion of the chain pump: we make the height of the plates $AB = h$, $BD = a$, the width of the plate $= b$, the angle of inclination $HFG = i$; deducting the space between the sides of the trough and the edges of the plates, the volume of water contained between two consecutive plates will be $\frac{1}{2}ab\ (2h - a \text{ tang. } i)$. Calling L the length of the trough, N the number of plates which pass upwards in a given time, observing that $L \sin. i$ is the height to which the water is raised, we shall have for the expression of useful effect produced in this time, Fig. 88.

$$\tfrac{1}{2}NabL \sin. i\ (2h - a \text{ tang. } i).$$

In the same chain pump, the effect will be proportional to $\sin. i\ (2h - a \text{ tang. } i)$; and that value of i which will render this quantity a *maximum*, will be that under which the chain pump will produce the greatest effect.

485. The inclined chain pump requires a greater motive power than the vertical chain pump, in proportion to the effect produced, by reason both of the friction of the plates, and of the great loss of water through the spaces; moreover, it is not used at present.

Perronet, however, had three at the draining for the bridge of Orleans. One was moved by a float-wheel, and raised 2401 cub. ft. 13.12 ft. high in one hour. Each of the two others was put in motion by a horse-gin, upon which twelve horses acted together: the plates were 0.66 ft. wide, 0.53 ft. high, and the same distance apart: the product was estimated at 4768 cub. ft. per hour, raised 16.40 ft. (*Perronet*, pp. 247 and 255.) This would only be 6488 cub. ft. raised one foot per hour by a horse.

From an observation made during the construction of the bridge "*de la Charité-sur-Loire*," an inclined chain pump, worked by six men, raised in one hour 723.9 cub. ft. of water to a height of 10.723 ft. This is, per man, 1289 cub. ft. raised one foot, which is but half the weight he could raise with a wheel.

ARTICLE FOURTH.

Persian or Cup Wheels.

486. Buckets or cups may also be fitted to the circumference of a float-wheel, with its lower part plunged in a current of water. They are open, and so arranged as, when at the foot of the revolution, to take up a certain quantity of water, which they deliver, when arrived at the top, in a trough or tank designed to receive it. There is no simpler or more economical method of raising water; the same current furnishes at once the force and the material needed. Thus, when the locality admits of it, this mode is frequently used, either for irrigations, or for different domestic purposes.

In great drainings, we construct separately a wheel with buckets and a float-wheel. The first consists of two circular plates, between which we suspend the cups or buckets by means of an axle passing through their upper part, and around which they can move. In this manner, they remain vertical, and retain the water which they have drawn, to the very summit of the wheel; then, by means of a quite simple contrivance, (examples of which may be found in *Architecture hydraulique* and in *Traité des machines,*) they incline, deliver their water, and then resume their original position. The float-wheel communicates motion to this wheel with buckets, either by a common shaft or otherwise.

487. Perronet also employed this machine, with great success, in the draining for the foundations of the bridge of Neuilly. The float-wheel was established in a place where the current had a velocity of 2.65 ft.; and the wheel with buckets was placed suc-

cessively on the site of different piers, even to a distance of 114.8 ft. The first wheel had a diameter of 19.18 ft., the width of its floats was 21.32 ft., and their height 3.18 ft. The second was 17.58 ft. in diameter; it bore sixteen buckets or boxes, containing 4.83 cub. ft., but arrived at the point of delivery with only 3.63 cub. ft. It raised, in one hour, 6532 cub. ft. from 10.66 to 12.79 ft.; its effect was equal to that of twelve vertical chain pumps, employed at the same bridge. (*Perronet*, pp. 66 and 114.)

488. We will include among wheels with buckets, a machine frequently used by the ancients, to which they gave the name of *tympan*. Tympan wheel.

It has the appearance of a drum, being formed of two circular plates, with a cylindrical envelope, which answers to the tube. It is divided, in the interior, into eight or a greater number of compartments, by partitions placed in the direction of the radii: the cylindrical surface is pierced with an opening for each of the compartments: the drum is traversed by a great axle, on the surface of which there are as many notches or grooves as there are compartments.

When this machine is suitably placed upon the water to be drawn, and is put in motion, each opening, passing beneath the level of the reservoir, there takes up a certain quantity of water, which enters in the compartment, and issues through the corresponding groove of the axle.

489. At the commencement of the last century, Lafaye curved the partitions conformably to the evolute of the circle of the nave, and abandoned the convex envelope. Fig. 89.

The celebrated engineer whose various observations upon draining machines we have already cited, has also made some upon this kind of tympan. That which he employed was 19.18 ft. in diameter, had twenty-four partitions, and raised the water 8.52 ft.: when plunged in the water a depth of 0.78 ft., twelve men, stepping upon a tread-wheel, fixed upon the same axle, caused it to make two and a half turns a minute, and raised 4343 cub. ft. in one hour. (*Perronet*, p. 252.)

The useful effect of each of these twelve men would be 3112 cub. ft. raised one foot in one hour; and we have seen that, upon a vertical chain pump, it was only 2016 cub. ft. But it was by means of the tread-wheel that the tympan was moved; the force of a man, when he acts upon such a wheel, depends upon his weight, and the effect produced is generally more considerable than that obtained by the use of the winch in the ratio of three to two. Notwithstanding this advantage, the tympan is seldom used: unless we give it extraordinary dimensions, it can raise the water but a small height; and even with its usual dimensions, it is bulky, hard to construct, and takes up too much room in the work-yard: in these respects, it is the reverse of the Archimedean screw.

APPENDIX.

As the method of reducing the formulæ from the metrical units to our unit of the foot, whether linear, superficial, or cubic, may not be generally understood, it is thought best to give an example of each, so that the reader, by referring to the original, may test for himself the accuracy of my reductions.

It is well known, that every algebraic expression, admitting of geometric construction, must have its terms of the same dimension: or, to quote from Young, "that each term must be either of one dimension, and thus represent a line; or, secondly, each must be of two dimensions, and so represent a surface; or, lastly, each must have three dimensions, and so denote a solid. It is plain, that if this uniformity of dimension does not belong to all the component terms of an algebraic expression, such an expression involves a geometric absurdity; for we can in no wise combine a line with a surface, or a surface with a solid. Nevertheless, it often happens that an expression, really admitting of construction, *does* appear under this unsuitable form; but such a result can arise only from the *linear* unit having been represented in the calculation by the numerical unit 1, thus causing every term into which it entered as a factor to appear of lower dimensions than the other terms. Whenever, therefore, for convenience of calculation, the linear unit is so represented, the result should be made *homogeneous*, by introducing it and its powers into the defective terms."

We have only to apply this principle to insure the correctness of the reduction. I would compare the process to the measurement of weights with scales. Calling the two members of the equations the weights, if they are of unlike dimensions, such operations are to be performed on them as shall render the literal factors homogeneous; so that if we have a line in one member, and the expression of a surface in the other, the coefficient of the second is to be divided by 3.2809, (or, for brevity, 3.28,) the value of a metre in feet, and the balance, so to speak, is restored.

Let us take the fundamental equation of the motion of water in canals. (Sec. 112.) It is expressed thus in metres:

$$p = 0.00036554\,\frac{c}{s}\,(v^2 + 0.06638v).$$

First, we ascertain the dimensions. p being a slope or ratio, we call it of the zero order, and the second member must be made the same; c, being the wetted perimeter, is of the first order or linear, v^2 of the second; the two multiplied give us a quantity of the third order. It is divided by s, which is an area, consequently, of the second order. The division $\frac{l^3}{l^2}$ gives us l, a linear quantity; therefore, to make it homogeneous with the first member, we divide its coefficient .00036554 by l, or 3.2809, which gives us .0001114155.

For the second term, we have $\frac{cv}{s}$, which is of the zero order: but inasmuch as its coefficient or multiplier has been divided by 3.2809, if we multiply .06638 by this quantity, we shall thus have the proper expression, in units of feet; or

$$p = \frac{.00036554}{3.2809}\,\frac{c}{s}\,(v^2 + 0.06638 \times 3.2809v) =$$

$$.0001114155\,\frac{c}{s}\,(v^2 + 0.21778v) =$$

$$.0001114155\,\frac{cv^2}{s} + .000024264\,\frac{cv}{s}.$$

Again, in Sec. 123, we have the expression, in metres,

$$p' = \frac{Q^2}{2g}\left(\frac{1}{s_n^2} - \frac{1}{s_0^2}\right) + \int\left(0.0003655c\,\frac{Q^2}{s^3} + .00002430\,\frac{Q}{s^2}\right)dz.$$

Here p', representing the fall of the surface from A to M, or the line EM (Fig.24), is linear; the first term of the second member is to stand as it is, for Q^2 is of the sixth order, or l^6, and $g \times s^2$ is of the fifth; it, then, is linear and homogeneous.

In the second term, we have for the first part $\frac{cQ^2dz}{s^3}$, or its dimension is $\left(\frac{l\times l^3\times l}{l^2\times s}\right)$ of the second order; therefore, we divide its coefficient by 3.2809, and have as before, .0001114155.

The dimension of the second part, or $\frac{cQdz}{s^2}$, is $\left(\frac{l\times l^3\times l}{l^2\times s^2}\right)$ linear, and it remains as it is. So we have the expression for feet given on page 131.

Another method, the reason of which may be more apparent to those unacquainted with these reductions, consists in supposing the literal expressions to be given in English feet, and then converting them to metres, in multiplying by .30479, or $\frac{1}{3.2809}$.

Thus (Sec. 186), suppose H, v, L and D to be given in feet, we have the metrical equation of this form, viz.:

$$H\times.30479-.051v^2\times.30479^2=.00137\frac{L\times.30479}{D\times.30479}(v^2\times.30479^2+0.055v\times.30479).$$

The expression is now good for metres. Dividing both members by .30479, the expression is good for feet, and will stand thus:

$$H-.051v^2\times.30479=.00137\frac{L}{D}\times.30479\left(v^2+\frac{.055}{.30479}v\right),$$

which, reduced, gives us the equation on page 206.

Sec. 109. — In the edition of Dubuat, (Paris, 1816,) vol. II., p. 87, we have, as Mr. James B. Francis, of Lowell, informs me, the expression $U=(\sqrt{V}-1)^2$, in which U is the velocity at the bottom, V the velocity at the surface, the unit being the French inch or "*pouce.*"

Sec. 113. — The coefficient 2736, under the radical, as D'Aubuisson has it, should be 2735.6; and in Sec. 114, instead of 51, it should be 52.3, unless the author intended to diminish the coefficient a little, to allow for — 0.033, omitted in his formula for practice.

Sec. 130. — D'Aubuisson has $H-h=\frac{130}{m^2}\cdot\frac{ps}{c}$. It is seen, in Sec. 107, that the resistance experienced by water moving in a canal is proportional to the wetted perimeter, to the square of the velocity, plus a fraction of the velocity, and is in the inverse ratio of its section; its expression is $a'\frac{c}{s}(v^2+bv)$.

Now, if, in great velocities, we are to disregard the fractional velocity, having the resistance proportional to the squares of the velocities, the expression for resistance then is $gp = a' \frac{c}{s} v^2$, or $p = \frac{a'}{g} \cdot \frac{c}{s} \cdot v^2$; calling $\frac{a'}{g} = .00036554 = a$, we have $v^2 = \frac{ps}{ca}$, and $v = \sqrt{\frac{ps}{.00036554c}} = 52.33 \sqrt{\frac{ps}{c}}$. If this is correct, the equation should be

$$H - h = \frac{v^2}{2gm^2} = \frac{.050974 \times 52.33^2}{m^2} \times \frac{ps}{c} = \frac{139.44}{m^2} \times \frac{ps}{c}\text{; or,}$$

$$H - h = 170.26 \frac{plh}{l+2h}, \text{ and } Q = 52.33lh \sqrt{\frac{plh}{l+2h}},$$

instead of the two coefficients 160 and 51 given by D'Aubuisson.

SEC. 163.—D'Aubuisson has $H = 0.1805 \frac{Q^2}{a^2}$. I make it $1305 \frac{Q^2}{a^2}$ for $Q = 0.625a \sqrt{2gH}$ gives $H = \frac{Q^2}{2g \times .625^2 a^2} = .13049 \frac{Q^2}{a^2}$ in metres $= .03977 \frac{Q^2}{a^2}$ in feet.

SEC. 173.—The author has made an important omission in not giving the value of L in the table. The second line in the first column in my edition has 452^{mm}. Mr. Francis says that in his edition (1840) it is 432^{mm}.

SEC. 213.—Page 250, fourteenth line of original, D'Aubuisson has for the lowering of the fluid in the piezometer, the expression

$$Q^2 \left(0.00222 \frac{L}{D^5} - \frac{0.0826}{D^4}\right) + \frac{0.000096L}{D^3}.$$

I think it should be

$$Q^2 \left(\frac{0.00222L}{D^5} + \frac{0.0826}{D^4}\right) + \frac{.000096LQ}{D^3},$$

and I have accordingly reduced it to the expression in feet given on page 299.

SEC. 218.—Page 257, fifth line, D'Aubuisson appears to have been in error in putting $1^{\text{m}}003$ for the *quantity*, when it is undoubtedly meant for the *velocity*. The diameter of the pipe being $.081^{\text{m}}$ and the velocity $1^{\text{m}}003$, we have for the quantity

$$Q = \frac{vD^2}{1.273} = 0^{\text{m}}00516.$$

Sec. 319. — We have $E < 250QH$, in metrical units;

$$\frac{E}{2.205 \times 3.28} < 250 \frac{Q}{3.28^3} \times \frac{H}{3.28}, \text{ in English measures;}$$

$$\text{or } E < \left(\frac{2.205}{3.28^3}\right) 250QH = 15.616QH.$$

Sec. 334. — We have this expression in metres:

$$\text{Force applied} = 6S \left(\sqrt{\frac{S}{s}} + 3\right) (\pm V \mp u)^2.$$

In this expression, the quantities are supposed to be given in metres and kilogrammes; we wish to use it for feet and pounds. The characters being in English measures, we have

$$\frac{\text{Force applied}}{2.205 \times 3.28} = 6 \frac{S}{3.28^2} \left(\sqrt{\frac{\frac{S}{3.28^2}}{\frac{s}{3.28^2}}} + 3\right) \pm \left(\frac{V}{3.28} \mp \frac{u}{3.28}\right)^2;$$

which, by reduction, becomes

$$\text{Force applied} = 6 \frac{2.205}{3.28^4} S \left(\sqrt{\frac{S}{s}} + 3\right) \pm (V \mp u)^2,$$

and is what we have given in Sec. 334.

Sec. 343. — In consulting with Mr. Francis as to what unit the author adopted in the formula $d = 0^{\text{m}}0013 \sqrt{\varpi}$, he favored me with a letter, from which I quote as follows: —

"As to the rule for '*tourillons*' at page 401 of D'Aubuisson. Morin gives the following formula, at page 350: $d^3 = \frac{Pc}{368156}$, P being the weight in kilogrammes, c the length in metres, usually equal to d the diameter; if so taken, the equation becomes $d^2 = \frac{P}{368156}$, then $d = \sqrt{\frac{P}{368156}} = .0016 \sqrt{P}$, which is not far from the rule given by D'Aubuisson; so you may be satisfied that he has used the metrical measures for the unit.

"In English measures, $\frac{d}{3.28} = .0016 \sqrt{\frac{P}{2.205}}$; or $d = .00364 \sqrt{P}$. That is, I get from Morin, *Aide Memoire*, page 350, $d = .00364 \sqrt{P}$;

you get from D'Aubuisson $d=.002872\sqrt{w}$, which is near enough, for such an uncertain matter.

"Water-wheel gudgeons here vary from six to ten inches in diameter.

"Suppose a wheel to weigh twenty tons on each gudgeon, or 40000 lbs.:

$$\begin{array}{rl} \sqrt{40000}\ \text{log.} = & 2.3010300 \\ .00364\ \text{log.} = & \bar{3}.5611683 \\ \hline .728\ \text{ft.}\quad & \bar{1}.8621983 \end{array}$$

say nine inches, which is about what we should make it."

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Fig. 5.

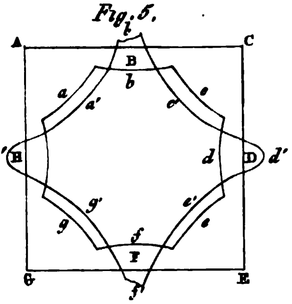


Fig. 6.

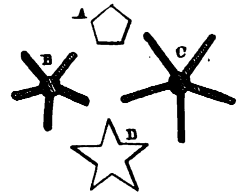


Fig. 7.

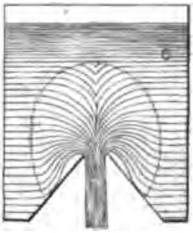


Fig. 12.

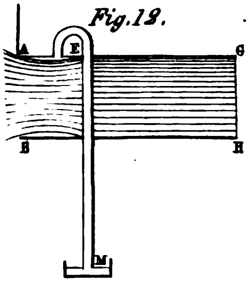
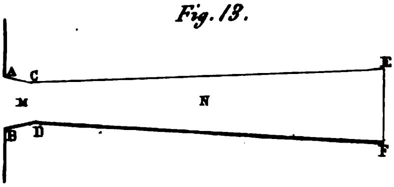


Fig. 13.



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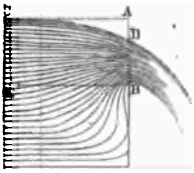


Fig. 18.

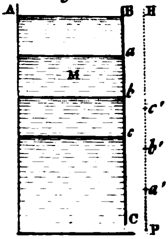


Fig. 19.

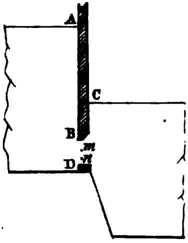
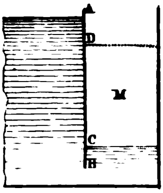


Fig. 20.



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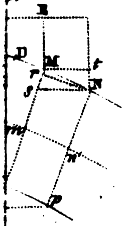


Fig. 25.

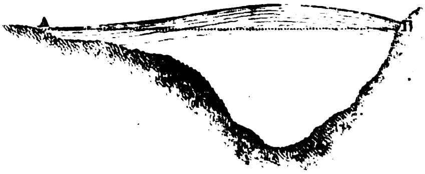
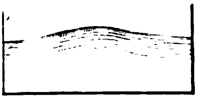


Fig. 26.



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